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OPTICAL INSTRUMENTS.

Introduction.

The construction of optical instruments has, in almost every instance, originated with eminent philosophers and mathematicians. Their gradual perfection has been a natural result of the difficulties which were presented to the progress of discovery, by the inefficient and inaccurate means which science possessed; and thus, the same great minds that have struck out and pursued vast and splendid ideas in their investigations of nature, have only been enabled to follow up their own conceptions by applying themselves to the practical improvement of the instruments with which they had commenced their discoveries. For instance, we are indebted to Newton for the construction of the first reflecting telescope that was ever made, although the idea had been previously suggested by Dr. Gregory. Leuwenhoek, one of the most assiduous naturalists of his day, carried on his curious researches in the animal and vegetable economy, with microscopes made by his own hands. The late Dr. Herschel, whose astronomical discoveries were the result of the profoundest mathematical knowledge, constructed the most powerful telescopes ever known, which, like the Gregorian, bear the name of their inventor. Indeed, the ordinary makers of optical instruments have been often men of considerable scientific attainments;—and it is from this union of a theoretic and practical knowledge, that these instruments, as is the case with almost every other important invention and improvement, have been conducted to their present very high state of perfection, in an almost unlimited adaptation to all the purposes of science, and all the wants and luxuries of common life.

In the limited extent of this treatise on Optical Instruments, the chief object will be to point out the principle of their construction, freed from *technical* mathematics, and to describe the improved state in which the more important are adapted to their several purposes. The various modes in which many instruments, not differing in their principle and end, are *mounted*, depend upon the varying taste of the artist, the caprice of fashion, or the demands of luxury. Any minute details of the *external* parts of the instruments would be therefore perfectly useless. Indeed, the external parts of the ordinary instruments can be much better understood by an inspection at the opticians' shops, than in volumes of description. So indifferent is the outward form of the common instruments, that makers living even in the same neighbourhood vary as much in their mountings as the artists of London or Paris. We shall, therefore, confine our observations to the *essential* parts in the construction of particular instruments, passing over their accidental varieties of mounting; —in the same way that the anatomist speaks of the various parts of the human body, without enquiring whether the *subject* upon which he has made his observations was dark or fair, tall or short.

The ancients seem to have been but little acquainted with dioptrical instruments, or those by which the light is refracted and transmitted; from their earliest history, however, they appear to have been conversant with the laws of the reflection of objects from the surface of water and polished metals, or that department of optical science called catoptrics. The first application of their knowledge of this branch of science, with which we are acquainted, is that of the burning mirrors employed by Archimedes, a philosopher of Syracuse, about 200 years before the Christian era, who, at the siege of that city, by Marcellus, the Roman Consul, employed them to destroy the besieging navy. The method by which this was probably accomplished is thus described by the historian Tzetzes:—"When the fleet of Marcellus was within bowshot, the old man Archimedes brought an hexagonal mirror, which he had previously prepared, at a proper distance from which he also placed other smaller mirrors of the same kind that moved in all directions on hinges, which when placed in the sun's rays directed them upon the Roman fleet, whereby it was reduced to ashes." We are also informed that Proclus in the same way destroyed the fleet of Vitellius at the siege of Byzantium.

CHAPTER I.—*Mirrors—Plane Looking-Glasses—Concave Mirrors—Burning Mirrors—Convex Mirrors.*

(1.) *Mirrors* are surfaces of polished metal, or glass silvered on its posterior side, capable of reflecting the rays of light from objects placed before them, and exhibiting to us their image. There are three classes of mirrors, distinguishable by the figure of their reflecting surface. These are *plane*, *concave*, and *convex*.—The reflection of light by either of these mirrors observes this constant *law*, that the angle which the incident ray makes with the reflecting surface, is equal to the angle of reflection. We may explain this law by a figure. Let *c d*, (*fig.* 1.) be a section of the reflecting surface: *o* an object before the mirror; and *O a* rays proceeding to the surface in the point *a*. The angle O *a c* which the rays make with the surface of the mirror is called the angle of *incidence;* and the direction in which an observer will see the object O in the mirror at *a*, must be I *a*, the angle of reflection I *a d* being equal to the angle of incidence *O a c*. If we suppose O I to be two persons viewing each other in a plane mirror or looking-glass *c d*, the direction in which each observeth the other at the point *a* is *a o* or *a i*, but the apparent place of their images will be behind the glass at the point *i* or *o*, the distance behind corresponding with that of their situation before the mirror. This deception proceeds from our common experience, which leads us to expect the object to be in the direction in which the rays come to our eyes, instead of in the real place of the object. The illusion is so complete, that domestic animals, when viewing themselves in a looking-glass for the first time, often have their passions strongly excited. When a person is viewing himself in a looking-glass, if he measure the size which he appears on the glass, the image will be one half his real magnitude, let his distance from the glass be in any manner varied. For, as it was stated above that the image appears behind the glass exactly at the same distance as the object is before it, it must be evident that as the mirror is half way between him and his apparent image, it will cut in half the cone of rays which proceed from his image to his eye. This is shown in *fig.* 2, where *c d* is a section of a plane mirror or looking-glass, and *a b* the face of a person viewing himself in it; now rays from every part of his face fall upon the reflector, from which they are sent to the eye, forming a cone of rays whose apex enters the pupil; consequently, as this cone is cut by the glass half way between the apex and its base, which base is himself, the measure of the line *e f* will be half of *a b*.

Fig. 1.

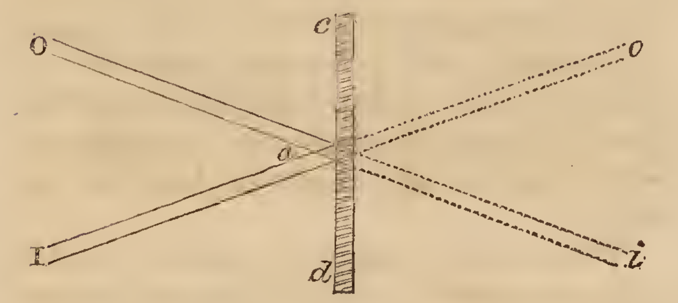
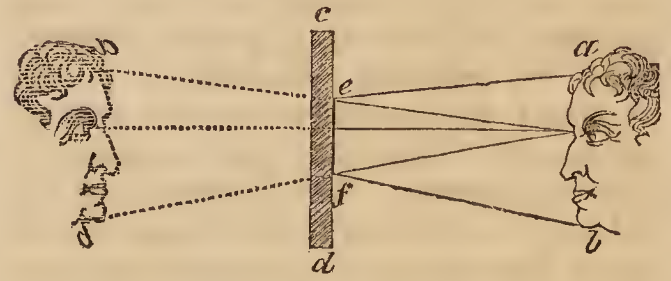
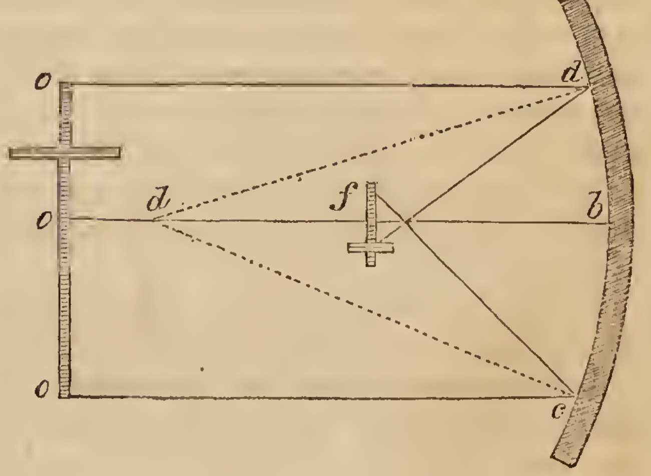


Fig. 2.



(2.) *Concave Mirrors* are those whose polished surfaces are spherically hollow. The properties of these mirrors may be easily understood, when we consider

Fig. 3.



their surface as composed of an indefinite number of small planes all of which make a determinate angle with each other, so as to throw all the rays to a point. (See *fig*. 3.) Let *a b c* be a concave mirror, and let *d* be the centre of curvature, and *o o o* rays of light from a distant object (the cross in the wood-cut must be supposed at a considerable distance) falling on the mirror at *a b c*, making different angles with its surface; these rays when reflected at equal angles to their incidence, as may be seen by the dotted lines which are perpendicular, will meet in a point *f*, called the focus of the mirror, where an image of the object will be formed in an inverted position. The distance of this focal point from the surface of the mirror, when the curvature is moderate, will be equal to half its radius *d b*. The importance of concave mirrors in the construction of reflecting telescopes, in which construction they are commonly called specula, will be shown hereafter under that head.

The employment of concave mirrors in collecting the heat of the sun's rays from the whole of its surface to a single point, thus accumulating a very great degree of heat, for the combustion and fusion of various natural substances that are infusible in the greatest heat capable of being produced from ordinary fire, may be exemplified, amongst those of modern date, by the burning mirror of M. de Villette. The diameter of this metal speculum was 3 feet 11 inches, and its focal distance or point *f* from the surface was 3 feet 2 inches. The composition of this metal was of tin and copper, which reflects the light very powerfully, and is capable of a high degree of polish. When exposed to the rays of the sun, by Drs. Harris and Desaguliers, a silver sixpence was melted in 7½ seconds when placed in its focus; a copper halfpenny melted in 16 seconds and liquefied in 34 seconds; tin was melted in 3 seconds; and a diamond, weighing 4 grains, lost ⅞ths of its weight*. The intensity of heat, obtained by burning mirrors or lenses, will always be as the area of the reflecting surface exposed to the sun is to the area of the small circle of light collected in its focus; thus, the diameter of the spot of light at the focus of *Villette's* mirror was 0.358 of an inch, and the diameter of the mirror 47 inches; hence the area of these circles was as 0.358^2 to 47^2, that is, the intensity of the sun's rays was increased 17257 times at the focal point. The loss of light occasioned in passing through the medium of which the lens is composed, together with that lost by reflection from the surface of mirrors, must, however, be deducted from this theoretical calculation. See *Photometers*.

Concave mirrors afford many curious and pleasing illustrations of their peculiar properties. For example:—when a person stands in front of a concave mirror, a little further from its surface than its focus (or half the radius of its concavity), he will observe his own image pendant in the air before him, and in an inverted position: this image will advance and recede with him; and, if he stretch out his hand, the image will do the like. Exhibitions have been brought before the public in which a singular deception was obtained by a large concave mirror. A man being placed with his head downwards, in its focus an *erect* image of him was exhibited, while his real person was concealed, and the place of the mirror darkened; the spectators were then directed to take a plate of fruit from his hand, which in an instant was dexterously changed for a dagger, or some other dangerous weapon.

(3.) *Convex mirrors* are chiefly employed as ornaments in apartments. The objects viewed in these are diminished, but seen in an erect position; the images appear to emanate from a point behind the mirror; this point, which is its focus, will be half the radius of convexity behind their surface, and is called the negative, or imaginary focus, because the rays are not actually collected as by a concave mirror, whose focus is called virtual.*

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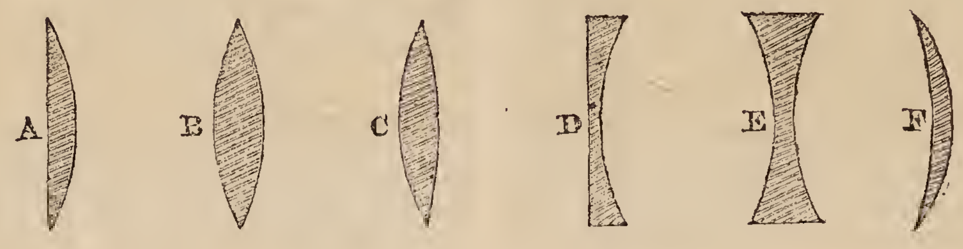
* The burning mirror constructed by Count Buffon, was a polyhedron, 6 feet broad and as many high, consisting of 168 small mirrors, or flat pieces of looking-glass, each 6 inches square. By means of this instrument, with the faint rays of the sun in the month of March, he set on fire boards of beech wood at 150 feet distance. This machine had the convenience of burning downwards, or horizontally at pleasure, each speculum being moveable, so as, by the means of three screws, to be set to a proper inclination for directing the rays towards any given point. It thus turned either in its greater focus or in any nearer interval, possessing this great superiority over common burning glasses whose foci are fixed and determined. Buffon at another time burnt wood at the distance of more than 120 feet, and silver was fused at 50 feet. See the article Burning Apparatus, in the Edinburgh Enclyclopædia.

* The reflecting surfaces of cylinders have been occasionally used in optical amusements, for rendering *anamorphoses* (distorted or deformed pictures) of their proper shape when reflected from its surface.

CHAPTER II.—*Lenses—Burning Lenses —Polyzonal Lenses.*

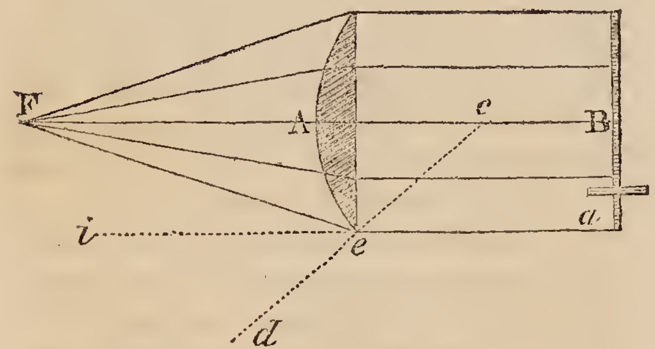
Lenses.—(4.) A lens is any transparent substance, as glass, crystal, water, or diamond, having one or both of its surfaces curved to collect or disperse the light transmitted by it. The lenses in general use are made of glass, and are usually called magnifying glasses. Glass, however, does not possess a greater share of the magnifying property than other transparent substances. In *fig.* 4, there are six differently shaped lenses, shown in section. A is called a plano-convex, from having one side flat, and the other spherically rounded. B is a double convex, and has both sides spherically rounded. When these sides are unequally curved, as at C, it is termed a crossed lens. D is a plano-concave, having one side spherically hollow. E is a double concave with both sides hollow. F is a Meniscus (so called from its moon shape), and has one side convex and the other concave.

Fig. 4.



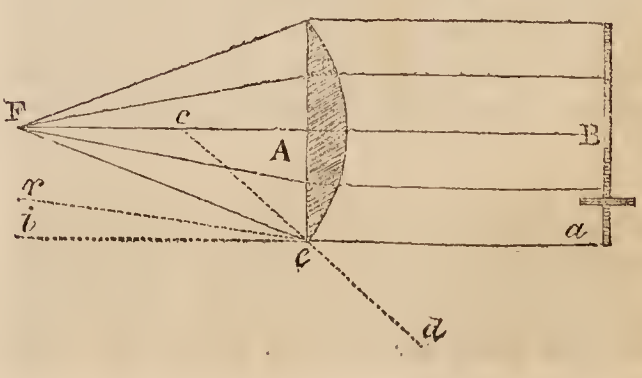
(5.) The passage of light, when transmitted by a plano-convex lens, is shown in *figs.* 5 and 6. Let A (*fig.* 5.) represent a section of the lens, and B an object at an infinite distance, as a star (in the figure a cross is placed to assist the conception). Now, the lines from B to A will represent the rays proceeding from every part of the object B, to every portion of the surface of the lens A, and from the distance of the object they will be parallel to each other. Only five rays are drawn in the figure, to prevent confusion; but it should be constantly remembered that the light strikes every part of the lens. The first surface of the lens next the object B is flat; and as all the rays fall perpendicularly on it, and the attraction on each side of the rays is equal, they will pass on in their right-lined direction, till they meet the curved surface of the lens, when all the rays will be bent or refracted, so as to meet in a point F, called the focus of the lens; but the central ray being perpendicular to the curved surface is not bent. This constant law of refraction, or bending, is always observed by rays of light in their passage from one medium to another of different density, whose surface is oblique to their direction; and the rays of light in passing from a dense medium, as glass, &c. into a less refractive medium, as air, will be bent so as to form a greater angle with a perpendiular to that surface, than it had at first. Let the dotted line *c d* be a perpendicular to the surface of the lens at the point *e*. Now, the angle *a e c*, called the angle of incidence, is less than the angle F *e d*, called the angle of refraction, as is shown by continuing the ray *a* to *i*.

Fig. 5.



(6.) But whenever light strikes an oblique surface in its passage from a rare medium, as air, to a denser, as glass, it will be refracted nearer the perpendicular, as shown by *fig.* 6, in which the lens is reversed. The ray of light *a e* is there bent nearer the perpendicular *c d*, as shown by the dotted line *e r*, and when this ray is transmitted by the plane side, it will be refracted to F,

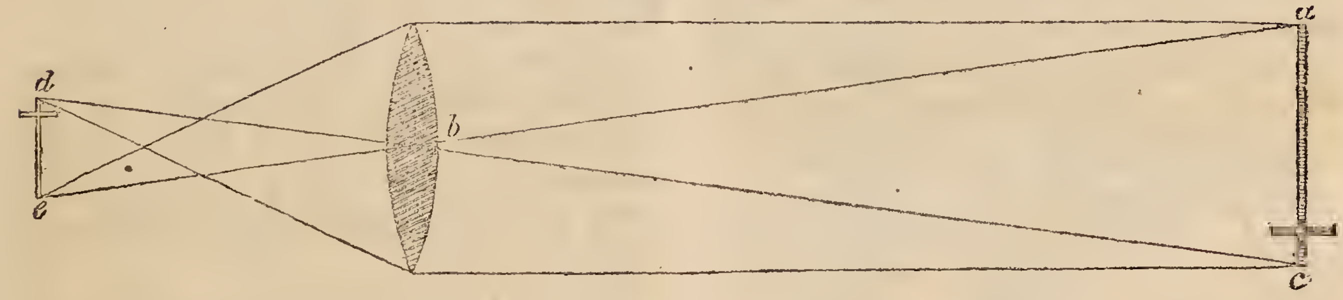
Fig. 6.



making a greater angle with the perpendicular to the flat side, as it now enters a rarer medium.

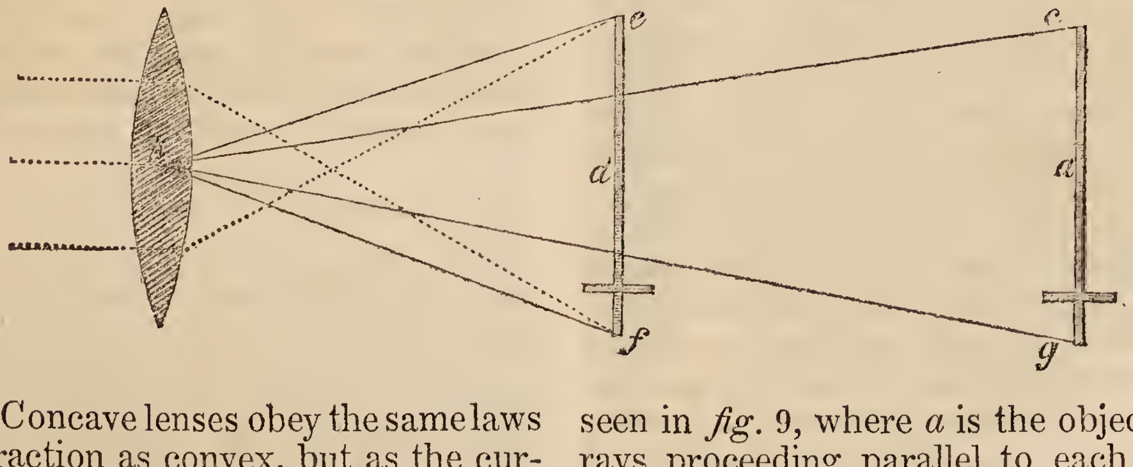
(7.) If, instead of supposing the object at an infinite distance, and consequently a point, we imagine it removed to some finite distance, a like action in a proportional degree will be observed, only the object will not be reduced to a point, but its image will be formed at F, whose size is equal to the angle under which the object would be seen without the lens. Let *a c* (*fig.* 7.) be the object, and *b* the lens: now, this object will subtend the angle *a b c* from every portion of the lens, and an inverted image *d e* will be formed under the equal angle *d b e*; for whenever right lines intersect each other, the opposite angles are always equal. This experiment may be proved by a common convex glass lens. Suppose the distance of the focal point is six inches, and the lens two inches in diameter, the image of a distant object may be seen on a wall when the lens is held a little more than its focal length from it; then let the size of the image be measured: now remove the lens, and measure the apparent size of the object, while the eye of the observer is in its place, taking the distance of the focus of the lens for the point of measurement, and it will be found of the same size in both cases. If a wafer is made to adhere to the surface of the lens, so as to stop a portion of the light, the size is not altered, but the image will be formed less bright.

Fig. 7.



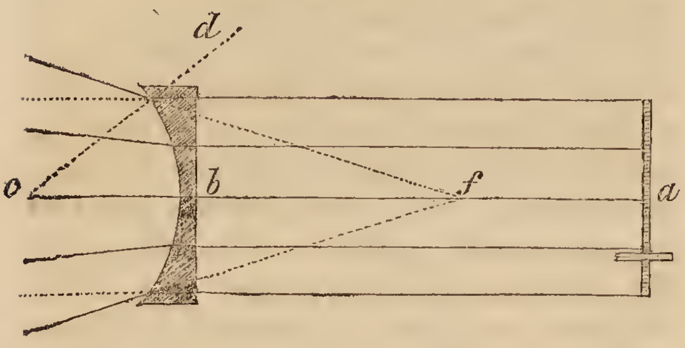
(8.) When an object is placed in the focus of a lens, the rays diverging from it will, by the action of the lens, be rendered parallel; this case, however, is only the reverse of the former, the place of the object and image being changed. But this astonishing circumstance will take place: when observed on the side for parallel rays, the objects will appear magnified or increased in size, should the distance of the object from the lens be less than the eye can see it without. Let *a b* (*fig.* 8.) be the nearest distance at which an object can be seen distinctly without the assistance of a lens, and *b d*, the distance of the object when seen through the lens, equal to half the distance *a b*; now the angle *c b g*, which the object subtended without the lens, is only half the angle *e b f*; and, therefore, the object will be magnified twice, when seen by the lens; and if the distance had been only $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{10}$, the object would have been magnified 3, 4, or 10 times in length and breadth, and, consequently, its surface increased 9, 16, or 100 times.

Fig. 8.



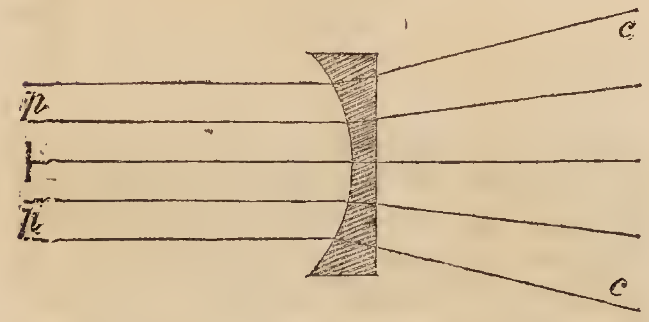
(9.) Concave lenses obey the same laws of refraction as convex, but as the curvature is reversed, the rays are bent outwards; hence a concave lens will render parallel rays diverging, as may be seen in *fig.* 9, where *a* is the object, *a b* rays proceeding parallel to each other from the object. These rays, when transmitted by the concave lens, are made to diverge, as if they came from the point f, the imaginary focus of the lens; the perpendicular cd shews that the rays obey the same law of refraction as in a convex lens. It should be here observed, that when the lens is concave, the focal point will be on the same side as the object, and it is termed negative, as objects are diminished by concave lenses.

Fig. 9.



When converging rays from a convex lens are transmitted by a concave, they are rendered parallel, as shewn in *fig.* 10, where the converging rays at c (from a convex lens) are brought parallel at p, after passing through the lens.

Fig. 10.



(10.) The manner in which the foci of lenses of different curves are calculated, and how the foci of combined lenses may be obtained, are as follows. When the lenses are made of plate glass, the focal distance is nearly the diameter of the sphere from which we may suppose a plano-convex lens to be cut, or it is equal to twice the radius of the circle that forms the convex surface of the lens. For example, if the globe of glass is one inch in diameter, and a portion is cut off to form a plano-convex lens, the focus will be one inch, or twice the radius of the circle. If the lens is double convex, the focus will be equal to the radius, or half the diameter. When the lens is crossed or unequally convex, the focal length will be twice the product of the two radii, divided by the sum of the radii. For example, let the radius on one side be 2 inches, and on the other side 6 inches; the focus of this will be $2 \times 2 \times 6 = 24$, divided by $2 + 6 = 8$ or 3 inches. The focus of the miniscus lens is found by dividing twice the product of the two radii by their difference. Example; let the radius on the convex side be 2 inches, and on the concave side 4, the focus is $2 \times 2 \times 4 = 16$ divided by $4 - 2 = 2$, or 8 inches, the focus of the lens*.

If two lenses are placed in contact, the compound focus, when each lens has the same power, will be half the focus of the single lens. When two convex lenses are in contact, having different focal lengths, then, as the sum of the two foci is to one of them, so is the other to the compound focus required. For example, let the foci of the lenses be 2 and 6; then, as $2 + 6 = 8 : 2 \therefore 6 : 1\frac{1}{2}$, the compound focus. Lastly, if two lenses are not in contact, the compound focus is found by dividing the product of the two lenses by the sum lessened by their distance. Example: let the foci of the lenses be 2 and 4, their distance 2; then $2 \times 4 = 8$ divided by $(\overline{2 + 4}) - 2 = 4$ gives 2 as the compound focus.

(11.) If lenses be made of different substances, although the curves may be the same, the focal lengths will vary; while, in like mediums, the action will always be equal. Let ab (*fig.* 11.) be a ray of light, and let it enter the medium $c\,d$ at the point b; instead of continuing in a right line to e it will pass on in the direction $b\,f$, should the medium cd be denser than the first ab; now if on the point b a circle be drawn, and a line $s\,i$, parallel to the surface of the medium, touching the incident ray ab be produced to e, this line will be the sine of incidence; and if another line $p\,r$ be drawn in the same manner to the refracted ray, it will be the sine of refraction. Now if the angle abc be varied to any degree, the sine $s\,i$ will always be in the same proportion to the sine of refraction, $p\,r$. If the dense medium is water, the sine $p\,r$ will be $\frac{3}{4}$ of $s\,i$. When glass is used, the sines are as 2 to 3 nearly, and in diamond as 2 to 5.

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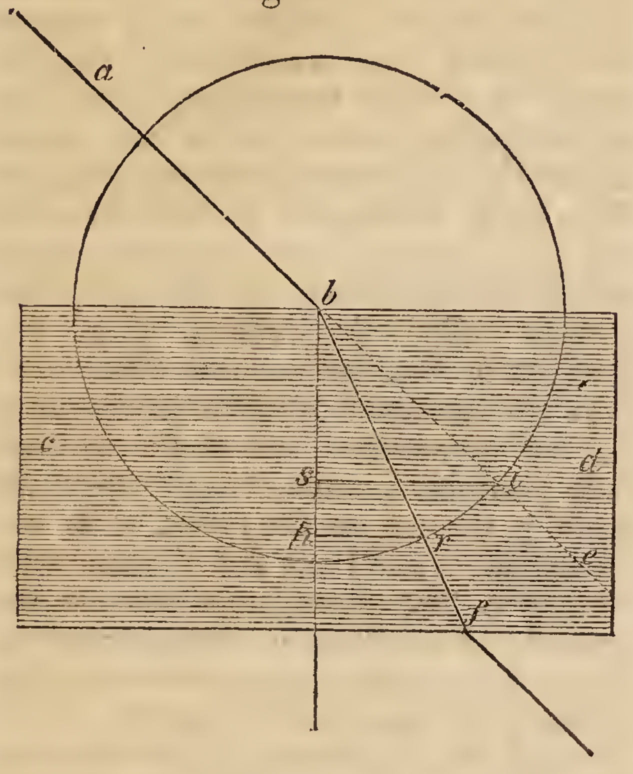
* In many cases, it is found advisable to ascertain the radii of the two surfaces of a convex lens, as well as its focus, by a more accurate manner. This may be effected by forming a reflected image by the posterior surface, which distance will be half of the radius of curvature (or one quarter the focus of a plano-convex lens); then, by exposing the other side, we obtain the radii of the opposite surface. This method was adopted by Professor Robinson, to measure the different radii of double and triple achromatic object-glasses.

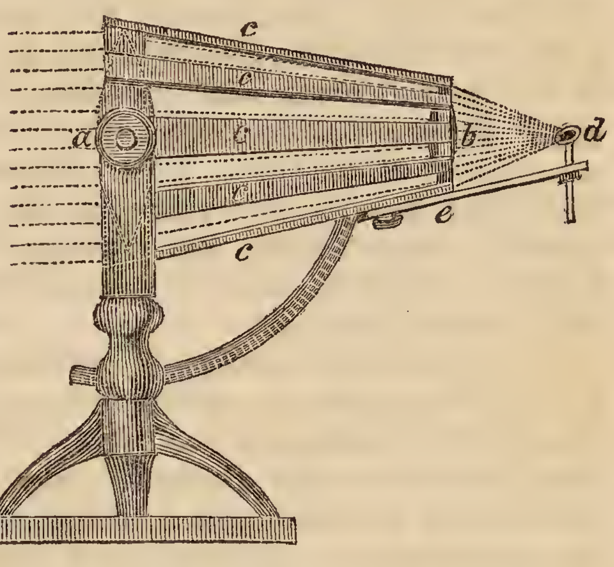
Fig. 11.



When a ray is passing out of a dense medium into a rarer, the direction will be changed, and the ray *bf* will now be bent further from the perpendicular, so as to make the sines the reverse of the former case. Out of water they will be as 4 to 3: from glass as 3 to 2, and from diamond as 5 to 2. The theorems just described for finding the foci of lenses are called geometrical, and will be nearly the same as the refracted, when the lens is made of plate glass. The refracted focus is only $\frac{1}{241}$st part less than the geometrical, when ascertained by accurate experiment. The refracted focus of lenses of other media may be obtained by dividing the geometrical focus by the quotient obtained when the sine of incidence (*i*), minus the sine of refraction (*r*), is divided by half the sine of refraction. $\left(\frac{i - r.}{\frac{1}{2}\,r}\right)$

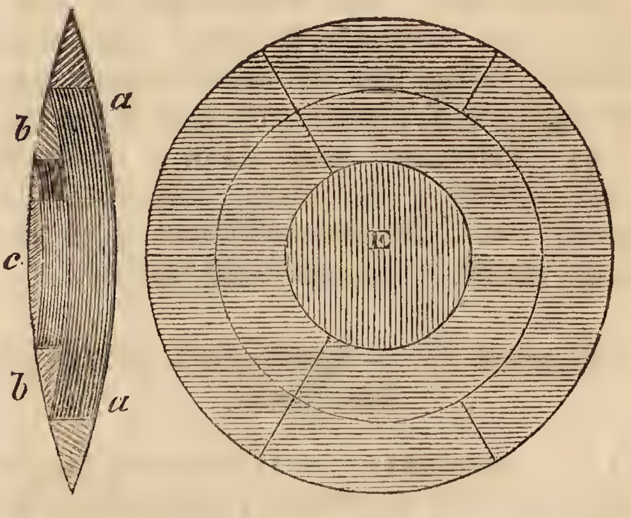
(12.) *Convex Lenses*, in their simple state, have been applied to collect the heat of the sun's rays for purposes similar to that of burning mirrors. One of the largest lenses that have been mounted for these purposes, was that made of flint glass by Mr. Parker. This lens was 3 feet in diameter, and when mounted, exposed a surface of 330 square inches to the sun's light; its focal distance was 3 feet 9 inches, and the diameter of the circular spot of light was one inch. But in order that the light might be condensed as much as possible, he employed another lens, 13 inches diameter, and of 29 inches focus, so as to decrease the diameter of the focal point to 3-8ths of an inch. The apparatus on which it was mounted is shown in *fig.* 12: *a* is the large convex lens mounted in a ring, and connected to the smaller lens *b* by wooden ribs *c c*; the lower rib has a piece *e* attached to it, capable of adjustment to or from the smaller lens: to this bar is fixed the holder *d*, having an universal joint. On this holder, the substance to be experimented on is placed. The following are some of its effects on bodies placed in its focus; 20 grains of pure gold was fused in 4 seconds; 10 grains of platina fused in 3 seconds, and a diamond, weighing 10 grains, exposed for 30 minutes, lost 4 grains. This lens, which is now in the posession of the Emperor of China, cost 700*l*.

Fig. 12.



In large burning lenses the weight of the glass employed becomes of considerable importance; and to effect as great a saving as possible, Count Buffon has proposed to construct them of circular rings, as shown in *fig.* 13, where the lens

Fig. 13.



is composed of three pieces, two rings, *a* and *b*, and a lens *c*. When, however, the size is very great, the rings may be composed of several pieces, as shown by the front view E, where the lens is built of ten pieces. These instruments have been denominated by Dr. Brewster, who suggested this division, *Polyzonal lenses.*

The following advantages of these lenses have been laid down by Dr. Brewster.

1. The difficulty of procuring a mass of flint-glass proper for a solid lens of great dimensions, is in this construction completely removed.

2. If impurities exist in the glass of any of the spherical segments, or if an accident happen to any of them, it can be easily replaced at a very trifling expense. Hence, the spherical segments may be made of glass much more pure and free from flaws and veins than the corresponding portions of a solid lens.

3. From the spherical aberration of a convex lens, the focus of the outer portion is nearer the lens than the focus of the central parts, and, therefore, the solar light is not concentrated in the same point of the axis. This evil may, in a great measure, be removed in the present construction, by placing the different zones in such a manner that their foci may coincide.

4. A lens of this construction may be formed by degrees, according to the convenience and means of the artist. One zone, or even one segment may be added after another, and at every step the instrument may be used as if it were complete, without the rest of the zone to which it belongs; and it will contribute, in the proportion of its area, to increase the general effect.

5. If it should be thought advisable to grind the segments separately, or two by two, a much smaller tool will be necessary than if they formed one continuous lens. But, if it should be reckoned more accurate to grind each zone by itself, then the various segments may be easily held together by a firm cement.

6. Each zone may have a different focal length, and may, therefore, be placed at different distances from the focal point, if it is thought proper.

Chapter III.—*Spectacles—Periscopic Spectacles.*

(13.) *Spectacles.** — When two lenses are mounted in a frame to fix before the eyes, they are denominated spectacles: the lenses are employed to render the objects before the wearer more distinct. The eye, which consists of a convex lens, called the *crystalline lens*, refracts the light proceeding from the object placed before it in the same manner as a convex glass: the image of the object is formed at the focus of the lens, where it is received on a screen at the back of the eye; this screen, called the *retina*, is an expansion of the optic nerve, which conveys the sensation of vision to the mind. As the crystalline lens of the eye will only produce distinct vision when the focus is thrown on the retina, it is obvious that should any defect occur with respect to that organ, indistinct and imperfect vision will arise. Thus, if the lens of the eye is not of a proper convexity to bring the image on the screen, an indistinctness must ensue. This is the case when the lens through age has become flattened; the image will then be thrown beyond the retina, and thus convey an imperfect representation of the object to the mind. To obviate this defect, we must make the rays pass through a glass of sufficient convexity to assist the eye, and enable it to form the image at the required place, which is in this instance done by shortening the focal distance of the crystalline lens of the eye. If, on the contrary, the eye should be too convex, or short-sighted, as is often the case with young persons, then the image will not be formed at a sufficient distance from the lens of the eye to reach the retina, and thus imperfect vision of distant objects is produced. To remedy this defect *concave lenses* must be resorted to, in order to diverge the rays before they enter the eye, and thus lengthen the focus of the crystalline lens to form an image on the retina. When the eyes are not directed near the centre of the spectacle-glasses, the obliquity of their surface to the rays will be increased, so as to occasion a confused appearance of the object. A great portion of this confusion is removed in the spectacles now usually made, when compared with those formerly employed, whose size, being very large, augmented the imperfection; for it may be observed that when objects are seen through spectacle-glasses, no more of the glass is employed at one view than a portion equal to the size of the pupil of the eye; this on an average may be reckoned at the *eighth of an*

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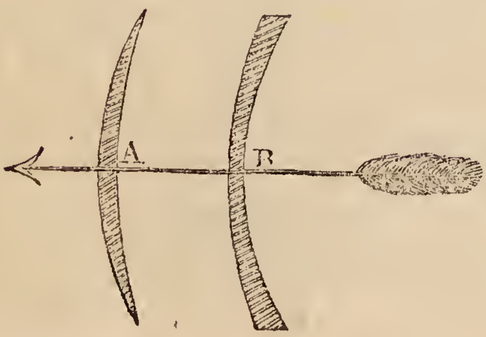
* These instruments are said to have been invented about the year 1290.

inch in diameter. Thus, we see how small a portion is used for the purposes of vision; but as it would be tedious to require the eye always to look through a small aperture, the glasses are left of a sufficient size to admit of a moderate degree of motion; and, as we require a greater latitude horizontally than vertically, their figure is made of an oval form.

In the selection of spectacle-glasses great care should be used in examining them, and the first point of importance is the goodness of the material of which they are formed; this should be free from all veins or small bubbles, for if one of these occur in the portion through which we look, it will greatly impair the eyes. The next circumstance is the colour of the glasses; the best adapted for general purposes is a pale blue. The figure of their surfaces should be perfectly spherical, for if they are curved more in one direction than in another, they will injure the sight, unless they are cylindrically formed, as for some particular disease. The polish should be clean, and free from flare, which too often arises from the manner in which they are usually polished on heterogeneous surfaces, producing what is technically termed a *curdled glass*. See the method of grinding and polishing lenses, described in (48).

(14.) Dr. Wollaston, in order to allow the eyes a considerable latitude without fatigue, invented a peculiar form of glasses, called by him *periscopic*, from two Greek words signifying *seeing about;* their form is that of a meniscus with the concave side always turned towards the eye. When they are intended for *long-sighted persons*, or *old age*, the anterior surface, or that next the object, is formed spherically convex, with a curve deeper than the concave, so as both to gain the required power, and compensate for the divergency occasioned by the concave side; this form is shown at A, (*fig.* 14.). The *periscopic* form employed for correcting the defect of a short or near sight is shown in section at B, having its anterior surface convex, as in the former case; but here the concavity on its posterior side is increased to procure the required divergency, and compensate for the convex side.

Fig. 14.



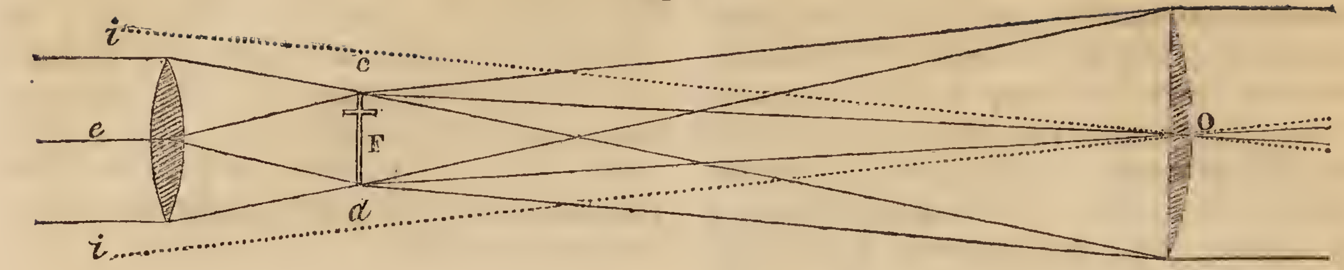
Chapter IV. — *Telescopes — Common Astronomical Telescope — Day Telescope—Dynameters.*

(15.) *A Telescope* is an optical instrument employed for viewing distant objects, by increasing the apparent angle under which they are seen without its assistance; and hence the effect on the mind of an increase in size, or, as commonly termed, *magnified* representation. The construction of the Telescope is, perhaps, one of the most important acquisitions that the sciences ever attained, as it unfolds to our view the wonders of the heavens, and enables us to obtain *data* for astronomical and nautical purposes.

The invention of this instrument is somewhat uncertain, and is ascribed to different individuals, as John Baptista Porta, Jansen of Middleburg, and Galileo. The time of its first construction was about the year 1590.

The simplest construction of this instrument consists of two convex lenses, so combined as to increase the apparent angle under which distant objects are seen. If we take a convex lens, and place it in a similar position to the object, as that in *fig.* 7, and another of shorter focus in the position *fig.* 8, with a distance between them equal to the sum of their foci, a telescope will be formed, and the magnifying power will be in proportion to the focus of the two lenses. Let O (*fig.* 15) be the object lens, and suppose it 8 inches focus, and *e* the eye lens, of 2 inches focus, the distance between these two lenses must be ten inches, if the object be at an infinite distance, as a star; but when the object is terrestrial, the distance between the two lenses must be increased to adjust for distinct vision: on this account the eye lens is mounted in a tube, sliding within another tube in which the object-glass is fixed, and, therefore, can be drawn out for near objects. As the size of objects is dependent on the angle under which they are seen, the image F, formed by the object-glass, in the focus of the eye-glass *e*, will subtend

Fig. 15.



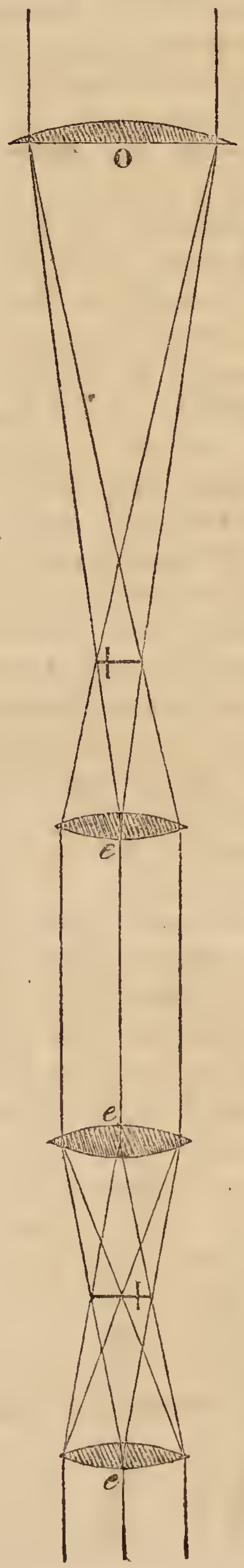
the angle *c e d*, which is four times the angle *c o d* that the object subtends, for the distance *F o* is four times *F e*: hence the magnifying power may be found, by dividing the focal length of the object-glass by the focus of the eye-glass, when the quotient will be the power. Objects seen through this telescope are inverted, and on that account it is inapplicable to land observation; but at sea it is occasionally used at night, and in hazy weather when there is little light; it is hence called a night telescope.

(16.) The common *astronomical telescope* is of the same principle of construction as the preceding. The inversion of the object is immaterial in its application to celestial observations; but the disadvantage of this instrument is felt when very high powers are required, for then the objects are rendered dark and obscure, and if the aperture of the object-glass is increased to admit more light, the formation of the object is confused. M. Huygens, however, made a telescope of this construction, in which he was enabled to use an aperture of 6 inches, by making the focus of the object-glass 123 feet in length, and, by changing the eye-lenses, any required power was produced. From experiments on different combinations, he found that to obtain the greatest distinctness and light, the focus of the object-glass, its aperture, and the power of the instrument, should be according to the following table:

Focus of the Object Glass.	Aperture of the Object Glass.	Focus of the Eye Glass.	Magnifying Power.
Feet.	Inches.	Inches.	
1	0.545	0.605	20
2	0.76	0.84	27.6
3	0.94	1.04	33.5
4	1.08	1.18	39.5
5	1.21	1.33	44
10	1.71	1.88	62
20	2.43	2.68	88
30	3.00	3.28	108
40	3.43	3.76	125
50	3.84	4.20	140
100	5.40	5.95	197
120	5.9	6.52	216

(17.) The common *day-telescope* is an instrument of this class, with the addition of two other lenses of the same power as the eye-lens *e*; these lenses will produce an erect image of the object when placed at a fixed distance from each other, equal to the sum of their two focal lengths. Let *o* (*fig.* 16.) be

Fig. 16.



the object-lens, which may be the same focus as that in *fig.* 16, *e e e* three lenses of equal power. Now, if the focus of each eye-lens *e* is two inches, as in the former case, then each eye-glass must be placed at a fixed distance of 4 inches from each other; and the distance between the object-lens *o*, and the nearest eye-lens, will be 10 inches, this distance increasing as the objects to be viewed approach the instrument. The power of day-telescopes may be calculated in the same manner as the astronomical; for the two additional lenses produce no effect in the amplification of the objects.

(18.) The magnifying power of telescopes may be ascertained without a knowledge of the foci of the glasses, by means of a *dynameter;* this apparatus simply consists of a strip of mother-of-pearl, marked with equal divisions, from the $\frac{1}{100}$ to $\frac{1}{1000}$ of an inch apart, according to the accuracy required; this measure is attached to a magnifying lens in its focus, in order to make the small divisions more apparent. When the power of a telescope is required, the person must measure the clear aperture of the object-glass; then holding the pearl *dynameter* next the eye-glass, let him observe how many divisions the small circle of light occupies when the instrument is directed to a bright object. Then by dividing the diameter of the object-glass by the diameter of this circle of light, the power will be obtained.

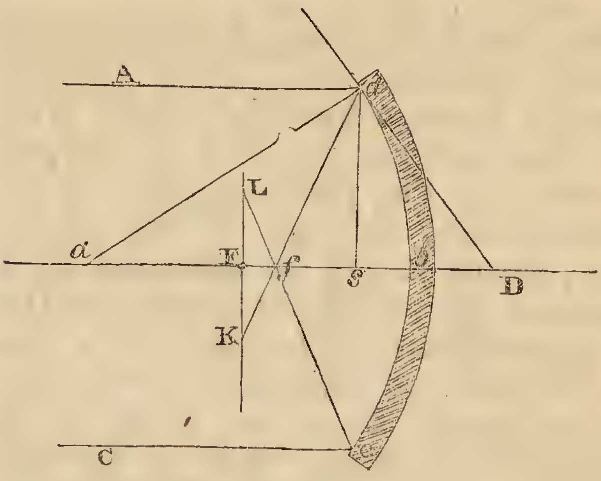
CHAPTER V.—*Aberration of Reflectors and of Lenses—Glass and Diamond compared—Huygens' Eye Piece—Ramsden's Eye Piece—Newton's Parabolic Lenses—Chromatic Dispersion.*

(19.) The field of vision, or number of objects seen by the telescopes, *figs.* 15 and 16, is very limited, the eye-lenses not being sufficiently large, as is shown by the dotted lines *i i* in *fig.* 15, which do not enter the eye lens *e*, and are not received by the eye. Now, if the diameters of these lenses were increased, the objects would be rendered indistinct, arising from the rays, spread over the surface of the lens from any point in the object, not being collected again in another point after refraction. This error is occasioned by the figure of the lens, and is called the spherical aberration by figure.

As a lens is formed with two surfaces, and, consequently, has two refractions, we shall first investigate the aberration of a spherical reflecting surface.

In (2) the focus of a concave spherical reflector was stated to be half the radius distant from its surface; this, however, is only the case with parallel rays near the centre. When we are desirous of employing specula for telescopes, they require to be made of the parabolic or hyperbolic form, to unite all the rays to one point: the rays that fall on the extreme parts of a spherical reflector, forming an image nearer the speculum than those that fall on its centre. In *fig.* 17, F is the focus of central rays, and the point *f* the focus of the extreme rays A C, while, along the axis from *f* to F, images from the different parts of the reflector will be formed of the same object; these, not coinciding, will confuse one another. The quantity F *f* is called the longitudinal aberration, and will be equal to half the aperture of the speculum squared $(a\,b)^2$, divided by 4 times the radius of curvature $(d\,b)$, or $\frac{a\,b^{2}}{4\,b\,d}$ nearly, in specula whose surface is spherical.*

Fig. 17.



This spherical aberration produces an indistinctness of vision, by spreading out every mathematical point of the object into a small spot in its picture; which spots, by mixing with each other, confuse the whole. The diameter of this circle of confusion, at the focus of central rays F, over which every point is spread, will be L K (*fig.* 17.); and when the aperture of the reflector is moderate it equals the cube of the aperture, divided by the square of the radius

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* The focus of rays reflected by any curve will be equal to half the distance of the tangent from the centre or half *d* D for A *a*.

$\left(\frac{a\ c_3}{b\ d_2}\right)$: this circle is called the aberration of latitude.

(20.) The aberration produced by a lens with a spherical surface is shown in *fig*. 18, where *a* B *c* is a section of a plano-convex lens. Let the plane side be exposed to parallel rays, and let *a* A be an extreme pencil of rays; D the centre of curvature; D *f* the axis of the lens; and F the focus of a slender pencil of incident rays, at an infinitely smaller distance from the centre. Now, as the extreme ray *a* A is perpendicular to the plane surface, it will pass directly through to the convex side, where it will be refracted to *f*, crossing the axis in that point, for D A is perpendicular to the curve at *a*, and *a* D the sine of incidence, *n* D the sine of refraction; hence, an image of the object will be formed at F by the central rays, and another image of the same object will be formed at *f* by the extreme rays; while, from F to *f*, images of the same object will be formed by the intermediate portion of the lens. The longitudinal aberration F *f* bears a certain ratio to the thickness or versed sine B P; and when the lens is placed in the position shown in the figure, it is equal to $\frac{9}{2}$ or $4\frac{1}{2}$ times B P; this quantity will be decreased, when the curved surface of the lens is exposed to parallel rays, that is, when the refraction of the first surface is made nearer the perpendicular, or when the ray is bent in passing from a rare into a dense medium, and this difference out of air into glass, will be in the proportion of 27 to 7; so that when the convex side is placed next the radiant, the longitudinal aberration will be only $\frac{7}{6}$ of the thickness B P, or 1.166.

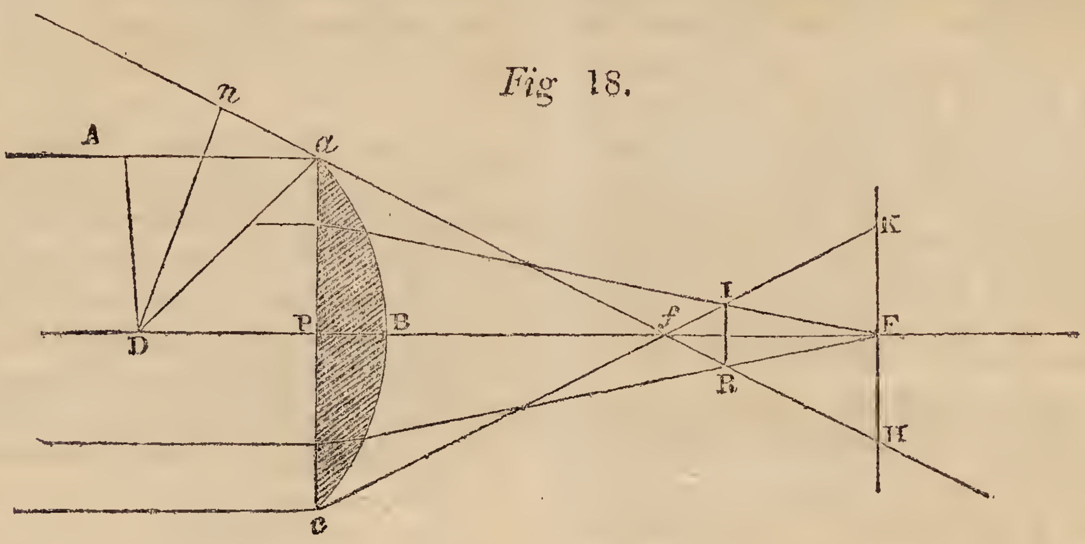


Fig 18.

When a crossed convex lens is used, the proportions of the radii of whose surfaces are as 1 to 6, and the most convex side is exposed to the distant radiant, the longitudinal aberration will be the least possible quantity; viz. $\frac{15}{14}$. or 1.0714 of the thickness of the lens, When the radii of a double-convex lens are equal, the aberration is $\frac{5}{3}$ of its thickness; therefore, this lens is not so good as a plano-convex of the same thickness, in its best position. *The longitudinal aberration* F *f increases as the square of the aperture, when the curvature of the lens is not altered;* and is *inversely as the focal distance, when the aperture is constant.*

The lateral aberration, which is the actual confusion of the image at the focus of central rays, is equal to the longitudinal aberration F *f*, multiplied by $\frac{\text{A C}}{\text{B F}}$, or the aperture of the lens divided by the focal distance, which is equal to K H. Now, if rays are drawn from the different parts of the lens, it will be found that they will be refracted through a small circular space I R, whose diameter will be $\frac{1}{4}$ of K H; hence, this point must be considered as the focus of the lens. *The lateral aberration of lenses increases as the cube of the aperture, if the radius remain the same, or inversely as the square of the radius when the apertures are the same.*

These laws may be considered as determining the relative aberration of all lenses; yet it is found that if we employ media of different refractive powers, and form each into lenses of like curvature, the separation or spreading out of the rays at their focal point will be different; that possessing the highest refractive power producing the least aberration, though its amplifying power will be greatest: thus, if three lenses were ground in the same tool, one of *plate glass*, and the others of *sapphire* and *diamond*, they would possess very different *magnifying powers*, *aberrations*, and *separation of colour*, or *chromatic dispersion*, (this latter error is explained in 23 :) their respective values are shown in the following Table :

Plano convex lenses with convex side exposed to parallel rays.	Magnifying powers.	Longitudinal aberration.	Chromatic dispersion.
Glass	150	1.167	48
Sapphire	250	1.005	26
Diamond	400	0.950	38*

But this difference in the longitudinal aberration would be much greater if the lenses were so formed as to give the same magnifying powers ; for this error always decreases as the squares of their respective radii, while the lateral aberration or area of the circle of confusion will be as their *cubes*.

Hence, in *sapphire* and *diamond lenses* of high magnifying powers, the indistinctness arising from their figure would barely be discernible in practice, thus producing a kind of *natural aplanatic magnifier*.

The valuable properties possessed by these stones were known to Sir Isaac Newton, and Martin, and have been more particularly pointed out by Dr. Brewster, in his Treatise on *New Optical Instruments*. But their *hardness* and *crystalline form* probably occasioning difficulties in the formation of spherical polished surfaces almost insurmountable, has retarded their adoption as lenses : however, we have lately learned that *Mr. Pritchard* has succeeded in forming these substances into lenses, and their application to the microscope is so favourable, that, if any new discoveries are to be made in the *minutiæ of nature*, they seem most likely to develope them.

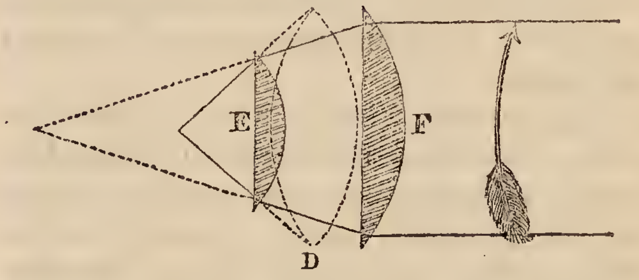
The process by which these lenses are worked, and their application to the microscope, are detailed in the Journal of Science of the Royal Institution, vol. ii. page 15 (New Series). This paper was communicated by Dr. Goring, who suggested to Mr. Pritchard the advantages which diamond lenses would most probably possess.

The adaptation of these lenses to telescopes in place of the ordinary eye-glasses would, in all probability, be attended with equal success, where every circumstance calculated to produce a perfect representation of the object is of the utmost importance.

21. The great advantage of duly considering the aberration of lenses will be evident, if we combine two lenses, of twice the focal distance, instead of one, to produce any given power, as the aberration will be decreased to one quarter of that of a single lens of equivalent power, and, therefore, the aperture of the compound lens may be increased, while the error will be less than in a single lens. In the common telescopes (*figs*. 15 and 16), if two lenses were used, instead of the single object and eye-glass, as there shown, the apertures of each might be increased, and, consequently, the instruments would be improved in light and field.

(22.) M. Huygens has demonstrated, that when the greatest possible distinctness is required for the eye-piece of a telescope, it may be obtained by two plano-convex lenses, placed as in *fig*. 19, with their plane sides outward, and the focus of the eye-lens E must be $\frac{2}{3}$ of that of the field-lens F, with a distance between them equal to the difference of their focal lengths. This combination, from the purpose it has been adapted to, is called the astronomical positive eye-piece ; and the telescope, by this addition, will have four times the distinctness of a single lens D, of equivalent power, while the distortion of the object will only be $\frac{1}{4}$ of that produced by a single lens ; for the refraction of the object-lens brings the image of the marginal rays nearer to itself than the central, therefore the image will be formed convex next the lens F, as shown by the arrow ; and as the radius of curvature of the lens F is twice that of the single lens D, the distortion will be decreased in the square of this ratio, or 4 times. On this account, a similar combination is used for the eye-pieces of telescopes for astronomical quadrants, and other graduated instruments, when the convex side of the field-lens is turned towards the eye-glass E, because equal divisions on a micrometer correspond with equal angles, subtended by objects measured by this instrument. This combination, which

Fig. 19.



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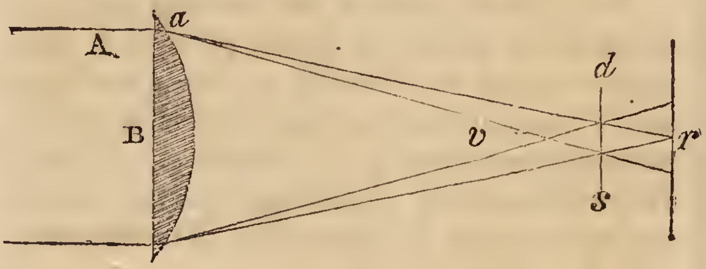
* When it is considered, that the refraction of diamond is nearly three times that of glass, it follows, that in equal refractions its dispersion will be only one-third of the latter.—See Optics, p. 24.

is called Ramsden's Micrometical Eye-piece, has one great disadvantage, viz. that it requires the eye to be placed exceedingly near to the eye-lens E.

(23.) Being now in possession of a combination that will diminish the aberration produced by the eye-piece of a telescope, our limit of magnifying power and light will arise from the errors occasioned by the object-glass; and this, we have seen, may be diminished by having the curves of the two surfaces as 1 to 6, with the most convex side outermost; for this lens has been shown to have less aberration than any other[*]. Secondly, by using two lenses of twice the focus in contact, to produce the required refraction, and thus diminish the error four times. But, although this error may, by the means here pointed out, be rendered very small and almost imperceptible, yet it is magnified in the same proportion as the objects; and when high powers are used, the indistinctness will become sensible.

Sir Isaac Newton conceived that the surfaces of the lens might be formed of some mathematical curve which would entirely obviate this error; and, by investigation he found that, if the surface were described by the revolution of a parabola, and the radiant or object be at an infinite distance, the rays would be collected to a point, and be free from all aberration. He afterwards formed tools to grind and polish lenses of this figure, but when made, although the error by figure was perfectly corrected, it was discovered that the white heterogeneous pencils of light (before that time considered as homogeneous) in their passage through the lens, were divided into their several constituents of red, orange, green, blue, and violet, in the same manner as by a prism, and hence lenses of this figure became useless.

Fig. 20.



(24.) To illustrate the chromatic dispersion produced by a lens, let *a* A, *fig.* 20, be a white compounded pencil of light proceeding from any luminous body, and falling on the lens B; parallel to its axis, at the point *a*, this pencil of light will not be refracted colourless, but the red rays will cross the axis at *r*, and the violet, which will be attracted by the lens more than the other colour, crosses the axis at *v*; and along the intermediate space from *r* to *v*, will be formed a coloured spectrum of orange, yellow, green, and blue; the proportional quantity of these colours, and the total length, will vary according to the substance of which the lens is formed.[*] Sir Isaac Newton, by most accurate observations, found that in common glass, when the sine of the angle of the incident rays A *a* was 50°, the sines of refraction of the red and violet rays were 77° and 78°, the mean refraction of the pencil being $77\frac{1}{2}°$. Now, if we call the sine of incidence *i*, the sine of refraction for red rays *r*, and of the violet *v*, it is found that the diameter of the circle of dispersion *d s*, through which all the colours pass, will be as $(v-r)$ is to $(v+r-2\,i,)$ or as 1 to 55, so that the diameter is $\frac{1}{55}$th part of the aperture of the lens, which is equal to half the diameter of the circle of dispersion at the focus of central rays *r*. The circle of dispersion that will comprehend any particular colour, or set of colours, may be easily calculated. Thus, all the orange and yellow will pass through a circle, whose diameter is $\frac{1}{200}$th of the aperture of the lens. When it is considered that an object-glass $5\frac{1}{2}$ inches in diameter has a circle of dispersion $\frac{1}{10}$th of an inch in diameter, it may be surprising that any picture of an object can be distinguished; but the superior vivacity of the orange and yellow light in comparison with the rest, make the effect produced by the confusion of the colours much less sensible, and will allow this aperture to be used when the focal length of the lens is considerable.

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[*] Dr. Brewster, in his Edition of Ferguson's Lectures (vol. ii. p. 299), states, that in order to render the common refracting telescope as perfect as possible without making it achromatic, the exterior surface of the object-glass should be ground to a radius equal to 5-9ths of its focal length; and the radius of the interior surface, or that next the eye, should be 5 times its focal length. In eye-glasses, the radius of the surface next the object should be 9 times its focal distance, and that of the surface next the eye 3-5ths of the same distance. By this means, the aberration arising from the spherical figure of the lenses will be nothing for objects placed in the direction of their axis, and the least possible for objects removed from the axis. According to Huygens, the spherical aberration was the least possible, when the radii of the surfaces are as 6 to 1. But though this be true for objects placed in the axes of the lenses, yet a considerable aberration remains when the objects are placed on one side of the axis.

[*] Sir Isaac Newton imagined, that the different colours divided the spectrum formed by all substances in the proportions of a musical canon. This is found to be a mistake: for when the spectrum is formed by a prism of crown glass, and another of precisely the same length is formed by the side of it with a prism of flint glass, the confines between the green and blue will be found precisely in the middle of the first spectrum; but in the second, it will be considerably nearer to the red extremity: indeed, there are hardly two substances that disperse the colours in the same proportions. Oil of cassia exerts the strongest action on green light, and sulphuric acid the least.

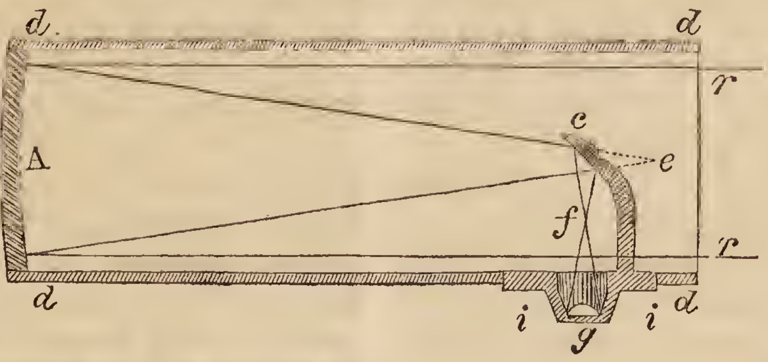
(25.) We may now compare the diameters of the circles of the chromatic and spherical aberration together. If we take a standard telescope of approved goodness, it has not been found possible to give more than 4 inches aperture to an object-glass of 100 feet focal distance, so as to preserve sufficient distinctness; and if the diameter of the circle of spherical aberration is computed for this lens, it will not exceed $\frac{1}{120000}$th part of an inch, while the chromatic, if restricted to $\frac{1}{250}$th of the aperture, which is hardly a fifth of the whole dispersion, (or diameter of the circle of orange and yellow light,) is $\frac{1}{62\frac{1}{2}}$ of an inch, and is therefore about 1900 times greater than the other. But when the aperture of a lens is increased to 30°, the spherical aberration will be found equal to the chromatic in a glass lens; but this aperture can only be used for eye lenses or microscopes.

Chapter VI.—*Reflecting Telescopes—the Newtonian—the Gregorian—the Cassegrainian—Sir W. Herschell's—Mr. Ramage's.*

(26.) With these disadvantages to contend against in refracting substances, Sir Isaac Newton, in the year 1666, turned his attention to reflected light, in which the angle of all the coloured rays are equal. By pursuing this idea he entirely obviated the chromatic error. In the first telescope he made by reflection, the distinctness with which objects were seen through it was surprising, when compared with the refracting telescopes of those times; for though the focal distance of the metal was only $6\frac{1}{3}$ inches, it would carry a power of 38 with equal distinctness to a 4 feet refractor. The form of the metal was spherically concave; but by investigation he ascertained that if the form had been that of a parabola, there would not have been any spherical aberration produced; and if we examine the spherical aberration by figure of a spherically concave metal, and compare it with that of a plano-convex lens ground in the same tool, the former will be 4, while the latter is 9. But when it is considered that the focus of the glass lens is 4 times that of the metal, (for the focal distance of a plano-convex lens is twice the radius, and that of a concave reflector half the radius,) to make their foci equal, the curvature of the lens must be 4 times that of the speculum; and it has been shown that the error by figure increases inversely as the square of the radius: hence the aberration of the lens will be to that of the reflector as $4^2 \times 9$ to 4, or as 36 to 1, and the distinctness will be inversely as the areas of these circles, which are as the squares of their respective diameters; so that the distinctness of a reflector will be 1296 times greater than that of a lens of the same focus and aperture.

(27.) The Newtonian telescope (*fig.* 21.) consists of a concave parabolic metal A, fixed at the end of the tube *d d d;* the plane speculum *c* is fixed to a wire, having its other end attached to a dove-tailed sliding-piece *i i*, and the face of the plane metal is inclined to the axis of the tube and the large speculum at an angle of 45°. In the sliding piece *i i*, opposite the small metal, is inserted a short tube to hold an eye-piece, which is a single lens with its flat side outermost, or the astronomical eye-piece (*fig.* 19.); but as the colour produced by these eye-lenses is not corrected, another combination, called the negative achromatic eye-piece, should be used, which will be described when

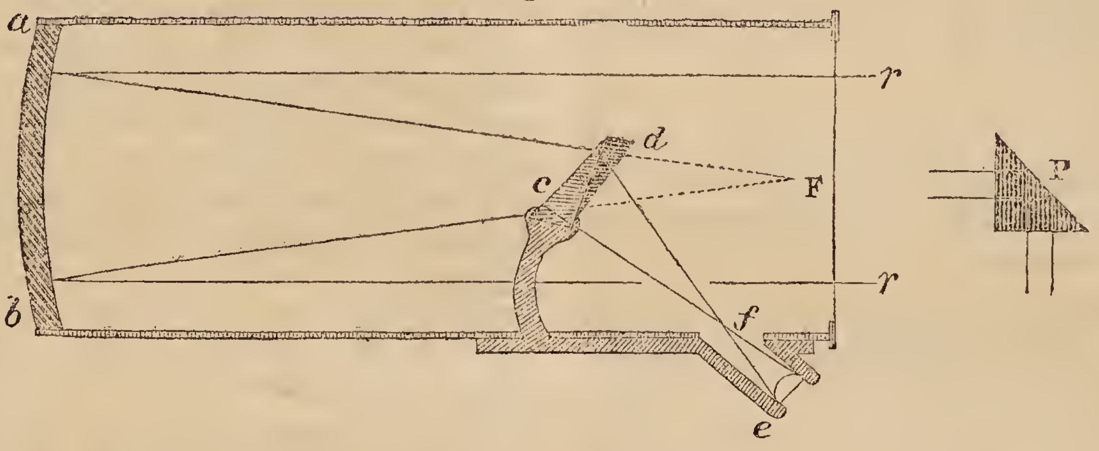
(*Fig.* 18.)



treating on the chromatic correction of lenses. The adjustment of this instrument to distinct vision is made by a rack and pinion attached to the sliding piece and great tube of the telescope, by which the eye-piece and small speculum is brought nearer or farther from the large metal. Let *r r* be the rays of light coming from a distant object, and falling on the large speculum A, these rays would be reflected to the focus *e;* but meeting with the oblique flat metal *c*, are reflected to *f*, where an image of the distant object will be formed, and is received by the eye-lens *g*, by which the rays are rendered parallel. The power of a Newtonian reflector is proportional to the relative focal distances of the concave metal and the eye-lens. For example, let the focal distance *A e* be 40 inches, and the focus of the eye lens *g*, half an inch, the power will be 80. It should here be observed, that the same instrument which is free from aberration for astronomical observation will not be so for terrestrial uses; for the rays in the former case are parallel, while they are divergent in the other. The curve, therefore, of the large speculum when required for the latter purposes, should be elliptical, having the object in one focus, and the focus of the eye-lens in the other.

This telescope, which is more simple than other reflectors, may be greatly improved according to the method of Dr. Brewster, who has proposed, (for telescopes of moderate size, where a front view cannot be used,) to employ two glass prisms in place of the small plane. By the experiments of Major Kater, it appears that one-third of the rays of light is lost when reflected by a speculum at a vertical incidence, and probably not more than 68 out of 100 are reflected at an angle of 45°, as in the Newtonian small metal; in addition to this, the imperfection of surface and figure in metals, which makes the rays stray 5 or 6 times more than the same imperfection in a refracting surface,[*] as well as the difficulty of working metals as perfect as glass, induced him to suggest this improvement. Let *a b*, (*fig.* 22,) be the great speculum, and *r a*, *r b* parallel rays from a distant object reflected to a focus at F; the cone of rays, however, is intercepted by the achromatic prism *c d*, and refracted to *f*, where a distinct image is formed in the anterior focus of the eye-glass *e* by which it is magnified. The double prism *c d*, being composed of a prism of crown glass *c*, and another of flint *d*, united by a cement of mean refractive power, the loss of light by transmission through the two prisms, says Dr. Brewster, will not exceed 600 rays out of 10,000, as the light transmitted through a lens of glass, according to Dr. W. Herschell, is 9,485 out of 10,000 incident rays. Hence, the light lost by the prism is only ⅙ of that lost by reflection.

Fig. 22.



The Newtonian telescopes made by Hadley had, in place of a plane metal a right angular prism P substituted, having its sides perpendicular to the incident and emergent rays. In this, as is accomplished by the two prisms of Dr. Brewster, the image will be erect, and a less quantity of light lost than by a mirror of the common kind.

(28.) Another class of reflecting telescopes was invented by Dr. Gregory, in 1660, but they were not made till some years after the Newtonian, from the difficulty of forming the metals. The Gregorian reflector is, however, preferred to the Newtonian, and is most commonly used, because the observer is stationed in a line with the object, whereas, in the Newtonian he is at right angles to it. *Fig.* 23 is a section of the Gregorian reflector. B D is a concave metal, whose surface

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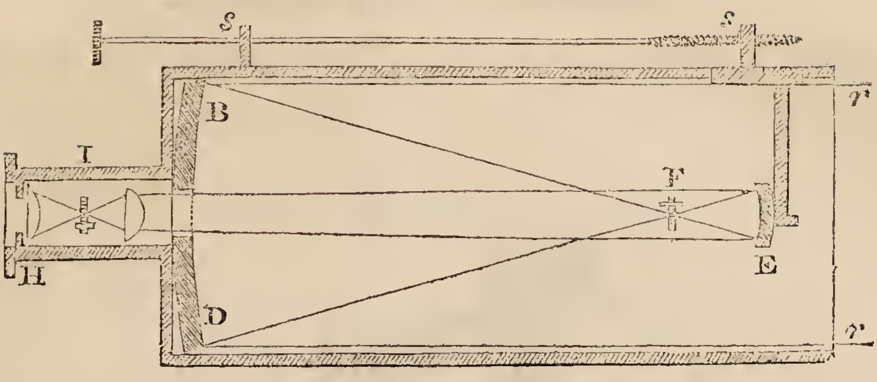
[*] See Newton's Optics.

Fig. 23.



should be formed by the revolution of the hyperbolic curve: this speculum has a small hole in its centre. E is another concave elliptical small metal placed in the axis of the larger one, at a distance from it a little more than the sum of their focal distances. H are the eye-lenses sliding in a tube fixed behind the large speculum: the adjustment is made by the screw *s s*, which moves the small metal to or from the great speculum. Let *r* B and *r* D be two parallel rays from a distant object, these will be reflected to the focus F of the large metal, where an image will be formed, and the rays, crossing each other, fall upon the small speculum E; and if the focus of this metal had coincided with the focus F, the rays would have been reflected parallel, but now they form a direct image at I, and this image is viewed by the eye-piece or a single lens at H. The magnifying power of this instrument may be computed thus:—suppose the focus of the large speculum B F is 9 inches, and the focus of the small metal 1½ inch: then will the angle be increased six times; but this must be multiplied by the ratio of the distances I H, the focus of the lens, and the distance I F; and if these are as 1 to 8, the amplification of the object will be 6 × 8 = 48 times.

(29.) The Cassegrainian reflector is constructed in the same way as the Gregorian, with the exception of a small convex spherical speculum, instead of one a little concave; and as the focus of this metal is negative, it is placed at a distance from the larger metal, equal to the difference of their foci, and only one image is formed, viz. that in the focus of the eye-glass; on this account, the distinctness is considerably greater than in the Gregorian. Mr. Ramsden, in the 69th volume of the Philosophical Transactions, states, that this construction is preferable to either of the former reflectors, because the aberrations of the two metals have a tendency to correct each other; whereas in the Gregorian, both the metals being concave, any error in the specula will be doubled. By assuming such proportions of the foci of the specula, as are generally employed in these instruments, which are about as 1 to 4, he asserts that the aberration or indistinctness occasioned by the figures of the reflectors (supposing each worked equally true) in the Cassegrainian construction, is to that in the Gregorian as 3 to 5.

(30.) In sidereal observations of nebulæ and small stars, abundance of light is necessarily required, and by whatever means a loss of light by reflection or refraction can be prevented, the adoption of such a construction would be advisable. Sir W. Herschel, from an investigation of the loss of light occasioned by the small speculum in reflectors, constructed an instrument which entirely obviated the use of the second metal, by what he called the *front view* telescope. The diameter of the polished surface of the speculum of his large instrument was 48 inches, and its focal distance 40 feet. This metal, which weighed when taken from the casting, 2118 lb., was placed at the end of an iron tube 4 feet 10 inches in diameter; the other end is elevated towards the object, and has attached to it a single eye-lens in the focus of the metal; the observer is mounted in a gallery moveable with the instrument, having his back to the object. The light obtained from so large a surface by this instrument was truly surprising, and enabled objects otherwise invisible to become extremely interesting. This telescope, erected at Slough, near Windsor, was completed on the 28th of August, 1789, and on the same day the sixth satellite of Saturn was discovered*. The frame of this instrument having greatly decayed, it has been taken down; and another, of 20 feet focus and 18 inches diameter, erected in its place, by his son Mr. J. Herschell, in 1822.

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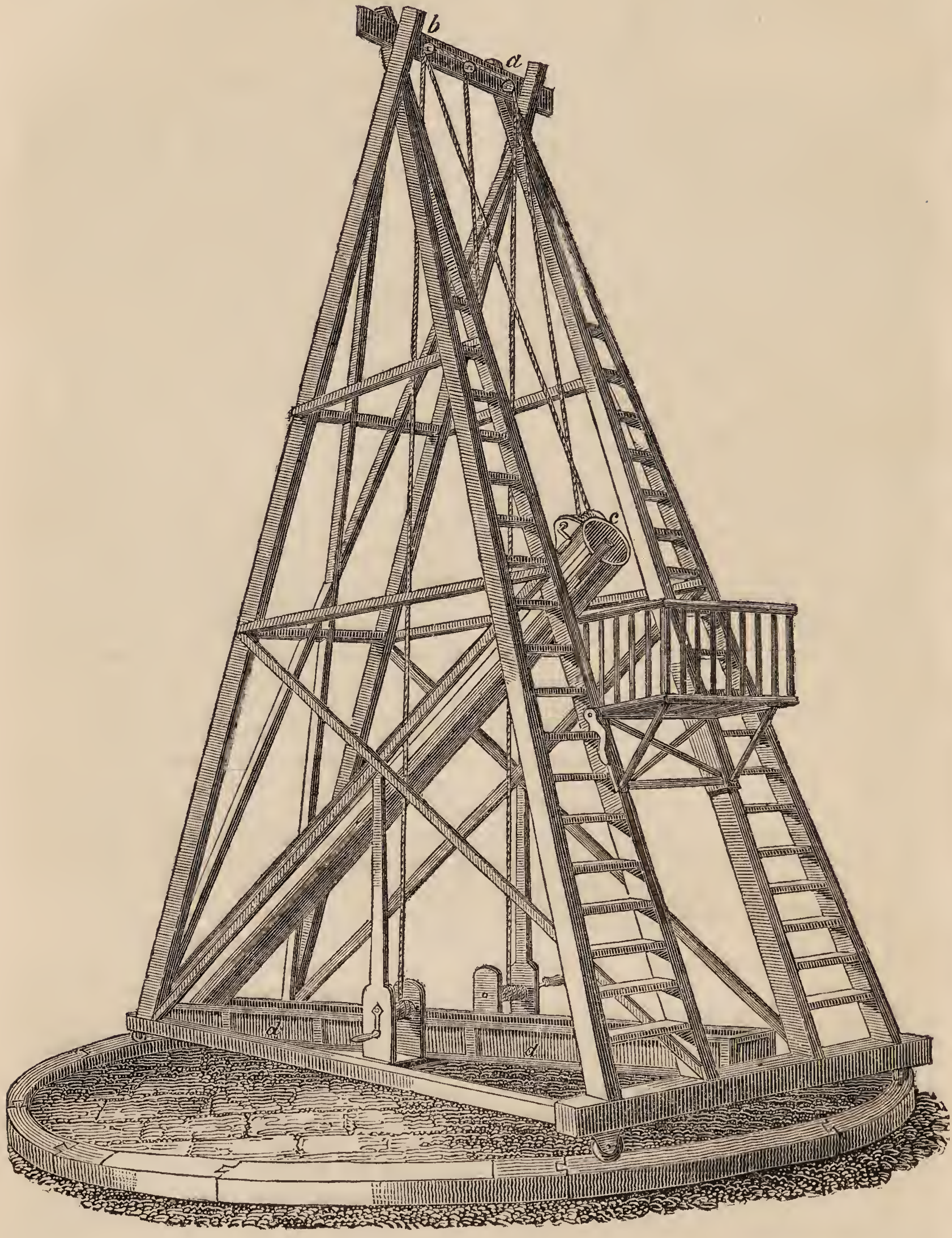
* A full description of this instrument will be found in the Transactions of the Royal Society for 1795, explained by means of 18 plates and 63 pages of letter-press; and an ample detail is given of every circumstance relating to the mechanical construction of this instrument.

Fig. 24.



Mr. RAMAGE'S REFLECTING TELESCOPE, ERECTED AT THE ROYAL OBSERVATORY, GREENWICH, IN THE YEAR 1820.

(31.) The largest front view reflecting telescope at present in this country, is that erected at the Royal Observatory, at Greenwich, by Mr. Ramage, in 1820. The diameter of the concave reflector is 15 inches, and its focus 25 feet; the mechanical arrangement of the stand is greatly simplified. A perspective view of the whole instrument is shown at *fig*. 24. The tube is composed of a twelve-sided prism of deal $\frac{5}{8}$ inch thick. At the mouth *c* is a double cylinder of different diameters on the same axis; around this a cord is wound by a winch, and passes up from the small cylinder over a pulley *a*, and down through the pulley *b*, on to the larger cylinder at *c*. Now, when the winch is turned to raise the telescope, the endless cord is unwound from the smaller cylinder, and wound on to the larger: the difference of the size of the two cylinders will be double the quantity raised, and a mechanical force to any extent may thus be obtained by duly proportioning the diameters of the two cylinders; by this contrivance the necessity for an assistant is superseded. The instrument, when not in use, is let down into the box *d d*, and covered with canvas, to prevent dust or moisture from tarnishing the speculum.

CHAPTER VII.—*Theory of Achromatic Telescopes—Double Object-Glass.*

(32.) Having noticed all the valuable modifications of the reflecting telescope, we must now return to the refracting one. The most obvious and important improvement in this instrument consists in the formation of object-glasses free from the errors of chromatic and spherical aberration, whence they were denominated achromatic telescopes. But as this word merely signifies freedom from colour, which in common telescopes is sometimes effected without a correction of the figure or spherical aberration, Sir W. Herschel has, therefore, very properly denominated a perfect telescope *aplanatic*, from two Greek words α *from* πλανος error, that is, *free from all errors*.

In (Note to 24) it was stated, that the length of the spectrum produced by lenses varied, when formed of different substances; thus, if two lenses are made of the same focal length, the one of flint glass and the other of crown, the length or diameter of the coloured image in the flint will be to that produced by the crown lens as 3 to 2 nearly. Now, if we make the focal lengths of the lenses in this proportion, that is, as 3 to 2, the coloured spectrum produced by each will be equal; but if the flint lens be concave and the crown convex, when placed in contact they will mutually correct each other, and a pencil of white light refracted by the compound lens would remain colourless. Unfortunately in the formation of such a lens, the dispersion of the flint glass is so variable, that trials on each specimen require to be made, before the absolute proportional dispersion of the substances can be ascertained*. As the achromatic object-glass is the most delicate test of the dispersion of the medium, it is best found by forming a piece of the flint glass into a concave lens, and combining it with a convex of crown glass whose focal length is known, and varying the curvature of the flint till the dispersions are corrected, *i. e.* till the purple or lilac fringe surrounding a white object on a black ground is observed on one side of the focus, and a green on the other, when converted into the object-glass of a telescope and using a powerful eye-glass. Now, if the compound focus be accurately measured, and the focus of the convex lens known, the proportionate foci of each may be ascertained, and, consequently, the proportion of their dispersive powers is found. This correction of the spectrum will not correct the error of each colour, for the proportional lengths of the blue, green, or red light are variable in different substances* : thus flint glass is found to refract green light considerably less than crown glass, in the proportion of the whole refraction of the red and violet light: so that when the divergency of the red and violet light, caused by the refraction of the two mediums, is equal, the divergency of the red and green light is always greater in the crown than in the flint, and the divergency of the violet is always less in the crown glass[*]. Hence it must be observed, that in order to have a complete correction of all the colours, more than two media must be used, and, therefore, the best telescopes have their object-lens composed of three kinds of glass. When the dispersive ratio is known, and the refractive focus of the compound lens is given, the refractive focus of each must be calculated, and to obtain the radius of the tools for working the lenses, their refractive foci must be converted into the geometrical shown as at (11), when the object-glass would be completed, and the task would not be difficult to perform; but the spherical aberration, although much less in quantity, is more troublesome to correct, and in making this correction, the proportion of the radii of the two surfaces of the convex lens must be assumed. When a suitable selection is made, the aperture of the lens being given, the spherical aberration must be calculated, when its thickness is ascertained[†]. And lastly, the curvatures and thickness of a concave flint glass must be found that will exactly balance the spherical aberrations produced by the convex glass; always keeping the foci of the two lenses in the *ratio of their dispersive powers.*

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* 1. Sulphuric acid exceeds all other transparent bodies in its action on the green rays, while oil of cassia exerts the least action upon them of all known substances.

2. It is ascertained that in all minerals in which a metal is the principal ingredient, those which have the greatest density have also the greatest faculty of producing colour, while in all the precious stones a high refractive power is attended with a low dispersive power.

3. The dispersive powers of *resins*, *gums*, *oils*, and *balsams* greatly exceed water, and correspond in some measure with their refraction.

4. The muriatic and nitric acids exceed water in dispersion, while the *phosphoric*, *citric*, *sulphuric*, and *tartaric* acids, surpassing them in refraction, possess very *low* dispersive powers. (See the Tables of Refractive and Dispersive Powers of different Substances in the Treatise on Optics.)

(33.) The radii of curvature of the different surfaces of the lenses necessary to form a double achromatic object-glass, when the sine of incidence is to the sine of refraction in the crown glass as 1.528 to 1, and in the flint as 1.5735 to 1, the ratio of their dispersive powers being as 1 to 1.524, and assuming the curvatures of the concave as 1 to 2, are shown in the following TABLE. The first column F is the compound focus of the object-glass in inches; r the radius of the anterior surface of the crown; R its posterior side; r' the radius of the anterior side of the concave lens of flint glass; and R′ its posterior surface.

F	r	R	r'	R′
12	3.	4.652	4.171	8.342
24	6.	9.304	8.342	16.684
30	7.5	11.63	10.428	20 856
36	9.	13.956	12.552	25.027
48	12.	18.608	16.684	33.369
60	15.	23.260	20.856	41.712
120	30.	46.520	41.712	83.424

In these computations it may be remarked that the radius of the anterior surface of the concave being less than the posterior side of the convex, admits of its approach without touching in the centre, which should always be a necessary practical condition.

CHAPTER VIII.—*Aplanatic Telescopes of Clairault's Construction—Mr. J. F. Herschell's Object-Glass—Triple Object-Glasses—Fraunhofer's and Tulley's Telescopes—Galilean Telescope and Opera Glass—Achromatic Opera-Glass—Dr. Brewster's Fluid Opera-Glass.*

(34.) The problem for the choice of the proportional curvature of the assumed convex lens is of the kind called *indeterminate,* or admitting of an infinite variety of solutions. In consequence of this, it allows an endless number of combinations of lenses, and each may be free from spherical aberration. It becomes therefore a matter of considerable delicacy to fix our choice among them, and numerous constructions have been calculated by different authors. Thus Clairault, a French mathematician, has given a construction in which the two internal surfaces are worked of equal radii, the one convex and the other concave, so as to admit of being cemented together, and thus avoiding the loss of light by reflections at the two surfaces. But having employed indices of dispersion in his computations, higher than what are usually met with in practice, and when those most likely to be obtained are used, the radii change so rapidly as to render this construction difficult to interpolate, where the artist is no algebraist; and hence it must lose much of its value to the practical optician.

(35.) Another construction has lately been proposed by Mr. J. F. Herschel, in which he states, that the destruction of the spherical aberration is ensured, not only for parallel rays from celestial objects, but also for those that diverge from objects situated at a moderate

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[*] Dr. Blair.

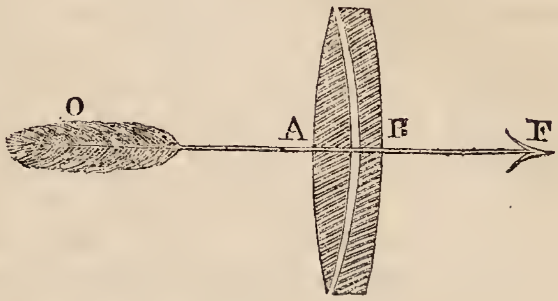
[†] The longitudinal aberration of a lens of glass may be found by the following general theorem, where r is the radius of the first surface, R the second surface, and T the thickness of the lens.

$$\left(\frac{27\,r^2 + 6\,r\,\mathrm{R} + 7\,\mathrm{R}_2}{6 + (r \times \mathrm{R})^2} + \mathrm{T}\right)$$

see Martin's new Optics, part vi, chapter 3; or Smith's Optics, book ii, chapter 18.

finite distance; and on these conditions he has rendered the problem determinate, while the radii resulting from the construction are such as will satisfy the following more important conditions:—1st. The curvatures assigned to each surface are more moderate than in any other theoretical combination. 2nd. The exterior surfaces of the compound lens vary within narrow limits by any variation in either the refractive or dispersive powers that generally occur in practice. 3rd. That the two interior surfaces approach in all cases so near to coincidence, that no sensible error can arise from neglecting their difference; finally, he states as a theorem, which will be found sufficiently exact in practice, that a double object-glass will be free from aberration, provided the radius of the exterior surface of the crown lens be 6.720, and of the flint 14.20, the focal length of the combination being 10.00, and the radii of the interior surfaces being computed from these data so as to make the focal lengths of the two glasses in the direct ratio of their dispersive powers. *Fig.* 25, is a section of this object-glass, the anterior glass A, or that which receives the incident ray, is an unequally convex lens of crown glass, the flatter side being placed outermost; the posterior glass B is a meniscus of flint glass.

Fig. 25.



The rule here stated is given only as an approximation, and will no doubt be sufficiently exact for ordinary practical purposes; but when object-glasses of great aperture and value are to be constructed, their radii must be computed more strictly, and for this purpose we shall subjoin Mr. Herschell's TABLE, calculated upon the rigorous formulæ as given in the *Philosophical Transactions* for 1821:—

Dimensions of an Aplanatic Double Object-Glass.

Refractive index of the Crown Lens......	1.524	Compound Focal Length, 10,000.
Ditto ditto of the Flint Lens........	1.585	

Dispersive Ratio.	1st Surface Convex.			3dSurface Convex.	2dSurface Concave.	4th Surface Convex.			Focal Length of Crown Lens.	Focal Length of Flint Lens.
	Radius for the above Refractive Indices.	Variation of Radius for a change of +0.010 in Refractive Index of Crown Glass.	Variation of Radius for a change of +0.010 in Refractive Index of Flint Glass.	Radius.	Radius.	Radius for the above Refractive Indices.	Variation of Radius for a change of +0.010 in Refractive Index of Crown Glass.	Variation of Radius for a change of +1.010 in Refractive Index of Flint Glass.		
0.50	6.7485	+0.0500	−0.0036	4.2827	4.1575	14.3697	+0.9921	−0.3962	5.0	10.0000
0.55	6.7184	+0.0740	−0.0011	3.6332	3.6006	14.5353	+1.0080	−0.5033	4.5	8.1818
0.60	6.7069	+0.0676	+0.0037	3.0488	3.0640	14.2937	+1.1049	−0.5659	4.0	6.6667
0.65	6.7316	+0.0563	+0.0125	2.5208	2.5566	13.5709	+1.1614	−0.6323	3.5	5.3846
0.70	6.8279	+0.0335	+0.0312	2.0422	2.0831	12.3154	+1.1613	−0.7570	3.0	4.2858
0.75	7.0816	−0.0174	+0.0568	1.6073	1.6450	10.5186	+1.3847	−0.7207	2.5	3.3333

The dimensions in the table are computed on the supposition of the focal length of the object-glass being 10; and to adjust them to any other assigned focal length, all that is required is to increase or diminish the radii here set down on the proportion of the assigned focal length (in inches, feet, or parts of any given scale) to ten parts of the same scale.

When the refractive powers of the two media are exactly 1.524 and 1.585 (which are nearly their average values) respectively, and the dispersive ratio is any one of the numbers in the first column, this table gives at once the exact values of the radii required; but when this is not the case, we must proceed as follows:—Suppose (for example's sake) we would find the proper radii for the surface of an object-glass of 30 inches focal length: the refractive index of the crown lens being 1.519, and that of the flint 1.589, the dispersive power of the former being to that of the latter as 0.567.1 or 0.567 being the dispersive ratio.

The computation must first be made as for an object-glass of 10 inches focus, and first, we must determine the focal length of the separate lenses, to this end.

1. Subtract the decimal (0.567) representing the dispersive ratio from 1.000, and the remainder multiplied by 10, is the focal length of the crown lens (in this case 10 × 0,433, or 4,330.)

2. Divide unity by the decimal above mentioned (0.567,) subtract 1,000 from the quotient and multiply the remainder by 10, and we get the focal length of the flint lens. In this case before us $\frac{1}{8.567} = 1.7635$ and $0.7635 \times 10 = 7.635$ is the focal length required. We must next determine by the *tables* the radii of the 1st and 4th surfaces for the dispersive ratios their set down, (0.55 and 0.60), next *less* and next *greater* than the given one. For this purpose we have

Refractive powers given	1.519	and	1.589
Ditto ditto in the table	1.524	and	1.585
Differences	− 0.005	and	+ 0.005

The given refraction of the crown being less and the flint greater, than their average value on which the table is founded. Looking out now opposite to 0,55 in the first column for the variations in the two radii corresponding to a charge of + 0,010 in each of the two refractions, we find as follows:—

	1st surface	4th surface.
For a charge = + 0.010 in crown	+ 0.0740	+ 1.0080
Ditto ditto = + 0.010 in flint	− 0.0011	− 0.5033

But the actual variation in the crown instead of + 0.010, being + 0.004, we must take the proportional parts of these, changing the sign in the case of the crown: Thus we find the variations of the first and last radii to be:—

	1st surface.		4th surface.
For − 0.005 variation in crown	− 0.0370		− 0.5040
For + 0.004 ditto in flint	− 0.3004		− 0.2010
Total variation from both causes	− 0.3374	and	− 0.7053
But the radii given in the table are	+ 6.7184	and	+ 14.5353
Hence the radii interpolated are,	6.6810	and	13.8300

If we interpolate (by a process exactly similar) the same two radii for a dispersive ratio 0.60, we shall find respectively:—

	1st surface.		4th surface.
For − 0.005 variation in crown	− 0.0338	and	− 0.5524
For + 0.004 ditto in flint	+ 0.0015		− 3.2264
Total variation	− 0.0323	and	− 0.7788
Radii in the table	6.7069		14.2937
Interpolated radii	6.6746	and	13.5149

Having thus got the radii corresponding to the actual refractions, for the two dispersive ratios 0.55 and 0.60, it only remains to determine their values for the intermediate ratio 0.567 by proportional parts thus:—

		1st radius.	4th radius.
For	0.600	6.6746	13.5149
For	0.550	6.6810	13.8300
Diff. +	0.050	−0.0064	−0.3151

We then say as 0.050 : 0.567 − 0.550 = 0.017 :: −0.0064 : −0.0022 and 0.50 : .017 :: −0.3151 : −0.1071, so that 6.6810 − 0.0022 and 13.8300 − 0.1071; or 6.6788 and 13.7229 are the true radii corresponding to the given data:

Thus we have in the crown lens:—

Focal length = 4.3300
Radius of first surface = 6.6788
Index of Refraction = 1.5190

From which data it is easy to compute, by rules familiar to every optician, the radius of the other surface, which will come out, 3.3868. Again in the flint lens we have for the

Focal length = 7.635
Radius of first surface = 13.7229
Index of refraction = 1.589

Whence we find 3.3871 for the radius of the other surface.

The four radii are thus obtained for a focal length of 10 inches; and to obtain them for 30 inches we have only to multiply them by 3, and we obtain finally. In the case proposed, the

Radius of 1st surface,	of 2nd,	3rd.	4th.
20.0364 inches,	10.1604 inches,	10.1613	and 41.1687.

So that here the radii of the two adjacent surfaces scarcely differ more than $\frac{1}{1000}$th of an inch, and may of course be cemented together, should it be thought desirable.

(36.) The triple object-glass is constructed in the same manner as the double, but as we have two convex lenses to produce the required refraction, the total spherical aberration will be less in a triple lens than in a double one, and, therefore, requires less correction by the flint; while the secondary spectrum may be greatly diminished by making one convex lens of crown, and the other of Bohemia, or Dutch plate glass. The theorems for finding the proportionate foci of the two lenses are indeterminate; for theoretically, it is immaterial whether their foci be equal, or in any other proportion, provided the compound lens be in the ratio of the dispersion of the flint. Again, the radii of the concave may be varied to any convenient curvature, as there are two convex lenses, and, therefore, the aberration cannot be greater than the convex lenses will correct*. When the radius of each surface is equal, and the ratio of dispersion and refraction is the same as in the double achromatic object-glass, and the compound focus of the lens is 30 inches, the radii of the first convex

will be $\begin{cases} r = 21.35. \\ R = 15.93. \end{cases}$

of the concave $\begin{cases} r' = 13.9. \\ R' = 13.9. \end{cases}$

second convex $\begin{cases} r'' = 21.35. \\ R'' = 15.93. \end{cases}$

(37.) The largest triple achromatic telescope ever constructed, has lately been erected in the observatory of the imperial university at Dorpat, on the 10th of November, 1824, and was made by Fraunhofer, the late director of the Optical Institute, at Benedictbauern, near Munich.

The concave is formed from a piece of dense flint glass made by Guinand, and has a greater dispersive power than any obtained before. It is perfectly free from veins; and the diameter of the object-glass exceeds that of any other telescope, having a clear aperture of $9\frac{6}{10}$ inches, and a focal distance of 25 feet. This instrument is mounted on a metal stand, and although of the immense weight of 5000 Russian pounds, is moveable in every direction with the slightest exertion, all the moveable parts being balanced by counter-weights. It has 4 eye-glasses, the lowest magnifying

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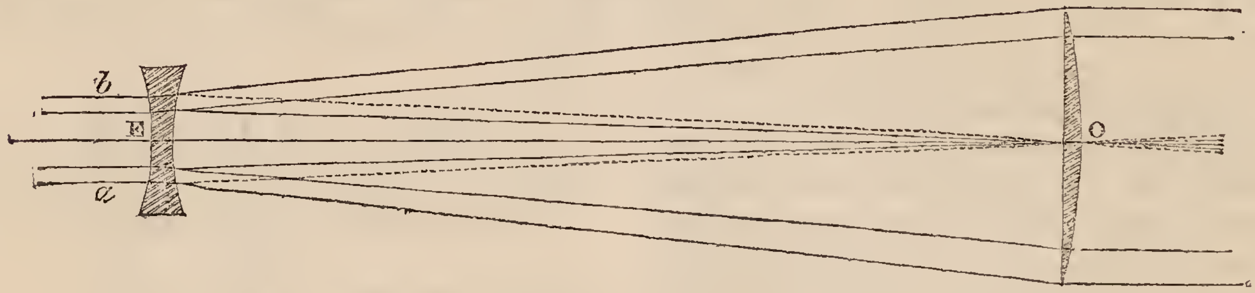
* In a double achromatic object glass, the convex lens should be assumed first, as the flint might be formed with more aberration than the convex could correct.

175 times, and the highest 700 times: the cost of this instrument was 1300*l.* sterling. Another telescope has lately been made of similar materials in England by Mr. Tulley; the aperture of the object-glass is $6\frac{8}{10}$, and its focal length is 12 feet. This instrument is now in the possession of Dr. Pearson.*

(38.) The Galilean telescope was invented in 1590, by the illustrious person from whom it derived its name; but as it is susceptible of little improvement from the nature of its construction, it is seldom used except for opera glasses, in which the shortness of the construction renders it available. It consists of a single convex or achromatic object-glass, whose focal length is usually from 4 to 8 inches, which it rarely exceeds. The eye-glass is a double or plano-concave lens from ½ an inch to 2 inches focus; the distance between the two glasses is equal to the difference of their focal lengths, and the power is in the ratio of their foci, as in the astronomical telescope. *Fig.* 26 is a section of the Galilean construction for an opera-glass. Let O be the object-glass of 6 inches focus, E the concave eye-glass 2 inches focus; the distance O E will be 4 inches, and the power will be expressed by 6 inches divided by 2 inches, equal to 3 times. The distinctness of the Galilean construction exceeds that of any other, and arises from the rays of light proceeding from the object directly through the lenses without crossing or intersecting each other; whereas, in the combination of convex glasses, they intersect one another to form an image in the focus of the object-glass; and this image is magnified by the eye-lens with its imperfections and distortions. With a power of 8 and a 12 or 16-inch object-glass, the satellites of Jupiter have been distinctly observed; while a common astronomical telescope of 4 or 5 feet focal length has scarcely rendered them visible. The area or field of view in this instrument is very limited, and, on that account, it cannot be used for high powers, as the objects seen at one view are always as the area of the pupil of the eye, and not as the area of the eye-glass, as in convex lenses. Thus, if *a b* (*fig.* 20.) is larger than the pupil of the eye they will not be seen, although refracted by the lens E; but it should be remembered, that as the rays, when they have passed through the eye-glass, are not converged to a focus, the nearer the eye is placed to the lens E, the more numerous will be the objects seen at one view.

Fig. 26.



(39.) The construction of opera-glasses might be achromatic, when made with only two lenses, provided the focal length of the object and eye-lens are in the ratio of the dispersive and refractive powers of the media from which they are formed. Thus, if an object lens be made of rock-crystal, whose focal distance is 5, and the eye-lens be formed of oil of cassia, fixed between two parallel pieces of glass, or other convenient substance so curved as to give a focus of 1.02, the combination will be achromatic, with an amplyfying power of $4\frac{9}{10}$ times. If the concave lens had been formed of flint-glass, with the same object lens, a magnifying power of two might be obtained, and the combination would be free from all colour. If oil of aniseed were used for the concave, the power obtained to make the instrument achromatic would be 2.82 times*.

The following TABLE exhibits the refractive and dispersive powers of different substances capable of producing achromatic combinations; and it should be remarked, that the medium used for the eye-lens must have the greatest dispersive power:—

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* See Astronomical Transactions, vol, ii.

* Dr. Brewster's treatise on new optical Instruments,

Substances to be employed for Eye-Lenses.	Refractive Power.	Dispersive Power.
1. Oil of Cassia	1.641	.139
2. —— Aniseed	1.601	.077
3. —— Cummin	1.508	.065
4. —— Cloves	1.535	.062
5. —— Sassafras	1.532	.060
6. —— Sweet Fennel Seed	1.506	.055
7. —— Spearmint	1.481	.054
8. —— Pimento	1.507	.052 .048
9. Flint Glass	1.616	.052 .048

Substances to be employed for Object-Lenses.	Refractive Power.	Dispersive Power.
Crown Glass	1.534	.036
Plate Glass	1.527	.032
Water	1.336	.035
Alcohol	1.374	.029
Sulphuric Acid	1.440	.031
Oil of Ambergris	1.368	.032
Rock Crystal	1.562	.026
Topaz	1.638	.024
Diamond	2.470	.038

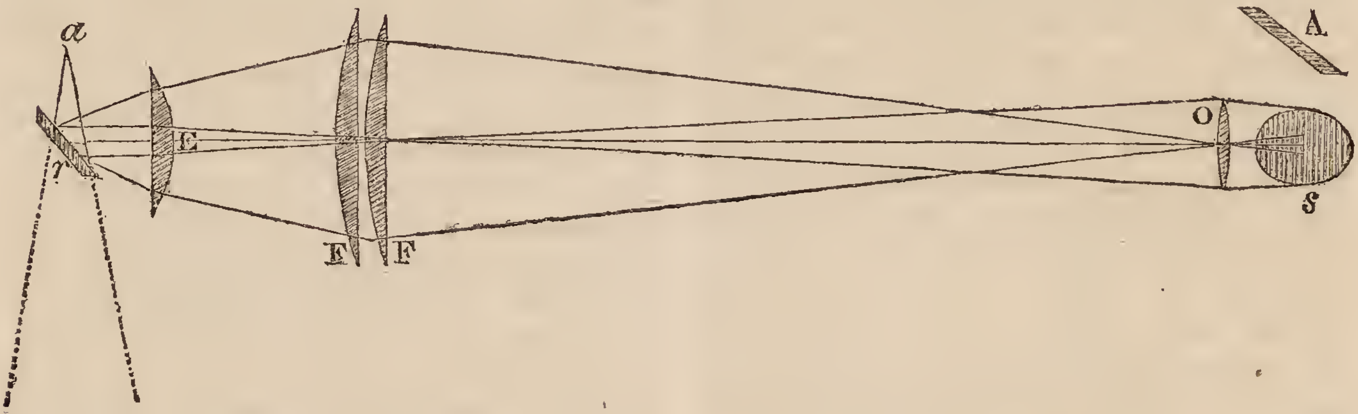
CHAPTER IX.—*Dr. Brewster's Telescope for measuring Distances — Double Image Telescope—Graphic Telescope —Achromatic Eye-Pieces.*

(40.) A telescope for measuring the distances of objects was invented by Dr. Brewster, for which he obtained a patent in 1810. When describing the telescope *figs.* 15 and 16, it was stated, that as the object approached the instrument, the eye-tube required to be drawn out to adjust the instrument for distinct vision: now the measure of the quantity required to be drawn out, if registered on the sliding tube by divisions, and by a corresponding mark on the outer tube, would determine the distances of the objects. The increase of the focal length of the object-glass may be found by dividing the square of the focal length of the object-glass by the distance of the object, minus the focus of the object-lens: thus, if the focus of the object-lens is 2 feet, and the distance of the object 50 feet, the tube must be drawn out $\frac{2^2}{50-2}$ or 1 inch from the solar focus, to adjust for distinct vision of the object; but the length of these divisions would be in a decreasing ratio, and the quantity is almost imperceptible for great distances: thus, if two objects be 100 feet apart, and the nearest 200 feet from the observer, the difference of adjustment for these two objects would be only $\frac{8}{100}$ of an inch. In the patent construction this quantity may be increased to almost any required length, by means of two object-glasses; the inner one is about $\frac{1}{3}$ of the focal length of the principal lens, and being fixed to the eye-tube slides along with the eye-piece. This instrument has been found useful at sea in determining the distance of head-lands, and in war, when an enemy's ship is gaining sail; this is determined by taking two observations, and noting the difference of the quantity required to be drawn out or pushed in to obtain a distinct vision; and when the time between each observation is known, the rate of the vessels sailing may be ascertained.

(41.) Another instrument, nearly for the same purpose as the one just described, but possessing more accuracy for measuring angles and distances, was constructed by the Abbé Rochon. This instrument is the ordinary telescope, with a double refracting prism of rock-crystal, placed before the eye-glass; this prism produces two images of an object. These images may be made to approach to or recede from one another by altering the distance of the prism from the eye-glass; but the object is better effected by a method proposed by Dr. Brewster, in which the first glass of the telescope, together with the prism, which may be cemented to it, is made to slide to or from the other glasses of the eye-piece, and the quantity moved is registered by divisions on the tube. This variable eye-piece, which changes the power of the telescope, is so adjusted, that the two images of the object shall be in accurate contact; when this is done, if an observation is made to find the angle, the index, by previous experiment, will give it; but when it is desired to know the rate of motion of the object, another observation must be made at a determinate interval, and the difference of adjustment required in the two observations to bring the images in contact will determine its distance, the scale being previously ascertained by experiment or calculation. The imperfect crystallization of quartz, and the difficulty of working it, led Dr. Brewster to adopt the *colourless topaz of New Holland* to form his double-refracting prisms, it being much freer from veins and imperfections, besides possessing the advantage of a lower dispersive power. In certain sections of the crystal, when we require only a very small separation of the images, we may preserve, on both sides, the natural surface of the cleavage. The other means of procuring a double image proposed by Dr. Brewster, we shall subjoin, as being well worthy consideration for micrometical purposes. 1. The double image may be produced by a small bisected plane speculum, placed between the eye-lens and the eye, and one of the halves may be made to move by a screw, not for the purpose of bringing the images in contact, but in order to vary the constant angle, according as it is wanted, for large or small discs. 2. The duplicature of the image may be effected by bisecting the eye-lens, or by placing a bisected lens between the eye-lens and the eye. 3. The two images may be formed by a slightly-inclined face, ground upon a highly-polished and parallel plate of fluor-spar, one image being seen by half of the pupil through the parallel plate, and the other through the inclined face. Fluor-spar is recommended because of its producing a less dispersion under a given angle than any other substance; and even this might be removed by the ordinary means.

(42.) The graphic telescope is employed for the delineation of objects situated at any distance from the instrument, which may be represented of any required size; and is used for drawing portraits, landscapes, and architectural subjects. It was invented in 1811, by Mr. C. Varley, who obtained a patent for it. This instrument consists of an astronomical telescope of low power, placed between two plane reflecting specula, with a particular construction of the eye-piece to correct the distortion of the image produced by the eye-glass. *Fig.* 27, is a section of the instrument.

Fig. 27.



O is the object-glass, E the eye-glass, and F F two meniscus field-glasses, whose form and distance from the eye-glass are so adjusted as to produce a flat and enlarged field of view. The rays of light proceeding from an object enter the side of the tube, and impinge upon the flat speculum *s*, placed at an angle of 45° with the axis of the telescope, as is shown in section at A; the rays will be reflected by this speculum along the axis and through the object-glass O, and are converged to a focus near the field lens F, where an inverted image of the object will be formed and may be received by the eye-glass E; lastly, they will strike the other inclined flat reflector, and be reflected up to the eye placed above at *a*. When a piece of paper is placed below the speculum *r*, the observer being supposed to look down on the speculum, keeping both eyes open, he will be able to see an image of the object on the paper when the instrument is adjusted to its proper focus. This representation may be traced with one or both eyes, and the size of the image may be varied by altering the distance of the paper from the speculum, or by changing the magnifying power of the instrument. If the first speculum *s* be taken away, the sides of the objects are reversed (as on an engraved copper-plate,) the speculum *r* forms the image erect, which was inverted by the eye-glass E, and prevents the telescope from intercepting a view of the paper.*

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* The inventor of this instrument has employed it very extensively in sketching from nature; and in the mountainous districts of Wales the views become of great value on account of their accuracy; he has also made with this instrument some very correct views of the seat of the late Lord Byron, at Newstead Abbey, Nottinghamshire. In drawing shipping and boats it is extremely valuable, as the numerous lines in the details of these objects are not of a geometrical figure. We have been informed that this instrument was the one employed in making the panoramic view of the metropolis from the top of St. Paul's, for the exhibition of which a building in the Regent's Park, called the Coliseum, has recently been erected.

(43.) The achromatic or negative eye-piece generally adapted to astronomical telescopes, is a combination of lenses intended to correct the dispersion produced by the eye-glass, independent of the object lens. Let O, *Fig.* 28, be the object-glass of a telescope free from chromatic dispersion, and E the eye-glass; let F be the focus of the object-glass O, where an inverted image is formed. Now the pencil of white light A *a b*, when transmitted by the lens E, will be divided into its component colours, so that *b* R will be the direction of the red rays, and *b* V the path of the violet, and the angle V *b* R in crown-glass will be $\frac{1}{27}$ of *a b* R. The rays B *a d* passing through a part of the lens whose surfaces are less inclined to each other, will be less refracted and less dispersed in the same proportion nearly, and *d* I will be the path of the red rays, and *d v* of the violet; therefore, the two violet rays will be very nearly parallel when the two red rays are rendered parallel. Hence it must happen, as coloured rays do not unite at the bottom of the eye, that the object will appear bordered with coloured fringes, and a black line seen near the margin on a white ground will have a ruddy and orange border on the outside, and a blue border within, and this confusion will increase nearly in the same proportion as the visual angle *b* I *c*.

Fig. 28.

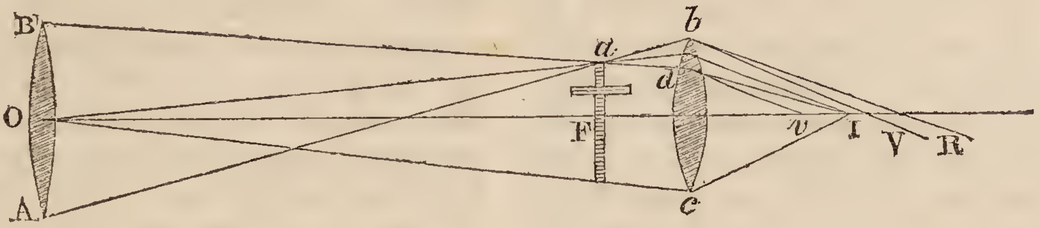
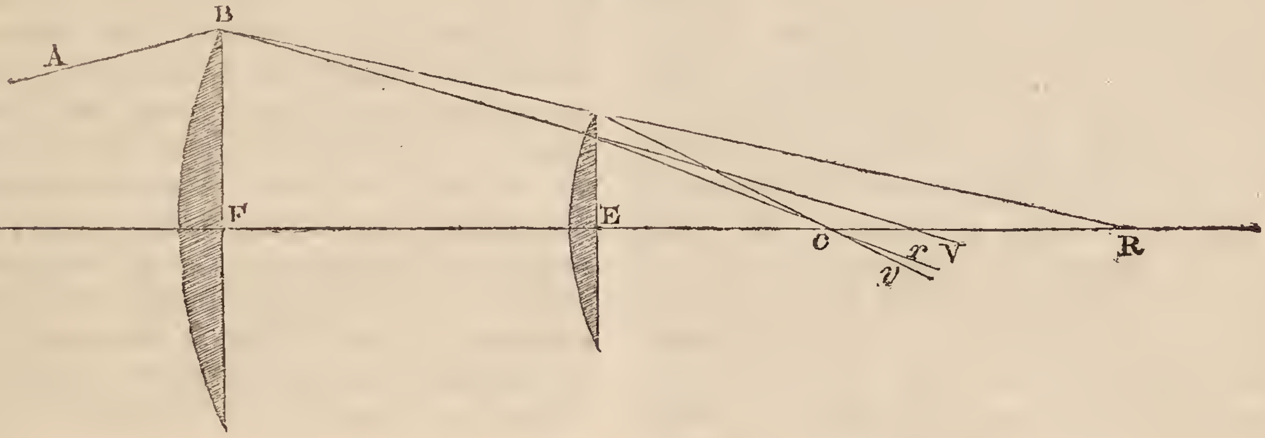


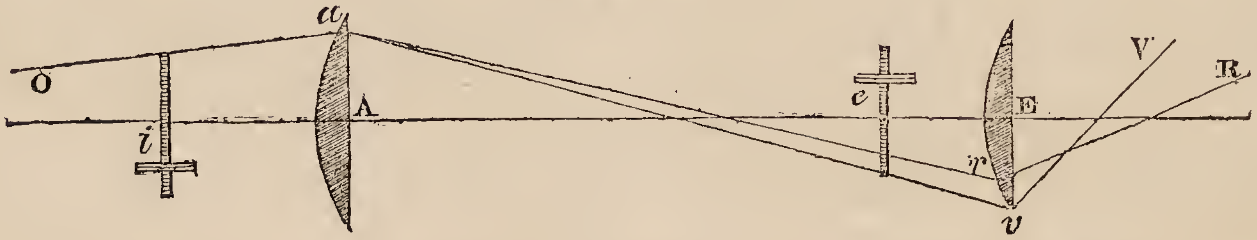
Fig. 29.



(44.) *Fig.* 29, is a section of the achromatic eye-piece. Let A B be a compounded pencil of white light proceeding from the object glass, B F a plano-convex field glass, with the plane side next the eye-glass E. Now the red rays of the pencil A B after refraction would cross the axis in R, and the violet rays in V, but meeting the eye-glass E, the red rays will be refracted to *c r*, and the violet to *c v*, when they will cross one another, in the axis at the point *c* and unite; for the violet ray being nearer the axis of the lens E, will suffer less refraction than the red, and when the eye is placed in the axis at *c*, the object will appear colourless. The distance of the two lenses F E, to produce this correction, when made of crown glass, whose dispersion is as 77 to 78, when the incidence is 50, must be equal to half the sum of their focal distances nearly; or, more exactly, the distance between the two lenses must be equal to half the sum of the focal distance of the eye-glass, and the distance at which the field-glass would form an image of the object-glass; for the point R is the focus to which a ray coming from the centre of the object-glass is refracted by the field-glass, consequently, this distance must be varied according to the distance of the objects, and also, as the length of the object-glass; for the same combination will not be correct for a long and a short object-glass, nor for celestial and terrestrial observations; for, when it is correct for one case, and the distance of the image or focal point is varied, the divergency of the rays will alter, and the dispersion will be either too much or too little corrected. The proportion of the foci of these two lenses should be as 2 to 3, to produce the largest field and the least distortion.

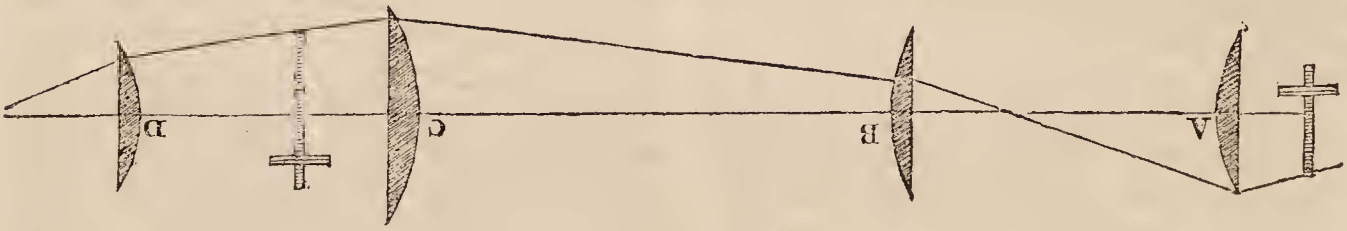
(45.) The eye-pieces employed to produce an erect image for terrestrial telescopes admit of numerous arrangements: the simplest was shown in *fig.* 16, composed of three similar lenses placed at equal distances; but as there is no field-lens in this construction, the view is limited, though the chromatic dispersion is in part corrected by the middle lens, and may be totally destroyed by using an eye-glass of shorter focal length than the other two, and at a less distance from the middle glass. Another combination may be produced by two glasses so placed that the lens which receives the inverted image formed by the object-glass shall form an image on the other side, inverted with respect to the first, but erect with the object, and this image may be viewed with an eye-glass. This construction is shown in *fig.* 30. Let *i* be the inverted image formed by the object-glass, *o a* a pencil of light from the centre of the object-glass falling on the lens A; this pencil will be dispersed after refraction, and *a r* will be the path of the red rays, and *a v* of the violet; at the focal point *e* an erect image will be formed, and may be viewed by the lens E. Now the red rays at *r* falling on this lens will be less refracted than the violet at *v*, and will cross the axis in R, while the violet *v* crosses the axis at V; hence the dispersion will be greatly increased both by reason of the spherical aberration, and their greater refrangibility, so that objects seen through this eye-piece will be more fringed with colours than in any other construction.

Fig. 30.



(46.) The last construction might be improved by using two lenses disposed in a similar manner to the negative eye-piece (44,) instead of the lens E. These would enlarge the field and correct the dispersion; but as the spherical aberration is very great, two lenses should be used in place of the lens A, in order to make the combination complete; thus a four-glass eye-piece would be formed that is perfectly corrected, if duly proportioned distances and foci are used. This is accomplished in the following combination (*fig.* 31.) where the lenses are in the order of the letters A B C D beginning with the lens next the object-glass, and the distances, &c. of each lens are as follows: focal length of A = $1\frac{7}{8}$ = B = $2\frac{1}{2}$, C = 2 and D = $1\frac{1}{2}$, and their distances A B = $2\frac{1}{2}$, B C = $3\frac{5}{8}$, C D = $2\frac{3}{8}$. This eye-piece will be nearly free from chromatic dispersion and aberration. In a very good eye-piece of Ramsden's, the focal lengths were found to be of A = .0775, B = 1.025, C = 1.01, D = .79; the distances A B = 1.18, B C = 1.83, C D = 1.105. Stops should be placed between the lenses A and B, and a larger one between C and D; these stops are to prevent any false light from passing through the lenses to the eye. The more black stops there are introduced into a telescope, provided they do not hinder the pencils of light proceeding from the object, the better will the instrument perform.

Fig. 31.



(47.) The brightness of any object seen through a telescope, in comparison with its brightness when seen by the naked eye, may, in all cases be easily found by the following formula. Let n represent the natural distance of a visible object at which it can be distinctly seen, and let d represent its distance from the object-glass of the instrument. Let m be the magnifying power of the instrument, that is, let the visual angle subtended at the eye by the object when at the distance n, and viewed without the instrument, be to the visual angle produced by the instrument, as 1 to m. Let a be the diameter of the object-glass, and p be that of the pupil. Let the instrument be so constructed that no parts of the pencils are intercepted for want of sufficient apertures of the intermediate glasses. Lastly, let the light lost in reflection or refraction be neglected.

The brightness of vision through the instrument will be expressed by the fraction $\left(\frac{a\,n}{m\,p\,d}\right)^2$ the brightness of natural vision being 1. But although this fraction may exceed unity, the vision through the instrument will not be brighter than natural vision. For when this is the case, the pupil does not receive all the light transmitted through the instrument.

In microscopes, n is the nearest limits of distinct vision nearly eight inches; but a difference in this circumstance, arising from a difference in the eye, makes no change in the formula, because m changes in the same proportion with n.

In telescopes n and d may be accounted equal, and the formula becomes $\frac{a^2}{m^2 p^2}$.*

CHAPTER X.—*Stands for Telescopes—Method of making Grinding and Polishing Specula and Lenses—Method of centering and adjusting Lenses.*

(48.) To describe the numerous varieties of stands or supports for telescopes would be both prolix and trifling, as different artists generally adopt such contrivances as they think most likely to please the fancy of the purchaser; but the chief consideration in a scientific point of view is, to obtain a steady and immoveable stand free from vibration. To effect this, the instrument should be supported at both ends, to give steadiness, and to prevent its being affected by the wind; for every vibration will be increased in the same ratio as the amplification of the instrument, and produce a tremulous or dancing motion in the objects. Thus a superior telescope badly supported may be rendered inferior to a common one fixed on an immoveable stand. The materials of which stands are composed should be capable of transmitting as little vibration as possible; thus, the vibration of a frame of cast iron in one piece, although perfectly steady, would be sufficient to destroy distinct vision. Wooden stands are preferable, provided firm diagonal braces are used, so as to form immoveable triangular frames. Where iron is required for durability, plates of lead should be screwed between each piece to stop the vibrations. In reflecting telescopes the difficulty of preventing vibrations greatly impairs their value; for the metals, particularly the small one, are easily set in vibration, and unless the arm of the small metal is damped, the vision is frequently indistinct. When a Gregorian telescope has been taken from its stand, and placed on a lump of soft clay, the distinctness has been such as to enable a person to read a bill placed at 900 feet; while on the stand it could be read only at the distance of 650 feet, although no apparent tremor could be discerned when on the stand.

(49.) A good composition for the specula of reflectors is one of the most important desiderata in the making of telescopes. The qualities most in request are, a sound uniform metal, free from all microscopic pores; not liable to tarnish by absorption of moisture from the atmosphere; not so hard as to be incapable of taking a good figure and exquisite polish, or so soft as to be easily scratched; and possessing a high reflective power. The various compositions employed for specula differ more in the admixture of minor ingredients than in their essential materials. Copper and tin (bronze metal) are the metals mostly employed, with small quantities of arsenic, silver, and brass. The proportions generally employed are, copper 32 parts, grain tin 15, with the addition of two parts of arsenic to render it more white and compact. The Rev. Mr. Edwards, in a treatise annexed to the Nautical Almanac for 1787*, says that if 1 of brass, and 1 of silver, be used with only one of arsenic, a most excellent metal will be obtained, which is whiter, harder, and more reflective than any other he ever met with. With respect to the practical value of this composition we cannot speak, but having made specula for reflecting instruments ourselves, we can vouch for the goodness of the following, both with respect to the exquisite figure and polish it is capable of assuming, and its freedom from pores. To make this composition, take two parts of copper, as pure as it is possible to be procured; (for the goodness of the speculum will depend on the purity of the materials employed) this must be melted in a crucible by itself; then put in another crucible, 1 part of pure grain tin†. When they are both melted, mix and stir them with a wooden spatula, keeping a good flux on the melted surface to prevent oxidation: this metal must be quickly poured into the moulds, which may be made of founders' loom; the intended face always being downwards. Where the speculum is required particularly good, the best mode of casting is to have an iron mould made with a vertical tube attached on one side, and the bottom of the tube to end in a bulb; the melted metal is then to be poured down the tube, and will fill the bulb and mould, leaving a sufficiency in the tube to give pressure. The bulb being lower than the mould will retain any dense impurities, and the tube the lighter ones, while the speculum will be uniform and dense.

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* Barlow.

Having thus procured the speculum, the next thing will be to grind it to the required figure; this is effected on a convex brass or hard metal circular tool, carefully turned to a gauge of the required curve. This tool is fixed on a post or upright, and the speculum is held in the hand by means of a convenient holder cemented on its back. The grinding is then commenced with coarse emery powder and water, when the roughness is taken off by moving the speculum across the tool in different directions walking round the post: finer emery is used in the same way, till the surface of the speculum has become uniform. The next step will be to smooth it by means of fine washed flour emery, gradually passing from one degree to the next finer, and washing the tool and speculum between each application of emery, to prevent any gritty particles from scratching the metal. When the speculum is completed, and of the required figure, it is next to be polished. This is done either by taking a convèx tool similar to the grinder, or the grinder itself, and covering it with pure pitch evenly spread over its surface; while warm a concave tool of the same figure as the speculum is then worked over its surface wet. When the proper figure is obtained, washed putty (*i. e.* combined oxide of tin and lead) is poured on the pitch, and the speculum polished thereon by moving it as before. During the process of grinding and polishing, the tools must be carefully examined by the *gauge*, and if they happen to get out of the true figure, the speculum must be worked more on the edge, or middle, as the case may require. Instead of the vertical *post* above mentioned, a *lap* is sometimes employed, which produces a much better figure and more expeditiously. A lap consists of a common lathe communicating a slow and regular motion to a vertical mandral, on which the grinding or polishing tool is fixed; in using the lap, the artist is enabled to stand in the same place, and has more command over the work.

Lenses are ground precisely in the same manner as specula, but the polishing is different. Here the concave or convex polishing tool is made of brass, and when turned of a proper curve, a smooth thick piece of felt (cloth) is stretched over the tool and cemented to it; the outer surface is then imbedded with washed putty powder. After this is done, the lens, or block of lenses, is worked on it with cross motions; if the powder be employed too wet the fibres of the cloth will rise up, and polish not only the surface, but also the small hollows left in the grinding. This effect, from the nature of the polishing surface being heterogeneous, generally takes place to a greater or less extent when viewed by a microscope; these cavities being polished admit the light and disperse it, instead of it being collected as with a uniform surface. When these faults are visible to the eye, the lens is called *curdled*. If we are desirous of procuring an uniform and perfect surface, the polishing tool must be homogeneous, and the best material for its formation is good clean *bees wax*, hardened by the addition of *red sulphate of iron*, dry and finely washed. This composition, when of the proper temper, is melted over the brass tool; and when cold can be turned to the required curve. The advantage of this improvement, besides its uniformity, is that should any hard scratching particles insinuate themselves between the tool and glasses, they sink and are imbedded in the wax, and thus their injurious effects are prevented. The polish of lenses made in this manner is clear and defined when examined by a microscope; when the shadow of a bar is brought across them. This method is now employed by one of the first opticians in the metropolis.*

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* This treatise which is now very scarce, is republishing in the *Technological Repository*.

† It is most probable that the best proportions would be as their respective *atomic* weight, that is, thrice 32 of copper, and 58 of tin, as the metal would then be more intimately combined.

Centering of Lenses.—The centering of lenses for accurate instruments is of great importance, more especially for the object-glasses of achromatic telescopes. Different opticians employ their own methods, but one of the best is done by reflection: let the lens to be centered be cemented on to a brass chuck, having the middle turned away so as not to touch the lens, but near the edge, which will be hid when mounted; this rim is very accurately turned flat where it is to touch the glass. When the chuck and cement is warm it is made to revolve rapidly: while in motion a lighted candle is brought before it and its reflected image attentively watched. If this image has any motion, the lens is not flat or central: a piece of soft wood must therefore be applied to it in the manner of a turning tool, till such time as the light becomes stationary. When the whole has cooled, the edges of the lens must be turned by a diamond, or ground with emery. This method of centering and adjusting object-glasses by their reflected images, was laid before the public by Dr. Wollaston, and has been used by our first opticians for a considerable time.

Concluding Remarks on Telescopes.

(48.) The applications of the telescope to the purposes of man are so numerous, that their details would far exceed the boundaries of our treatise. Amongst its principal uses, however, besides those accompanying the descriptions of the various modifications of that instrument, may be enumerated the following:—The accurate determination of the longitude of the various places on the earth's surface, is ascertained by the telescope, by observing the immersions and emersions of the four satellites of the planet Jupiter; from thence, by the aid of a good chronometer, with the time of any known place, the situation of the unknown spot is determined. Before the invention of the telescope, navigators were compelled to keep within sight of the coast in sailing from one country to another, and thus were often endangered while passing an hostile or rocky shore; by the assistance of this instrument, the voyage is made direct to the intended place without fear or danger.

To the astronomer the telescope is his principal and most important guide. It enables him to determine, with precision, the transits of the planets and stars across the meridian. The computation of astronomical and nautical tables, to determine the revolution of the planets on their axes, and their relative polar, and equatorial diameters, is derived from observations by the telescope. We are by this instrument enabled to discover the analogy between the laws which govern the motions of the planets and those of our earth; their parallax, and from thence their distances. The aberration of light and the motions of the *siderial* systems in space, unfold wonders which must excite the imagination of the most profound philosophers in the highest possible degree. The harmony and simplicity displayed in such immense worlds prove the design and wisdom by which they were created; and the wonderful facts thus ascertained raise the most ordinary mind up to a sublime contemplation of the great Creator.

In surveying of land, the telescope is highly useful, and for this purpose is mounted on a stand, with an horizontal and vertical motion registering by divisions the degrees and minutes of inclination or position of the instrument. For the more accurate *reading off* these divisions, the two limbs are furnished with a *Vernier's scale.* Spirit levels and a magnetic needle are usually attached to the instrument; and from the purposes to which it is applied, a telescope with this mounting is called a *Theodolite,* derived from two Greek words θεομαι *to see*, and οδος *the way or distance,*—that is, an instrument *for seeing or determining distances.* The method by which the distances and heights of remote objects are ascertained is by measuring the angles subtended by the object, and computing trigonometrically therefrom.

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* The method of grinding and polishing lenses from diamonds for microscopes, by Mr. Pritchard, will be found in the Quarterly Journal of the Royal Institution, vol. ii, page 14, new series.

The continental writers have much exaggerated the powers and penetration of the telescope; indeed the time is not far distant, when it was gravely asserted, that works of art had been recognised in our satellite the moon. The fallacy of this circumstance may be easily shown to our readers by the following simple considerations.—Let a person direct the tubes of a telescope (without the glasses) to any celestial object, and there fix them; he will soon find that in a short space of time, the object will have removed from before the mouth of the tube. Now this motion of the celestial bodies, which is only *apparent,* arises from the revolution of our earth on her axis; and the quantity of this motion may be determined with facility, thus:—the earth is known to revolve once about her axis in 24 hours, and as every circle is supposed to be divided into 360 equal parts or degrees, the apparent time any celestial body takes to describe one degree, will be found by dividing the 24 hours by 360, which gives us 4 minutes as the time an object would pass the mouth of the tube if it only takes in one degree of the heavens.

Now, if we suppose the glasses to be placed in the tubes, the magnfying power of the instrument being 60, and we direct it (as before) to an object, as the moon, whose diameter is about half a degree, the time of her passing or transit will be one minute, if the field of view be, as in the ordinary telescopes, about 30°, which the moon would exactly occupy. If the power of the telescope be increased 10 times, the eyepiece having the same angle of vision, only $\frac{1}{100}$ part of the moon would be seen at once, 100 being the square of 10, the increased power of the instrument; and the time this portion of the moon would pass the telescope is 6 seconds. Again, if we increase the power 10 times, so that its linear amplification of an object is 6000 times, only a $\frac{1}{10000}$ part of the moon's surface could be seen in the field of view; or the planet Saturn, whose apparent diameter is 10 seconds, would just fill it, and the time of their passing the instrument would be only $\frac{6}{10}$ of a second.*

Having thus shown the amazing velocity a planet passes the mouth of a telescope with these high powers, we shall next proceed to point out the apertures and amplification necessary for observing some given measure on the surface of the moon. First, we must determine the angle every object must subtend to the eye, in order to render it visible: this is found on an average for different sights to be one minute, that is, when an object is removed from the eye about 3000 times its own diameter it will only be just distinguishable. From this we can now determine the extent of the smallest part of the moon's surface discoverable by the unassisted eye. Its real diameter being 2100 miles, which divided by the number of minutes which its apparent diameter subtends, (viz. 30,) gives us 70 miles as the measure of the least distinct spot seen by the naked eye; therefore, we know that, if a telescope magnifies 70 times, we can just discern a spot one mile in diameter on the moon's surface; and to recognise any object 10 feet in diameter, we shall find by this rule the magnifying power of the telescope must be 37100 times, and the diameter of an object-glass or metal for such an instrument may be found by the method described in (18); which if we suppose a pencil of rays $\frac{1}{50}$ of an inch in diameter will admit sufficient light to the eye, the diameter of the speculum must be 62 feet, and its focal distance 309 feet, when an eye-glass of $\frac{1}{10}$th of an inch is employed. These calculations must convince the reader of our inability for making such observations; for if the impossibility of procuring such enormous instruments were overcome, they would be so unwieldy as entirely to prevent our using them.

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* It is necessary for us to state, that, for the observation of small stars and nebulæ with reflecting instruments this power is occasionally employed; as even under these circumstances they possess no sensible diameter, and a regular motion is communicated to the telescope in the plane of the equator by means of a clock or other mechanism.

CHAPTER XI.—*Microscopes.—History of.—Theory of Single.—Magnifying Powers of.—Illumination.—Forms of Lenses for Microscopes.—Globules.—Globule and Lens compared.—Mr. Pritchard's Diamond and Sapphire Microscopes.—Table of their Magnifying Powers.*

(49.) THE history of the microscope, like that of many other valuable inventions, has been veiled in considerable obscurity by the lapse of time: and the discovery, amongst the moderns, of so useful a class of optical instruments, has been claimed by several individuals. But it seems certain that the ancients were acquainted with the microscope, at least in one of its forms, as appears from the following passage in Seneca:—"Letters, though minute and obscure, appear larger and clearer through a glass bubble filled with water." Although we have no account that they understood the laws by which the magnifying power of the spherule was effected, yet their acquaintance with its application appears certain.

The invention of the microscope is attributed by the celebrated Dutch mathematician, Huygens, to a countryman of his, named *Drebell*, (for it must be observed that it was entirely lost in the middle ages.) He constructed them about the year 1621, or 31 years after the invention of the telescope. According to Borelli, the microscope was invented by Jansen, the reputed contriver of the telescope, who presented some instruments of his first construction to Prince Maurice, and Albert Archduke of Austria. These instruments were six feet in length, and consisted of a tube of gilt copper, one inch in diameter, supported by thin brass pillars in the shape of dolphins, on a base of ebony which was adapted to hold the objects to be examined. Of the internal construction of this microscope we have no precise account; though there is reason to think that it was nothing more than a telescope converted into a compound microscope.* Viviani, an Italian mathematician, also expressly informs us in his Life of Galileo, that this great man was led to the construction of the microscope from that of the telescope; and in the year 1612 he actually sent a microscope to Sigismund, King of Poland. The honour of making a microscope of two double convex lenses, like those at present in use (without their field-glass,) seems to belong to F. Fontana, a Neapolitan, who, in a work published in 1646, claims it as his own, and dates the invention from the year 1618.

The numerous forms of microscopes which have at different intervals been constructed, may be included in *three* distinct classes, however varying as to their external appearances;—these are, *Single*, *Compound Refracting*, and *Compound Reflecting* Microscopes.

(50.) *Theory of the single Microscope.*—It must have been observed by most persons, that when the distance of an object from a spectator is decreased, we are enabled to define its parts more readily, and that it appears larger; thus if we look at two men, the one 200 feet, and the other only 100 feet from us, the former will appear only half the height of the latter; or the angle which the latter subtends to the eye of an observer will be twice that of the former. Hence we must conclude, that the nearer we can bring an object to the eye the larger it will appear. Now, if we have to examine a very minute object, and in order to render its parts distinguishable if we bring it very near to the eye, (suppose one or two inches,) it will become very indistinct and confused. This effect is produced by the great divergency of the rays of light from the object, and the power of the crystalline lens of the eye not being sufficient to collect the rays, whereby an image of the object may be formed on the retina, at the proper distance on the back of the eye. Now if we employ a single microscope, which consists of a convex lens usually made of glass, (though it would have the power of magnifying or increasing the angle, if made of any other transparent substance, but in a different degree,) and mounted in a brass setting, and place it between the object and the eye, the former being in the focus of the glass, the diverging rays from the object will be refracted and rendered parallel by the lens: and thus we shall obtain a distinct and near view of the object.

To exemplify this fact more simply, let A (*fig.* 32) be a small object which we desire to examine. Now, bundles of rays will diverge from every part of this object in all directions, as shown by the lines A *r*, A *r*. And if there be placed before this object a convex lens B, whose focal distance is B A, those rays diverging from the object which fall on the

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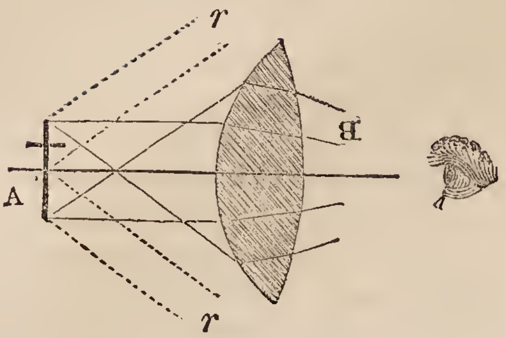
[*] See *Edinburgh Encyclopedia*, art. *Microscope*, for a more enlarged history of this instrument.

Fig. 32.



surface of the lens will be refracted at its two surfaces, and emerge from it nearly parallel to each other; consequently the object is capable of being received by the eye on the side B, under a greater angle than it could be seen without the lens.

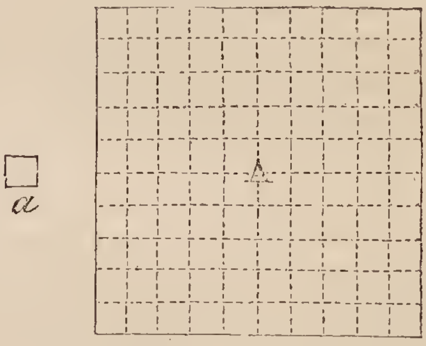
(51.) Having shown that the magnitude is dependent on the distance of the object, we must now investigate the increase obtained by the employment of lenses; this we shall find to depend on the difference of the distance of the object from a lens, and the *distance* when seen without its assistance. This latter distance, however, (that is, the nearest distance at which we can distinctly discern a minute object,) is variable in different persons, and indeed in the same individual at different periods of life; therefore it becomes necessary to assume some measure as a standard, before we can express the amplifying power of a lens so as mutually to comprehend the same idea of the real magnitude of an object. In old optical works the distance at which the authors usually reckon the place of distinct vision is *eight* inches, but in latter works a shorter distance has sometimes been assumed. These distances are, however, variable, and are often determined by the writer to suit his own vision.

Dr. Brewster, in his optical works, has taken *five* inches as the distance for minute objects; "for," says he, "if we examine a small coin or insect, we naturally hold it nearer than an ordinary object, in order to inspect it more carefully." Again, we find that Professor Amici, of Modena, in determining the amplifying powers of his instruments, has adopted a standard of *ten* inches as the focus of the eye under ordinary circumstances. This latter number has also been employed by some authors, principally for the reason, that being a decimal number it offers greater facilities as a multiplier or divisor in optical investigations. We shall therefore assume this as our standard in subsequent calculations. Those who may think proper to adopt a shorter standard, as that of Dr. Brewster, will simply have to divide the magnifying powers hereafter given by *two*, which will give the required power according to that standard.

With this *decimal standard* we can now determine the magnifying power of lenses of any focal length, or formed of any substance (*media.*) Thus, if we are in possession of a lens which requires, for distinct vision, the object to be *one inch* from its centre (in a double convex,) we must divide the standard *ten* by *one*, which will give ten, therefore such a lens will increase the diameter of an object seen through it ten times: again, if we have another lens which requires the object to be the *twenty-fifth* of an inch distance from it, the power will be 250, which is found by simply adding a cipher (0) to the denominator of the fraction which expresses its distance (viz. $\frac{1}{25}$), and removing the numerator.

Now, in the first case, when we employed an inch lens, we called its magnifying power ten, from the number of diameters or increase of length and breadth of an object by the use of that lens. This, which is its usual denomination, is very defective; for we shall find, that if we draw the real object and its magnified view, the latter will occupy 100 times the surface of the former, so that the most proper expression of the magnifying power of this lens would be 100. This will be manifest from an inspection of *fig.* 33, where *a* is the real size of a square object, and A its magnified view. Now, the dotted lines will show that it is ten times longer and broader than the object *a*, but it will be found to contain 100 such dotted squares all equal in size to the original square *a*: hence the proper magnifying power is 100, or the length squared.

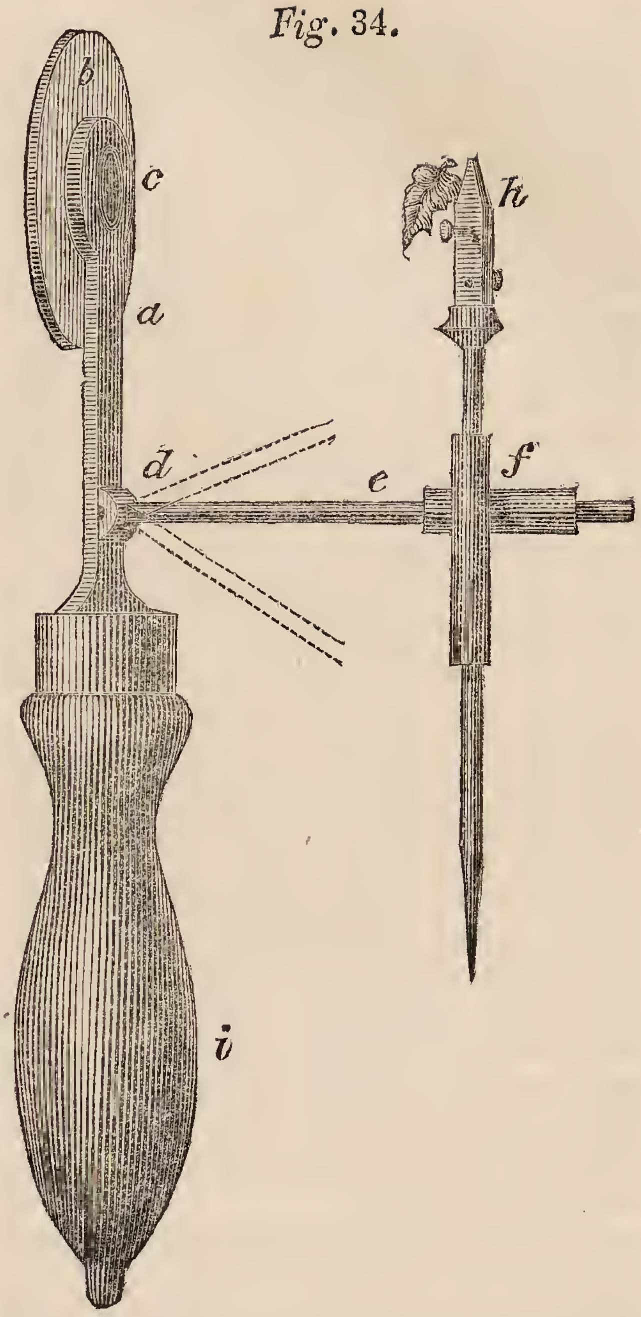
Fig. 33.



The following table will exhibit the magnifying powers of lenses from two inches focus to the $\frac{1}{10}$th of an inch, according to the *decimal* standard; the column marked superficial is the true increase in size of an object by a lens of the focal length shown in the first column, formed from any transparent substance.

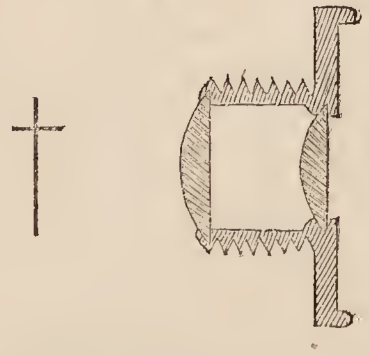
Focal lengths in inches.	Magnifying Powers.	
	Linear.	Superficial.
2	5	25
1½	6.6	43.5
1	10	100
¾	13.3	176.8
½	20	400
¼	40	1600
⅛	80	6400
$\frac{1}{10}$	100	10000

Fig. 34.



(52.) In a single microscope the circumstance requiring the greatest consideration, is the formation of the lens. The grinding of microscopic lenses is effected in a similar manner to that described in page 30, except that a quicker revolving motion of the grinding tool is necessary, and in the polishing great care is required to keep them of their proper figure (spherical.) When completed, they should be as thin as they can possibly be procured with a sufficient aperture, the reason for which will be hereafter explained. In the mounting or fitting up of the lenses, the intention of the instrument, whether it is to be used as *a hand, a pocket, or a table microscope*, must be considered. From the arrangement of the mountings to suit the nature of the objects to be examined, microscopes often derive their name, though the principle in all of them is similar, as *Botanical, Mineralogical, Anatomical, Natural History, Aquatic, Transparent*, or *Opaque Microscopes*, all of which are either *single* or *compound.* The focal distances of the lenses mounted in the single microscopes are usually 1½ inch, 1 inch, ½, ¼, $\frac{1}{10}$th, and $\frac{1}{20}$th of an inch, though the two last lenses seldom accompany the *pocket botanical.* One of these microscopes is shown in *fig.* 34, which is capable of having either lens adapted to it, and from its extreme simplicity is not liable to get deranged: *a* is the brass stem, having an internal screw cut in it to receive the cell *b* containing the lens *c*; at *d* there is attached an arm *e*: this arm being jointed at *d* is capable of lying flat when out of use, or can be altered to suit any other convenient position for holding the object, as shown by the dotted lines; on the round arm *e* is a sliding tube *f*, fixed to another sprung tube at right angles; this latter tube carries the forceps *h* moveable in every direction with respect to the lens *c*, either as to distance or position; the handle *i* is screwed to the stem *a* when in use. If a lens of a different power is required to be employed, the cell *b* may be removed, and another with a deeper or shallower lens screwed into its place. If low power is wanted, as for the inspection of flowers, the cell with the lenses (shown in *fig.* 35) should be used, as that combination is superior to a single lens of equivalent power. This cell contains two plano-convex lenses, with their plane sides turned to the eye, the focus of the nearest being 2 and the other 3, with a distance between them equal to 1 of such parts; or their sidereal focal distances are usually about 1 inch

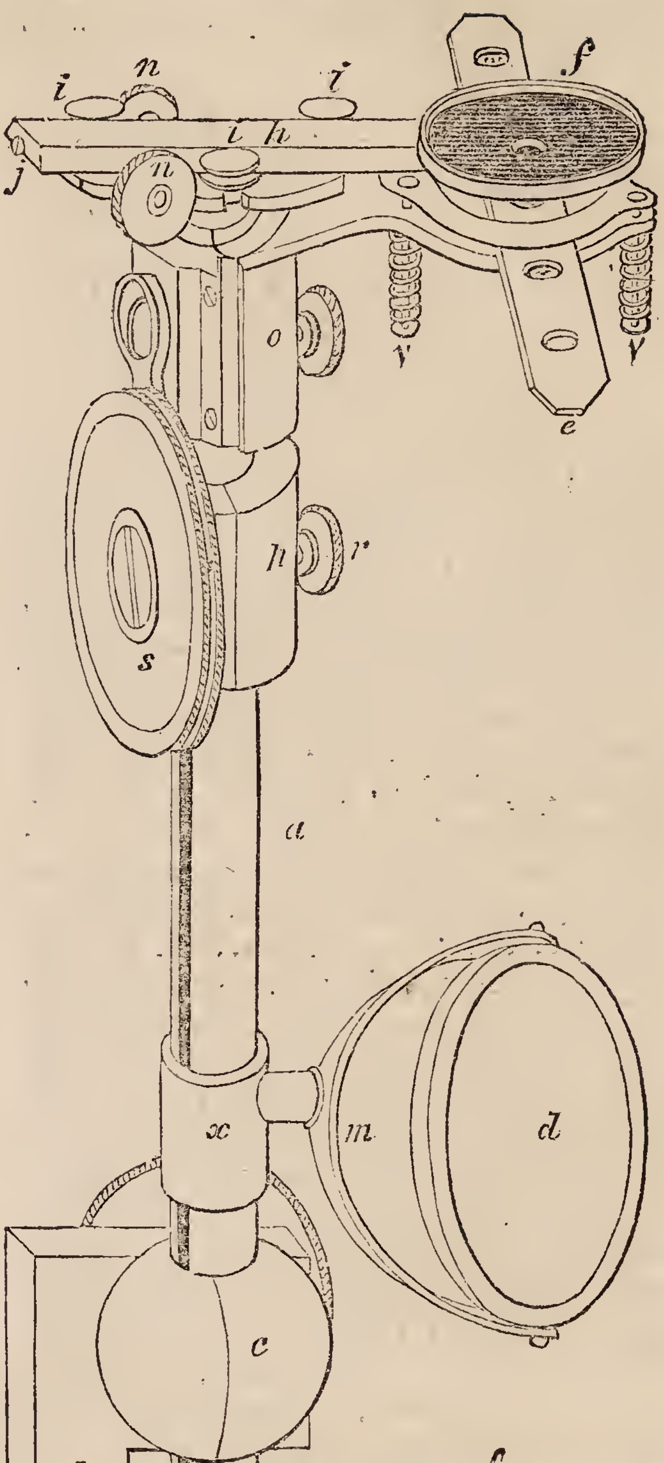
Fig. 35.



and 1½ inch. The advantage of this combination is, an increase of field (or number of objects seen at once,) and the diminution of aberration, as explained at page 13, (22.)

(53.) When microscopes are required for general purposes they are mounted on a stand, and usually called table microscopes. These, as we before remarked, are various, and accompanied with different implements for the purposes of dissecting and holding the various objects to be examined. The moveable parts of these, and of all microscopes, should be free from shake; while all their adjustments should be easy, and without *jolts* or *starts* when moving. The lenses should be so mounted as to be easily approached by the objects; and at the same time not too remote from the eye, as the field of view will always be increased with the nearness of the lens to the eye. Lastly, all shining or bright surfaces should be blackened on the anterior side of the lens, so that no glare or false light can be reflected towards the observer. For adjusting the object to the focus of the lens, a rack and pinion, or screw adjustment, is generally employed. These should be very fine, and sensible of the smallest motion.

Fig. 36.



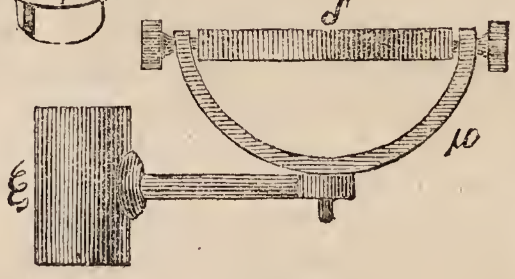


Fig. 37.

(54.) With these preliminary remarks, we shall now describe a single microscope, suitable for the examination of any kind of object, in which the adjusting motions are of a novel and simple construction, as neither screw nor rackwork are employed. This instrument has recently been designed and executed by Mr. Holtzaffel, jun., on the basis of the microscope of Mr. Pritchard, who constructed the optical part. It consists of a circular brass stem *a*, (*fig.* 36,) capable of being fixed in every possible inclination, by the ball *c* attached to the clamp *b*, by which the whole may be firmly fixed to a table or board; the circular mirror *d*, working in the arm *m*, is attached to the tube *x* in the usual way. But it would be more convenient if applied in the manner shown below at *fig.* 37, where δ is the mirror, μ the arm, and ξ the tube: by this means the mirror is moveable in any position, to throw the reflected light on the slider or object *e*, (*fig.* 36,) by the screw-heads fixed into the mirror case, which is loose in the arms. The whole slides upon the stem, that it may be placed nearer to, or farther from the object, according to the intensity of the light required for the various subjects under examination. The back of this reflector is flat and polished, so that a monochromatic light reflected from the brass may be employed when necessary.

The lenses are mounted in cells as shown at *f*, and are screwed into the dovetail bar *h*, sliding between three stout pins *i*, *i*, *i*; the nearest one having a strong spring on the under side, which keeps the bar in close contact with the other two, without any shake. The bar is moved across the object by either of the milled nuts *n*, *n*, which, instead of having a pinion as usual, have a spring wound round their axis attached at each end to the bar *h*, with an adjusting screw to regulate the tension at the end *j:* the bar may also be turned round on the central pin, fitted in the top of the stem *a*, and thus a traversing motion in every direction may be given to the bar and lens, without disturbing the object, or altering in the least the distance between it and the lens.* The adjustment of the focus is first made by sliding the stage pieces *o*, *p* by the hand, until the object is seen nearly distinct; the thumb screw *r* being then turned, fixes the lower piece *p* to the stem *a;* then, by means of the large milled head *s*, the final adjustment is made by the intervention of a connecting bar *t*, attached to the stage piece; this bar works on an elastic eccentric movement under the milled head *s*, so that an adjustment of any small quantity can be obtained with extreme precision (similar in principle to the infinite lever.) The slider containing the object is kept close to the stage by two heliacal springs *v v*. A condensing lens and a pair of forceps are made to fit in the piece *o*, and can be employed either with or without the stage plate, which may be entirely removed by the thumbscrew in front, when necessary.

(55.) The quantity of light necessary to be employed in using a microscope, is dependent on the nature of the object under examination, and on the magnifying power of the lenses necessary for its developement.

The light, therefore, from the sun or a lamp may be condensed by a convex lens attached to the stage; or a condensing lens may be substituted in place of the reflector, and the light from the lamp allowed to fall on it direct. This light, in order that it may not produce the injurious prismatic colours, may be made of one uniform colour. In the monochromatic lamp introduced by Dr. Brewster, spirit of wine mixed with a little water is used. This produces a yellow flame, which he states is less injurious to the eye than that of the ordinary light. The glare of the common light may be removed by placing a plate of greyed glass under the object, by which means a cloudy light is obtained. This effect may likewise be produced by a circular disc of plaster of Paris used to reflect the light instead of the mirror; this contrivance, which was suggested by Dr. Goring, forms an excellent reflector for many interesting objects.

(56.) Having described the mechanical arrangements belonging to a microscope, it remains for us to treat of the magnifiers and the apertures necessary to be used. But before so doing, it may generally be remarked of all microscopes, that if the magnified representation of any object be not truly depicted on the retina of the eye, with its various parts and markings, of their true form and proportion, their outline distinctly defined, having a sufficient degree of brightness, and of their proper colour, the instrument cannot be depended upon to give a correct idea of the nature of the objects to be examined.

(57.) The common form of the magnifiers employed for microscopes are double-convex, and from what is stated in (20.) page 12, they should be made as thin as possible; for the *wandering* or *spreading out* of the rays proceeding from an object when refracted by a lens with spherical surfaces, whereby an indistinctness is produced in its image, will be decreased as the square of the thickness of the lens employed, while the loss of light in passing through the substance of the lens is increased in the same proportion as its thickness. But in page 12 the object was considered at an infinite distance, and the image was formed in its focus. Now in the microscope the *object* is placed in the focal point of the lens, and the rays are rendered parallel when refracted by it; hence, in order to diminish the aberrations * as much as possible when we employ lenses of unequal curvatures or plano-convex, the flattest side of the lens must be turned towards the *radiant* (object,) when the aberrations will be as there stated; or, according to the results of Mr. Herschel's recent investigation, they are as follows, calling the aberration of a lens of the best form and position, that is, with the convex side turned to parallel rays, and the radii of its surfaces as 1 to 6, *unity*.

Best form	1.000
Double convex or concave .	1.567

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* The nuts *n n* might have a short ivory handle fixed to their axis, so that the same hand might turn it right and left, or backwards and forwards, at the same time.—See *Quarterly Journal*, vol. ii.

* The word *aberration* is derived from two Latin words, *ab*, *from*, and *erro*, to *stray* or *wander;* that is, the wandering or strayed rays of an object from the true image.

Plano-convex or concave, with the spherical surface exposed to parallel rays	1.081
Plano-convex or concave, the flat surface being exposed to parallel rays	4.2

In all these cases the lenses are supposed of equal thickness.

(58.) Now where the lens is of short focus, the proportional quantity of curvature required to be used with the necessary aperture will be greater than in lenses of long foci; besides which, as the quantity of light admitted by small lenses when thrown on the retina is attenuated in proportion to its magnifying power, it therefore becomes important to make it pass through glass of the smallest possible thickness, and of the utmost transparency. To show that this quantity is considerable, we shall give the thicknesses of two lenses of equal focus, each of which has an aperture sufficient for the vision of any class of objects.

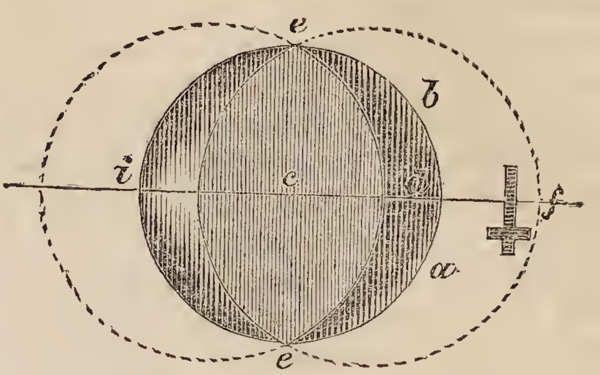
Double Convex Lenses.	Thickness in hundredth of an inch.	Focus. Inch.
Ordinary manufacture	9	$\frac{1}{10}$
Executed in the best manner	$4\frac{1}{2}$	
Difference in thickness	$4\frac{1}{2}$	

From this statement it appears, that if we suppose the transparency of the glass in both lenses to be equal, the light lost in passing through the first will be double that of the last.[*]

(59.) When lenses of higher magnifying powers than those of the $\frac{1}{20}$th of an inch are required, small glass spherules are frequently mounted for microscopic purposes instead of them. These glass spherules or globules were first substituted for lenses, by Dr. Hooke, who describes the method of making them in his *Micrographia*, published in 1665.

(60.) The effects of employing globules practically, independent of the increase of aberration arising from their thickness, may now be considered. Let *d f* be the focal distance of a double convex lens *c d e* of crown glass, from its surface *d* to an object at *f*, (*fig*. 38,) which may be supposed equal 48,[†] or $\frac{1}{48}$th of an inch focus, and let its thickness be equal to the radius or 60. Now, when a globe of glass[‡] is employed, the focal distance will be half the radius measured from the surface nearest the object, that is *a f*, therefore, the radius of a globe of equal magnifying power to the lens must be $\frac{2}{3}$ of *c f* or $\frac{\overline{30 + 48} \times 2}{3} = 52$.[*] Hence the distance between the object and the surface of the globe *i*, *c*, *b*, *a*, *e* will be only 26 parts (or $\frac{1}{92}$th of an inch,) that is only half the distance a lens of equal power would be removed from the object nearly. Therefore, in adjusting the spherule to distinct vision, the liability of touching the object is doubled, and the danger of destroying it altogether by too near an approach is increased in the same proportion. While for viewing animalcula or moist objects, which require a thin plate of talc or glass over them to prevent the lens from being dewed by evaporation, and likewise to keep the surface flat, these disadvantages would in many cases render them useless. And the aberration is increased in the globule, both by its increase in thickness and the diminished radius of curvature, which also increases the distortion of its figure in the same proportion, (*viz.* inversely as the squares of their radii.)

Fig. 38.



(61.) The magnitude of the apertures used in small lenses is dependent on their aberration, and the nature of the object under examination; while the brightness of objects seen through a microscope (in regard to illumination) will be directly as the areas of their respective apertures. Hence, by whatever means the aperture can be increased without indistinctness, the instrument will always be improved.

For this purpose various contrivances have been suggested by many scientific individuals to improve the single microscope, by reducing the spherical and chromatic dispersions, and increasing the field of view. And here it may be remarked, that these improvements become of considerable importance in the investigation of minute natural objects, from the fact, that in the vision of objects through single microscopes we look at the *real* object; but in all compounds, however perfect their construction, we only see a magnified representation or *picture* of the object, which it is evident in point of accuracy cannot be relied upon with the same confidence as a magnified view of the object *itself*.

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[*] The first lens here measured was taken from those made in the shops; the latter was executed expressly to examine the difference of the performance of lenses of different thicknesses, and experimentally proved the goodness of thin lenses,

[†] This number is here taken to avoid fractions.

[‡] The focus of an hemisphere of plate glass with the convex side to parallel rays is $\frac{4}{3}$ of the radius.

[*] See *New Elements of Optics*, by B. Martin.—Theorems 429 and 552 for demonstrations.

(62.) Dr. Brewster, in his *Treatise on New Philosophical Instruments*, speaking of single microscopes, says, "We cannot expect any essential improvement in that instrument, unless from the discovery of some transparent substance, which, like the *diamond*, combines a high refractive power with a low power of dispersion."[*] This substance has subsequently been formed into lenses by Mr. A. Pritchard, at the suggestion of Dr. Goring, who caused Mr. P. to commence the undertaking in July, 1824. The first diamond lens was completed at the end of that year. The difficulty of working this substance into a perfect figure was subsequently overcome. Mr. Pritchard finished the first diamond microscope in 1826; the focal distance of this magnifier, which is double convex, is about $\frac{1}{30}$th of an inch. Of the value and importance of the introduction of this brilliant substance for the formation of single microscopes, Dr. Goring states, "I conceive *diamond lenses* to constitute the ultimatum of perfection in the single microscope."[†]

The principal advantages of employing this brilliant substance in the formation of microscopes, arise from the naturally high refractive power it possesses, whereby we can obtain lenses of any degree of magnifying power, and that with comparatively shallow curves; the indistinctness occasioned by the figure of the lens is thus greatly diminished, and the dispersion of colour in the substance being as low as that of water, renders the lens nearly achromatic.

The curvatures of this substance and that of glass to produce the same amplification are shown in the following figures, where is also represented a lens of sapphire of the same power. This, next to the diamond, possesses all the qualities requisite for the formation of perfect magnifiers, and presents less difficulties in their construction; hence their expense is considerably lessened.[*] The lenses shown in the figure are plano-convex, having their flat sides exposed to the radiant (object,) which is the best position for divergent rays: their diameter is equal to their focal lengths, which is absolutely necessary for most transparent objects when the lens is less than $\frac{1}{20}$th of an inch focus. *Fig.* 39 is the diamond magnifier, which is the thinnest lens of the three; *fig.* 40, the sapphire; and *fig.* 41, the glass lens of the same power, but whose thickness being greater than the others has considerably more error by figure, (spherical aberration.)

Fig. 39.

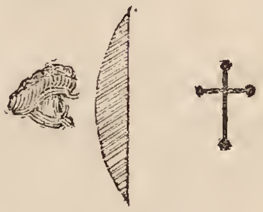


Fig. 40.

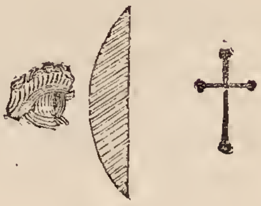
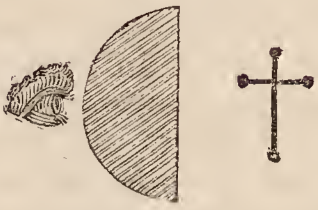


Fig. 41.



The aberration or indistinctness of lenses of the same aperture and substance are inversely as the squares of their radii. But when we compare a lens of diamond and glass of equal thickness their longitudinal aberrations are different.[†] And as the actual confusion is as the areas of the little circle into which every mathematical point on the surface of the object is spread over, it will be as their squares, or as .912 for the diamond, and 2.755 for the glass; and by multiplying these numbers by the squares of their radii inversely, it will be found that the indistinctness of a diamond lens of equivalent power is about *one twentieth* of that of a glass lens.

The magnifying powers of Mr. Pritchard's sapphire microscopes are shown in the following table, where their focal lengths, with their linear and superficial amplification, are given according to the decimal standard.

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[*] Page 402. 1813.

[†] *Quarterly Journal*, vol. xxii. p. 280.

[*] The same artist has also formed lenses of the other precious stones, but without any peculiar advantage, many of them producing two magnified images by double refraction, when their axis is not the axis of double refraction. The diamond has a very peculiar structure different from the other precious stones, some of which, when made into lenses, give a kind of treble image.

[†] See p. 13.

Table of Magnifying Powers of Sapphire Lenses.

Focal Lengths.		Magnifying Powers.	
Parts of an Inch.	Hundredths of an Inch.	Linear.	Superficial.
$\frac{1}{10}$	10	100	10000
$\frac{1}{15}$	..	150	22500
$\frac{1}{20}$	5	200	40000
$\frac{1}{25}$	4	250	62500
$\frac{1}{30}$	..	300	90000
$\frac{1}{40}$	2½	400	160000
$\frac{1}{50}$	2	500	250000
$\frac{1}{60}$	..	600	360000
$\frac{1}{70}$	..	700	490000
$\frac{1}{80}$	1¼	800	640000
$\frac{1}{100}$	1	1000	1000000

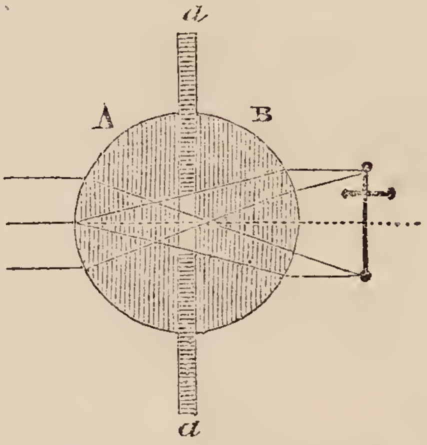
(63.) This table, which contains the foci of a complete set of sapphire microscopes, will also exhibit the amplification of a single microscope made of glass, or any other substance, by finding its focal length, and observing the number that stands against that measure. In glass, the curvatures being required so much greater than for the sapphire lenses, to produce the same magnifying power, very few have been made to exceed the $\frac{1}{60}$th of an inch focus when double convex, or the $\frac{1}{25}$th of an inch when plano-convex. In mounting the sapphire and diamond lenses, there are advantages which glass lenses do not possess. Their extreme hardness enables them to be burnished into brass settings, which is very difficult with those of glass. This facility of mounting renders them also more extensively useful in experimental research, from their capability of being applied in every possible way, with regard to the *object*, the *light*, or the *eye*.

CHAPTER XII.—*Periscopic Microscopes.—Dr. Wollaston's.—Dr. Brewster's.—Mr. Herschel's for Mineralogy.—Aplanatic.*

AMONGST the various contrivances that have been employed for the improvement of the single microscope, by diminishing its spherical aberration and increasing the field of vision, the following deserve particular attention:—

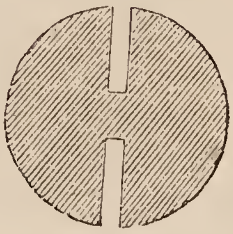
(64.) Dr. Wollaston, when treating of the single microscope, says, "The great desideratum in employing high magnifiers is a sufficiency of light, and it is accordingly expedient to make the aperture as large as is consistent with distinct vision. But if the object to be viewed is of such magnitude as to appear under an angle of several degrees on each side of the centre, the requisite distinctness cannot be given to the whole surface by a common lens (*of glass*,) in consequence of the confusion occasioned by the oblique incidence of the lateral rays, excepting by means of a very small aperture, and proportionable diminution of light."* To effect this purpose with glass lenses and render them *periscopic*, he unites two plano-convex lenses together by their plane surfaces, interposing a circular plate of brass between them, with a circular aperture in its centre, as shown in an enlarged section (*fig.* 42), where A and B are the two lenses fixed on each side of the perforated plate *a*, *a*. The diameter of the aperture in this plate which Dr. Wollaston found to answer best, was *one-fifth* of the focus of the combined lenses, and the visible field of view was about 20 degrees in diameter.

Fig. 42.



(65.) It has been remarked, that by doubling the number of surfaces in this combination a loss of light is occasioned. To remedy this inconvenience, and at the same time extend the limits of distinct vision, Dr. Brewster has ingeniously suggested the interposition between the lenses of a fluid of nearly the same refractive power as the glass. This fluid may either be oil of turpentine, castor oil, or Canada balsam; or the same effect might be produced more perfectly by grinding a groove in the circumference of a sphere or thick double convex lens, as shown in section *fig.* 43.

Fig. 43.



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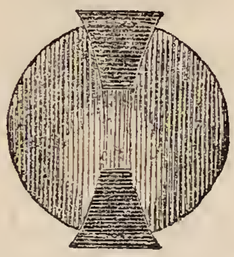
* *Philosophical Transactions*, 1812, vol. cii. p. 375.

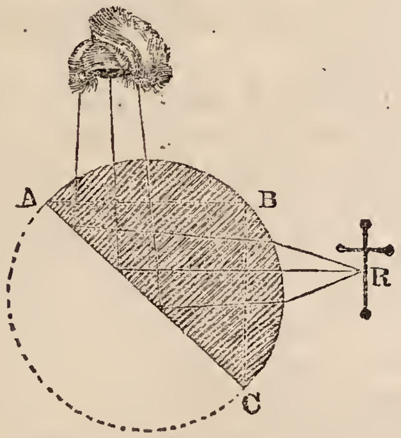
(66.) Dr. Brewster has also suggested another improvement in this combination when we wish to diminish the aberration of colour; this is effected by means of two double convex lenses, having their internal surfaces of such radii as to form a concave lens in the aperture at the centre, which, when filled with a fluid of a different refractive and dispersive power from the glass, shall correct both kinds of aberrations. This combination is shown in *fig*. 44.

Fig. 44.



(67.) A plano-convex lens is well known to possess only one-fourth the superficial magnifying power of a double convex of like curvature. But Dr. Brewster has proposed a method of applying the former lens so as to produce the same magnifying power as the latter, by using it in the position shown in *fig*. 45, where A B C is a hemispherical lens of half an inch radius, of which the inclination is as shown, with a microscopic object R before it, and the eye placed above. Now the rays issuing from the object R will, after refraction at the surface B C, fall upon the plane side A C, where they will be reflected up to the surface A B and suffer a second refraction. As the incidence of the rays upon A C is about 45° when the lens is hemispherical, and when the apertures are small, the rays will always be reflected at a greater angle than that at which total reflection takes place;* so that none of the rays will be lost by transmission at the surface A C. In this lens the source of error which arises from bad centering in a double convex lens is entirely obviated, as the surfaces through which the incident and emergent rays pass are ground simultaneously in one tool. To adapt the periscopic principle to this lens, we have only to remove the polish from an annular space of the plane side to exclude the lateral or obliquely incident rays.

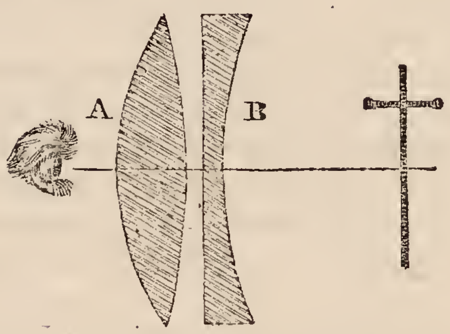
Fig. 45.



(68.) The reflecting lens A B C Dr. Brewster also proposed to be employed as a *diagonal eye-piece;* for it may be considered as composed of a right-angled prism (shown by the dotted lines) A B and B C, and instead of an ordinary prism and two plano-convex lenses having five plane surfaces and two convex ones, the hemispherical lens has only one plane side and two convex, thus avoiding a great loss of light by reflection from such a number of surfaces.

(69.) Mr. Herschel's periscopic combination of lenses consists of a double convex lens of the best form (that is with the radii as 6 to 1,) but in its worst position. This lens is to be employed for that next the eye, as A, *fig*. 46, and a plano-concave lens B, with its concave side next the object; the proportion of the foci of the two lenses being as 5 to 13. By this construction a very extensive field of view is obtained with moderate distinctness. In *reading* glasses and magnifiers of low power, a distinct field of 40° on each side the centre may be obtained. In the investigations of *Entomology* and *Mineralogy*, or in the examinations of *Crystals*, this combination is of great value, from the extent of objects seen at one view; and although the spherical aberration is increased,* yet by the contrary prismatic refractions of the two glasses the edges are nearly freed from colour.

Fig. 46.



(70.) When the rigorous destruction of aberration for the central rays is required, Mr. Herschel has proposed the aplanatic combinations shown in *figs.* 47 and 48, where the aberration of rays parallel to the axis is entirely destroyed. The lenses C in each of these cases are double convex, with the radii of their surfaces as 1 to 6, having the most convex side next the eye. The radii of the meniscus lenses D D are in the first case as 1 to 1.70, and in the other case as 1 to 3.957, with their concave sides exposed to the object; the focal lengths of the two lenses in *fig.* 47 are as 10 to 17.82, and in *fig.* 48, as 10 to 5.497. The compound focus of the lenses being 6.407, and in (*fig.* 48) 3.474.

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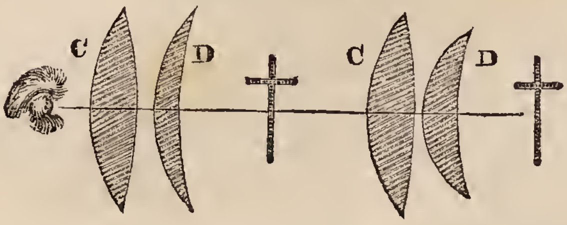
* The sine of the angle of total reflection being equal to $\frac{1}{\text{index of refraction}}$ it will be 41° in crown glass. *Edin. Phil. Jour.* vol. iii. p. 75.

* 22,302 times greater than that of a single lens of equivalent power of the best form and position.—*Philosophical Transactions*, 1821, p. 248.

Figs. 47 *and* 48.



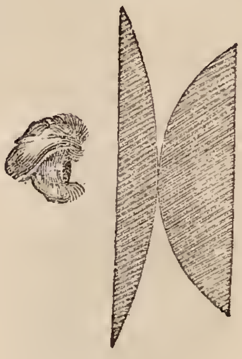
(71.) The superior effect of these two latter combinations (which offer little impediment in their practical construction) has been sufficiently tried to warrant their application, where the rays are required to be as perfect as the case will admit. As object-glasses for compound microscopes when executed of short focus, they are, in many respects, equal to an achromatic; indeed quite so when the latter has only the same aperture as this combination. The details of their application will, however, be pointed out in treating on that instrument.

(72.) In the construction of burning glasses, this combination appears from an experiment made with one of 3-inch diameter, by Mr. Herschel, to be far superior to a single lens, as the temperature was raised much greater than by any, even of the best form, although it is certain some portion of heat must have been lost in passing through two lenses. Hence it is evident, that by far the greater portion of heat is lost by the aberration of the lens.

(73.) When we are desirous of decreasing the aberration in a lens, Mr. Herschel has shown that by employing two plano-convex lenses of equal focus with their convex sides in contact, the aberration will be only 0.603, while that of a lens of equivalent power is *unity*.

But in order to diminish the aberrations as much as possible, without employing a correcting lens, the foci of the two plano-convex lenses should be in the proportion of 1 to 2, 3, when the aberration will be 0.2481, or about one quarter of an equivalent lens of the best form. This combination, which is shown in *fig.* 49, has likewise the advantage of decreasing the distortion of the object, and diminishes the prismatic dispersion.

Fig. 49.



CHAPTER XIII.—*Fluid and single Reflecting Microscopes.*

(74.) FLUID microscopes have, in the absence of more permanent instruments, been resorted to with unexpected success. The first account of these microscopes is given in the *Philosophical Transactions*,* by Mr. Stephen Grey. They consist of a drop of water placed in a perforated plate of brass, the edges of the hole being made as thin as possible by forming a cavity on each side of the plate, so that the fluid forms a convex lens.

Dr. Brewster has proposed other fluids in the place of water, to render fluid microscopes more perfect. The disadvantages attendant on employing water is its low refractive power and speedy evaporation, by which their focal length, and, consequently, their power, are continually varying. In place, therefore, of this fluid he employs sulphuric acid† or castor oil, both of which possess a refractive power considerably greater than water. And as far as their optical properties are concerned, oil of ambergriese or alcohol might be used, but their volatility renders them less manageable.

(75.) Dr. Brewster describes the following as the best method of constructing fluid microscopes:—Take Canada balsam, balsam of capivi, or pure turpentine varnish, and drop either of them on a parallel piece of glass, when a plano-convex lens will be formed, (see *fig.* 50.) Their power may be varied by the quantity of the fluid employed, or by allowing the plate of glass to be horizontal with the drop above or beneath it; thus if the plate be uppermost, the gravity of the fluid will make it more convex, as in *fig*. 51. If the drop be above the plate, the lens will be flattened, as in *fig*. 52.

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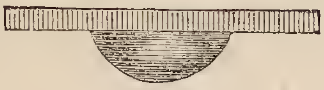
* *Philosophical Transactions*, No. 221 and 223, or Dr. Smith's *Optics*, vol. ii. p. 391.

† It should be observed, that in using sulphuric acid great care must be had to prevent its contact with the eye, as it might destroy that organ.

Fig. 50.

Fig. 51.



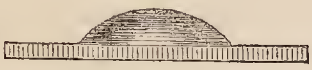


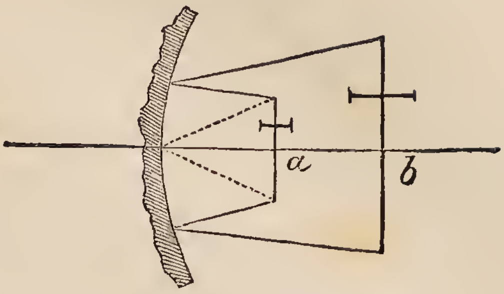
Fig. 52.

When the first of these substances is used it soon becomes indurated, and, if kept from dust, very durable. Dr. Brewster informs us, that he has made both object and eye-lenses of compound microscopes in this manner, which performed extremely well, and lasted a considerable time.

(76.) A single reflecting microscope may be formed by a concave speculum, having the object placed on its axis, and nearer to the surface of the reflector than the focus, as at *a*, *fig*. 53, when an enlarged view of the object will be formed in the same position as the object at *b*. This simple instrument is employed to enable a person to view his own eye, and show a magnified representation of the ramifications of the blood vessels, the pupil, and the iris.

When the object is placed between the focus and the centre, an enlarged image thereof may be formed on a screen or wall, but the image will be inverted with respect to the object,* and when the object is not very luminous, the room should be darkened.

Fig. 53.

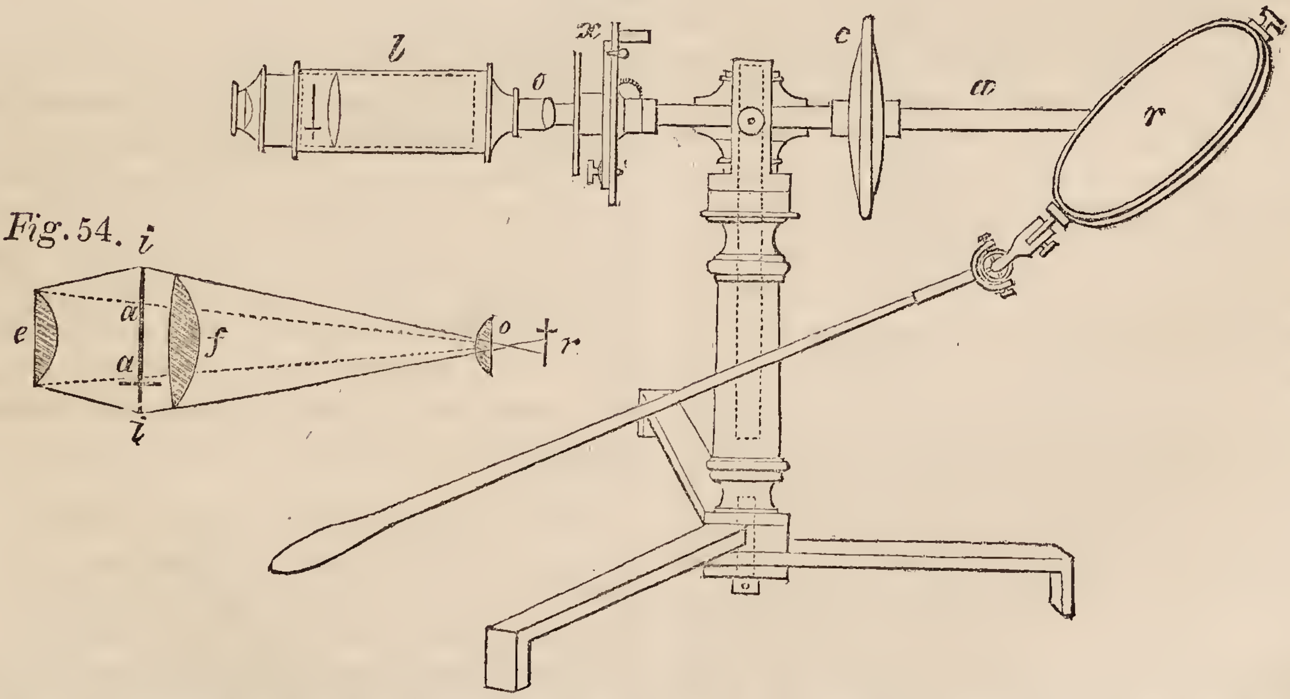


Chapter XIV.—*Theory of Compound Microscopes.—Mechanical arrangements of.—Eye-pieces.—Magnifying powers of.—Object-glasses.—Dr. Goring's.—Mr. Tulley's.—M. Chevalier's.—S. Amici's.—Mr. Herschel's.—Dr. Brewster's.—The Amician Reflecting Microscope, with its Improvements.*

(77.) A compound refracting microscope is an instrument consisting of two or more convex lenses, by one of which an enlarged image of the object is formed, and then, by means of the other employed as an eye-glass, a magnified representation of the enlarged image is obtained. The distance at which the two lenses of a compound microscope are placed from each other, must always exceed the sum of their focal lengths, in order that the image may be formed by the object-glass in the anterior focus of the eye-glass. *Fig*. 54 is a section of a compound instrument, *r* is the object intended to be magnified, which is placed in the focus of the object-glass *o*; by this lens an enlarged and inverted image is formed at *a a*, in the focus of the eye-glass

Fig. 54.

Fig. 55.



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* See p. 2.

e, by which the diverging rays from the image are rendered parallel, in the same manner as when a single lens was employed to magnify an object. Hence the great distinction, as before observed, between single and compound microscopes is, that in the latter we only view a magnified *image* of the object, while in the former we see the *object itself*. From this it must be evident, that unless the image formed by the object-glass is a perfect representation of the object in every particular, its imperfections, however small, will be increased by the eye-glass, in the same ratio as we magnify the image, or inversely as the focal length of the eye-glass.

With this disadvantage the compound instrument had been entirely laid aside, by the most distinguished naturalists and philosophers in their experimental researches, till very recently. For popular and general application it is preferred, on account of the extent of field obtained by it, which is far greater than that obtained by ordinary single glass lenses of equivalent power: and for these purposes there is usually introduced a field-glass *f*, by which the extent of the view is increased from *a a* to *i i*, by the rays being bent by this lens so that a greater portion may be refracted by the eye-glass *e*.

(78.) The compound microscope may be mounted nearly in the same manner as the single, or indeed the same stand may be employed for both. *Fig.* 55 is a compound microscope, having all its moveable parts sliding on a triangular bar *a*, which, from its mechanical properties, is better adapted for a microscope than any other form, provided the sliding pieces are sprung on one side only; *b* is the compound body, *o* the object end, *x* the slider attached to the stage, *c* the condensing lens, and *r* the reflector with a *hook's jointed* handle.

(79.) With the instrument just described, in order to bring the various parts of the object under the magnifier, the body of the instrument is moved by a rack and pinion working a triangular bar at right angles to the bar *a*, though it is often preferred to move the stage, as being of the least weight; for which purpose it is made to consist of *three* plates: the upper one carries the holder for the sliders, and is moved across the middle plate by means of a screw; while the middle plate is moved in a similar manner across the lower stage plate, but at right angles to the upper plate. In this way any position can be given to the object, by turning first the one and then the other screw. Different mechanical means have likewise been resorted to for effecting the same purpose. But when live objects, as *animalculæ*, are to be examined, it is preferable to make the stage a fixture, and to move the magnifier over the objects, as the most trifling motion will often disturb them for a considerable time.

(80.) Microscopic eye-pieces may be constructed in the same manner as those used for telescopes, (§ 22, 43—45) so as to correct the spherical and chromatic aberrations. But as the rays proceed from an object lens whose aperture is less than the field-glass, the distances of the two lenses will differ in a small degree from those employed for telescopes; this will also be the case if the distance between the object-glass and eye-piece of a microscope be varied.

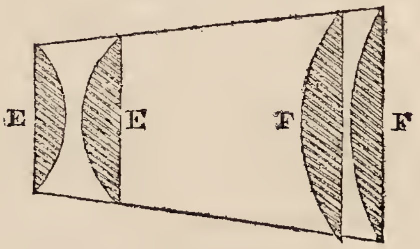
(81.) If the field-glass *f* (*fig.* 54) were exactly in the focus of the eye-glass *e*, it would have no effect on the magnifying power of the microscope; in this case the amplification of an object might be determined, by dividing the distance from the object-glass at which the image is formed by the distance of the object from that glass, which gives the diameter of the image; by multiplying this number by the power of the eye-glass, we obtain the magnifying power of the instrument. Example: let the sidereal focus of the object-glass be one inch, and the distance *o i* be 6 inches; then $6 \div 1 = 6$, and 6×5 (the power of the eye-lens *e* of two inches focus) will give 30 times as the increase in the diameter of the object, or 900 its increase in surface. But here we have divided by one inch the sidereal focus of the object-lens; this, however, would not be its focus when it has to form an image at 6 inches distance on the other side, but to obtain distinct vision the object will require to be $1\frac{1}{5}$ from the lens, (see 40.) this would reduce the actual magnifying power to 25, or 625 in surface.

(82.) Having thus shown how the power of a compound microscope may be obtained with only *two* glasses, it is necessary to point out the means of determining the amplification when a *field* lens is introduced. If we proceed for this purpose by calculation, the process will be rather complex, and unless the *data* are very accurate the result will be incorrect; these data, which are the foci of the several lenses and their distances from each other, offer many practical difficulties. We shall therefore describe a practical and simple method by which it may be found with equal accuracy; either when the instrument is a compound *refractor*, or *reflector*, of any construction. This novel mode of Dr. Goring, for determining the amplification of a microscope, is as follows:[*]—*As the aperture of the object-glass or metal is to its acting focus, measured from the radiant point*, (*i. e.* distance of the object from the lens when seen distinctly,) *so is the size of its visual pencil* (*i. e.* the small circle of light described in § 18) *to the focus of a single lens equivalent in power to the compound instrument.*

By this method the sidereal focus is not required to be known for any of the lenses, however numerous; the acting focus of only one, *viz.* the object-glass or metal must be ascertained; and then by measuring its true acting diameter or aperture, together with the diameter of the small pencil of light seen through the eye-glass when the observer is at a little distance from it, the whole is accomplished.[†]

(83.) The eye-pieces of some compound microscopes consist of four or five glasses, the makers having considered the perfection of the instrument to consist in obtaining a *flat* and enlarged field of view with a high power and shallow object-glass; but the brilliancy and sharpness of the image is, however, destroyed by this arrangement, and the introduction of considerable fog and flare is occasioned by the reflection of so many surfaces: for it may be remarked, that each surface will produce a reflected image of the object in the axis of the instrument, and thus occasion a dark and undefined spot in the middle of the object, while the edges are bright and fringed with colour; from these defects the instrument is rendered unfit for the vision of all but the most common objects, and these cannot be seen of their natural figure. One of these eye-pieces is shown at *fig.* 56,

Fig. 56.

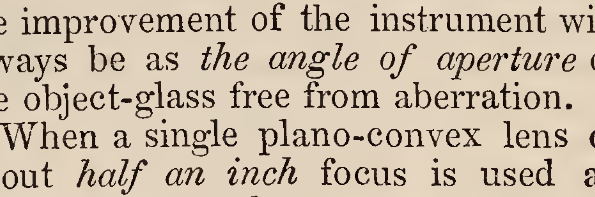


where E E are the eye and F F the field glasses.

(84.) The brightness and *penetration* of compound microscopes will be in proportion to their apertures under a given focus, from which circumstance the improvement of the instrument will always be as *the angle of aperture* of the object-glass free from aberration.

When a single plano-convex lens of about *half an inch* focus is used as an object-glass, the aperture must be about the $\frac{1}{15}$th of an inch, with the flat side outermost; but if we employ *two* lenses to produce the required refraction, the aperture should be between them, which may be made larger than can be employed with a single lens of equivalent power. The most advantageous combination of convex lenses for low-powered object-glasses, is that contrived by Dr. Goring,[*] and shown in *fig.* 57. The outer lens

Fig. 57.



a is plano-convex, with its flat side next the radiant, whose focal distance is $\frac{1}{2}$ or $\frac{2}{3}$ of the crossed or plane lens *c*; the stop *d* is placed in the focus of the lens *a*; when the focus of the lens *a* is not less than $\frac{1}{2}$ an inch, the combination has been employed with considerable advantage both as regards distinctness and aperture.

(85.) The most improved object-glasses for compound microscopes are those that are rendered achromatic, as a much larger angle of aperture can be obtained. These were first constructed at the suggestion and expense of Dr. Goring, by Mr. W. Tully, in the summer of 1824,* and more recently in France, by M. Chevalier.† The aplanatic of Mr. Tully consists of a concave of Guinard's flint glass placed between *two* convex lenses of *crown* and *Dutch* plate glass. The apertures obtained in these object-glasses are equal to half their focal distances, which are from 0.2 to 1.0 inch focus. By the increase of aperture which is here attainable, the penetrating power is augmented, so that the varied markings on objects are rendered visible with these large apertures, which with common object-glasses of the same power are wholly undiscernible. This property of the penetration of a microscope being always as the angle of aperture, was first pointed out by Dr. Goring, and is fully borne out by experiment. In the telescope the penetrating power necessary to observe *stars* and *nebula* is dependent on the area of the object-glass or metal, without relation to its focal distance; but in the microscope it is the result of *the aperture in proportion to their focal distance.* The difficulties that present themselves in correcting the aberrations of these object-glasses when of short focus is considerably increased, though their penetrating powers are always improved by a diminution of their focal length. The spherical aberration is greater and more difficult to correct than in telescopes, (the convex seldom having sufficient aberration to correct the concave,) and as the rays enter diverging from the object, the calculations of their radii are more abstruse than in a telescope for parallel rays. The curvatures and dimensions of one of Mr. Tully's object-glasses are as follows, and *fig.* 58 is a section twice its real size.

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[*] *Quarterly Journal*, vol. xxi.

[†] To determine the magnifying power of a compound microscope by calculation, let D = distance of the radiant from the object-glass; d = distance of the image from the same; Δ = the distance of distinct vision; and f = the focal length of the eye-glass: then, if only two lenses are employed, the amplifying power $= A = \frac{d}{D} \times \frac{\Delta}{f}$. If a field-glass is introduced, as in most compounds, this formulæ must be multiplied by the fraction $\frac{L}{\varphi}$; φ being the focal length of the field lens, and $L = \frac{\delta^2}{\delta - \varphi} - \overline{d' - f}$; δ being equal the distance of the first and second glasses, and d' the distance of the first and third glasses. If the construction of the eye-piece is *negative*, the power is diminished by the introduction of the field-glass, but when positive (§ 22) the power is augmented.

[*] *Quarterly Journal*, vol. xvii. p. 202.

	Inch.
Sidereal focus of the object-glass	0.933
Total diameter of the lenses	0.55
Clear aperture	0.5

		Inch.	
Radius of	1st surface	0.825	convex crown
Ditto	2nd	0.525	
Ditto	3rd	0.5	concave flint
Ditto	4th	0.5	
Ditto	5th	0.575	convex Dutch plate
Ditto	6th	0.575	

	Inch.
Specific gravity of crown	2.527
Thickness of ditto	0.15
Specific gravity of flint	3.627
Thickness of ditto	0.164
Specific gravity of Dutch	2.519
Thickness of ditto	0.175

(86.) The object-glasses of M. V. Chevalier are constructed on the principles laid down by Euler,* and consist of a double and equally convex lens of plate glass cemented to a plano-convex of Guinard's flint. The construction is remarkably simple, the radii of all their surfaces being alike. The measures of one of these achromatics obtained by Mr. Lister are as follows:—This object-glass is shown in section at *fig.* 59, of twice its real size.

Fig. 58.

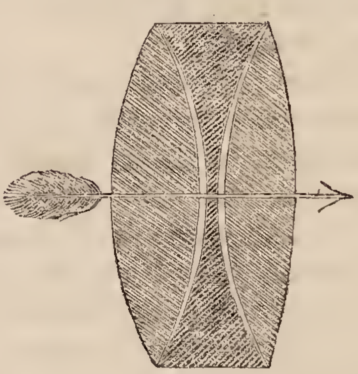
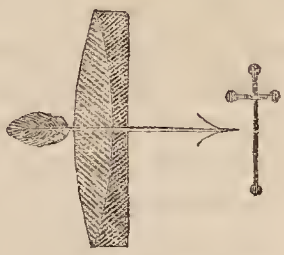


Fig. 59.



	Inch.
Radii of the isosceles convex	0.66
Radius of the plano-concave flint	0.66
Thickness of the concave at the edge	0.074
Thickness of the convex at the centre	0.072
Diameter of the lenses	0.36

When these object-glasses came first into this country, although achromatic, and two of them were combined, which is a considerable improvement on the single ones, they could not show the test objects, the aperture being cut off by a small stop; but on removing this stop Dr. Goring was enabled to make them *effective* on most of them.

(87.) The combination of three *sets* of achromatic lenses for an object-glass has recently been executed by Professor Amici, who, in order to have the body of the microscope horizontal and object-glasses vertical, introduces a right angular prism to change the direction of the rays, as at P, *fig.* 22. But the introduction of so many surfaces, *viz.* fifteen, has a tendency to weaken the outline of the objects. Mr. Dollond has, however, lately combined two triple object-glasses of one inch focus, and about half an inch aperture, which perform extremely well: they can also be used separately. It is necessary for us to remark here, that as the ray after passing through the first aplanatic combination is in a different state to what it is on entering the second, due correction ought to be made when these glasses are intended to be used separately and combined; indeed, in those of short focus such correction is absolutely requisite, so that if they are perfect when combined, only one can be so if employed singly. When a triple one and a double one are combined the performance seems best of all.

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* *Quarterly Journal*, vol. xxii. p. 265.
† Ibid. vol. ii. N. S.

* This theory was published at St. Petersburg, in 1774, and lately in the *Bulletin de la Société d'Encouragement de Paris*, No. ccliv. for August 1825.

(88.) If we are desirous of producing an aplanatic object-glass without using the achromatic combinations above described, that of Mr. Herschel, in *fig.* 48, will be found of great value, as it is free from all spherical aberration in the axis. One has been executed (which is in our possession) of only one-sixth of an inch focus, with an aperture of the $\frac{1}{15}$th of an inch, which brings out all the *test objects* and exhibits opaque ones with facility. This combination is made of crown glass. When it is not thought advisable to employ a correcting lens, (*i. e.* one with a concave surface,) it must be evident, from what has been stated respecting single microscopes, that the diamond or sapphire will be the most aplanatic as an object lens.

(89.) A compound microscope for viewing objects of natural history has been proposed by Dr. Brewster. In the ordinary instrument, from the objects being employed in a dry or shrivelled state, their "natural polish and brilliancy is impaired; the minute parts, such as the hairs and down, adhere to one another, and the general form, as well as the disposition of its individual parts, can no longer be distinctly seen." To remedy these defects, "several small glass vessels must be provided, having different depths from 1 to 3 inches, and having their bottoms composed of a piece of flat glass, for the purpose of admitting freely the reflected light which is intended to illuminate the object." The objects to be viewed must be preserved in a fluid which will cause them to retain their plumpness and impart that freshness of colour which they possessed when alive; these with the fluid are to be put into the vessel, "and being placed upon a glass stage, or, if necessary, fixed to it, the arm of the microscope and the lens is then brought into contact with the fluid in the vessel. The rays which diverge from the object emerge directly from the fluid into the object-glass, and therefore suffer a less refraction than if it had been made from air; but the focal length of the lens is very little increased;" as the radius of the immersed surface should be about 9 times its focal distance, and the side next the eye (or field lens) about three-fifths of the same distance.

(90.) This method of fitting up and using a compound microscope, says Dr. Brewster, enables us in a simple manner to render the object-glass achromatic, without the assistance of any additional lens. This may be effected by employing a fluid whose dispersive power exceeds that of the object-glass, and by accommodating the radius of the anterior surface of that lens to the difference of their dispersive powers, the image will then be formed free from any of the primary colours of the spectrum. The fluids most proper for this purpose are

Oil of cassia.
——— anise seeds.
——— cummin.
——— cloves.
——— sassafras.
——— sweet fennel seeds
——— spearmint.
——— pimento.

In order to render the object-glass achromatic, when it is made of crown glass, and the fluid in which the objects are immersed is oil of cassia, the radius of the surface next the object should be to that of the surface next the eye, as 2.5 to 1, or the radii may be made as 2.2 to 1, and the dispersion of the fluid let down (by any of the other oils which are here placed in the order of their dispersion) till the chromatic aberration is corrected.

(91.) Compound microscopes have been constructed of almost every possible dimensions, from a few inches in length to that of 20 feet; but from practical experience it appears evident, that when their magnitude is augmented beyond a certain point the performance of the instrument is deteriorated, though we suppose the amplifying power of both microscopes the same; indeed, those of the larger description, which have been made to examine objects of considerable dimensions, cannot be considered of any use except for amusement. The reason, however, why the smaller instrument produces a better representation of the object, has not been determined; for in both instruments (mathematically) the proportional errors are the same, but it may be that the errors in the smaller instrument, though existing, are not so divided as to be discoverable to the organ of vision. At present no certain proportion between the length of the body of the instrument and the focus of the object-glass has been determined, except that the body requires to be lengthened when the focal length of the object-glass is increased, to give its maximum effect. The average length of body for different object-glasses is from five to ten inches; for when the rays after once being acted upon are allowed to travel any considerable distance to form the image in the focus of the eye-piece, they will be so much separated and weakened that the eye-piece will not be able to collect them truly. And it may be further remarked, that the power should always be obtained by the object-glass in preference to the eye-piece; and although it is well known, that a flat and more extended field is obtained by the latter, yet when these are procured, no distinct idea of the structure of the object is discoverable, and thus the very intention of the microscope is destroyed, which ought to unfold to our perception the nature and functions of bodies too minute for ordinary inspection.

(92.) The Amician reflecting microscope was invented at Modena, by Professor Amici; it consists of a concave ellipsoidal reflector whose focal distance is $2\frac{6}{10}$ inches with an aperture of one inch. In the axis of this reflector, and at about half its focal distance, is placed a small flat reflector whose surface forms an angle of 45° with the axis of the instrument, (similar to the Newtonian telescope;) the object is placed opposite to the plane metal, and is adjusted to the focus of the concave reflector.* The rays, after reflection from the concave metal, form an image of the object in the other focus of the ellipse, which is viewed by an eye-piece attached to the tube at about 8 inches from the metal. The performance of this instrument was held in high estimation on the continent, being considered superior to any compound refracting microscopes then made; this induced the introduction of the Amician construction into England. But when carefully made of the proportion here stated it only equalled the compound refracting instruments, and like them it was unable to show the delicate markings on various test objects, particularly the lines on the dust from the wings of a certain class of moths and butterflies. This instrument, however, has since been improved in this country, so as to show in a distinct and satisfactory manner, free from chromatic and spherical aberration, almost every class of test objects. These improvements were made by Mr. Cuthbert, under the direction of Dr. Goring.* The most important improvement is the formation of the concave metal of very short focal distance, whereby less assistance is required from the eye-glass to obtain the required power, while its aperture is increased to more than half the focal distance; thus an increase of penetrating power is obtained. In the original Amician construction, the small flat metal occupied so large a portion of the central pencils of rays, as to produce a kind of nebulosity around the object, the image being formed on the retina of the eye by a narrow ring of reflected light, as represented at *fig.* 60. In Goring's improved construction the small metal is lessened, and, consequently, it does not occasion so much impediment to the central rays as shown at *fig.* 61.

Fig. 60.

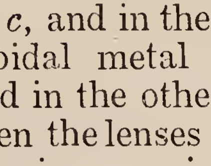
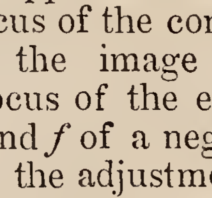


Fig. 61.



In the section of this *Engiscope*,† (*fig.* 62,) *a* is the object opposite the flat metal *c*, and in the focus of the concave ellipsoidal metal *d*; the image *i* is formed in the other focus of the ellipse between the lenses *e* and *f* of a negative Huygenian eye-piece; the adjustment of the object is effected by means of a rack and pinion *g*, attached to the tube at right angles to its axis. When the power is required to be changed, the metals are unscrewed from the body and others put on in their stead. Their usual apertures and foci are as follows:—

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* The above measurements are those of the original instrument; the Professor having subsequently found, that, by shortening the focal length of the concave metal, the performance of the microscope is improved, less assistance being required from the eye-piece.

* See *Quarterly Journal*, vol. xxi. p. 34.

† This term has been applied by Dr. Goring to designate a microscope in its perfect form.

Focus. Inch.	Apertures. Inch.
1.5	0.6
1.0	0.3
0.6	0.3
0.3	0.2

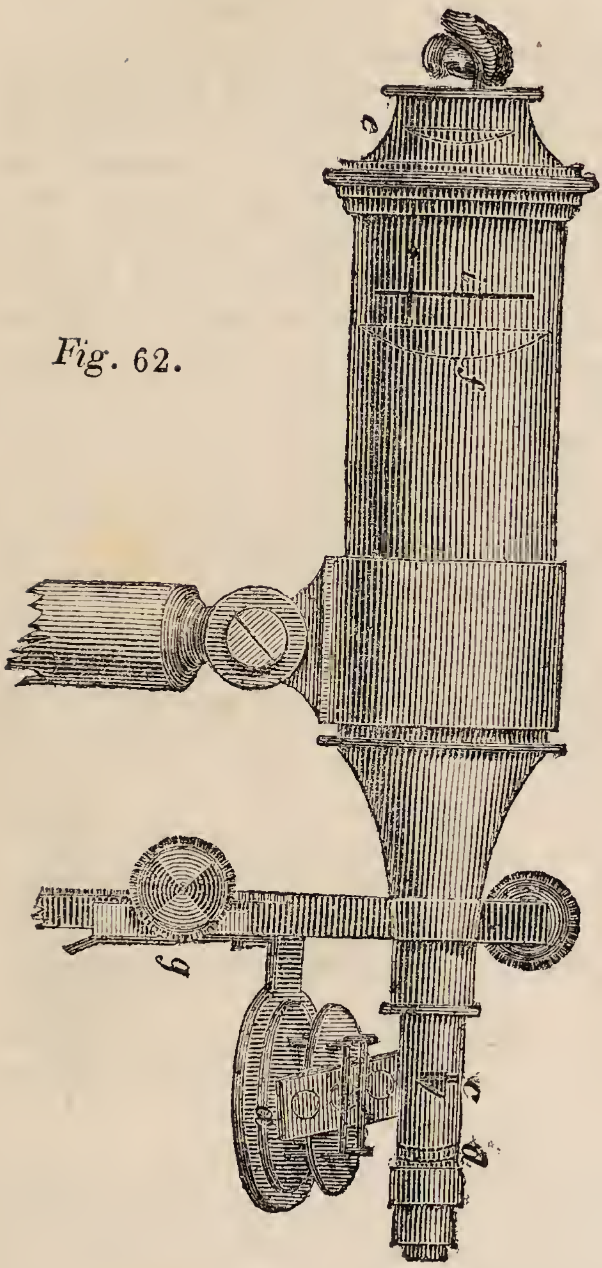


Fig. 62.

CHAPTER XV.—*Test Objects.—Opaque Microscope. — Solar Microscope. — Improved.—Lucernal.—Magic Lantern.—Phantasmagoria.*

(93.) *Test objects.*—It is perhaps one of the greatest requisites in the selection of a microscope, to be able to ascertain whether it will be efficient for the purposes intended. This can only be known by its capability of exhibiting those objects submitted to it: till very recently it was not ascertained that certain objects, in order to render their various markings or texture distinctly apparent, required the instrument to be of the best construction, whether single or compound, and possessing a considerable quantity of distinct light. These objects have therefore been denominated *tests*, by their discoverer, Dr. Goring. In order that our readers may be able justly to appreciate the efficiency of any microscope that may come within their observation, and determine its *penetrating* and *defining* powers, whether the instrument be single or compound, we shall describe the principal test objects necessary for that purpose. The objects best adapted to determine the penetrating power, are the dust or scales from the wings of certain classes of Papilio, (butterflies and moths.) Of these the *Menelaüs*, shown in *fig.* 63 and 64 (magnified) is a very useful object. The dust from the under side of the wing of the male papilio brassica (white cabbage butterfly,) shown in *fig.* 65, is a good proof-object, and a very peculiar one of the same kind is shown in *fig.* 66, (both magnified.) In viewing these objects a large angle of aperture is required (at least equal to half the focus,) in order that the lines and markings may be distinctly seen. There are, however, many of the scales from some kinds of papilio, on any of which the lines can be seen by an ordinary instrument. But the objects here selected, as well as the lines on the scales from the *small, brown, house moth;* the lines on the scales taken from the foreign curculio (diamond beetle) (*fig.* 67,) require a more perfect instrument to develope them. Lastly,

Fig. 63.

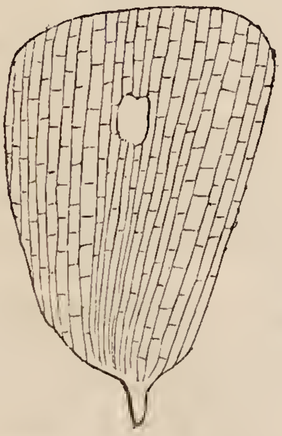


Fig. 65.

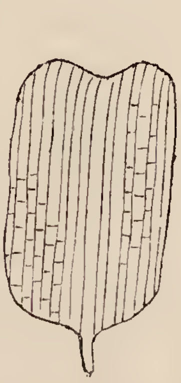


Fig. 67.

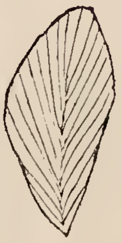


Fig. 66.

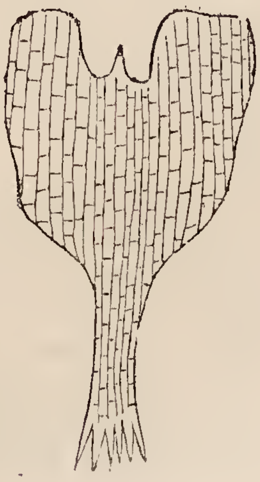
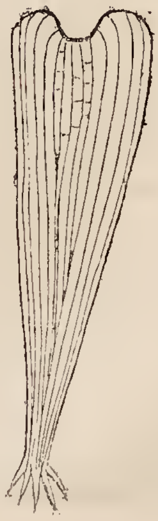


Fig. 64.



the most difficult of all the *test* objects are the lines on the scales from the *Podura springtail,* recently discovered by T. Carpenter, Esq., and on which the markings are only just discernible by the most perfect instruments. When the penetrating power is thus ascertained, its defining power may be determined by inspecting a leaf of the moss of a species of the genus *hypnum,*[*] which requires a considerable penetrating as well as *defining* power, fully to develope the lozenges which constitute its fabric, making out a luminous nucleus to each, which should be sharply defined and of the same shape with the outer lozenge.[†] As opaque test objects, the bat's hair shown in *fig.* 68 and 69, and the mouse hair (*fig.* 70 and 71,) may be considered excellent tests, when the outline and markings are well defined. These objects may be also examined with transmitted light with the same advantages.

Fig. 68.

Fig. 69.

Fig. 70.

Fig. 71.

(94.) The white letters on a black ground seen on a piece of enamelled watch plate, is perhaps one of the best tests to determine the quantity of chromatic or spherical aberration in a lens: indeed to detect the latter error an artificial star [‡] may be used with advantage, which requires considerable *defining* power to show well.

(95.) In examining these test objects the direction and quantity of light must be carefully attended to, nor must it be injured or mutilated by the reflector, condensing lens, or other diaphanous body through which it may pass to the object. When an instrument can show these proof objects, it may with certainty be pronounced *effective.* It should be remarked, that when the objects are used as *opaque,* a smaller aperture will do best, *viz.* about $\frac{2}{5}$ of its focus, (for any power less than 300 *decimal standard,)* but the magnifier requires to be more free from aberrations. For transparent objects a larger aperture is absolutely necessary; and for some *tests* it should be equal to its focal distance, to show the cross *striæ* between the lines on many of the scales, when the power of the instrument or lens is considerable. It is worthy of remark, that the same aperture that with advantage will develope one class of objects will not show another with the same success.

(96.) *Opaque Cups.*—When the focal distance of a magnifier, either employed singly or as the objective to a compound instrument, is too short to admit of an opaque object being illuminated by a light thrown between it and the lens, the magnifier is mounted in the manner shown in *fig.* 72, where the lens is set between two thin pieces of metal, at such a distance from the centre of the concave polished reflecting cup *a, a,* that the object *o* shall be in its focus and in the focus of the lens at the same time. When thus arranged the light is thrown on to the reflector (at the back of the object,) whence it is reflected to its focal point to illuminate the object placed therein. If the rays of light are not sufficiently condensed by this concave mirror, it may be assisted by a condensing lens *c,* by which the proper quantity of light required by the magnifier or object is obtained. When opaque objects are to be viewed in the Amician engiscope described in (92.) the silver reflector is made to be attached to the tube opposite the plane metal *c,* (*fig.* 62.)

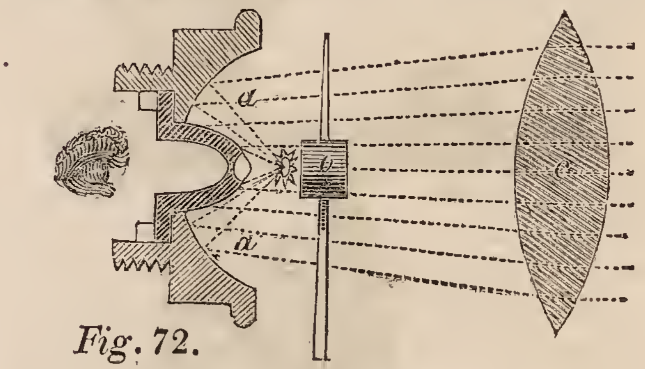


Fig. 72.

(97.) *The solar microscope* consists of a common microscope connected to a reflector and condenser, the former being used to throw the sun's light on the latter, by which it is condensed to illuminate the object placed in

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[*] The generic name of this moss was recently discovered by Mr. Carpenter.

[†] J. Pond, Esq., the Astronomer Royal, on recently examining this object with a deep sapphire-lens, discovered that they are not lozenges but hexagons, the two sides being very short in comparison to the others; he has also observed with a sapphire of the power of 250 all the lines on the *podura.*

[‡] An *artificial star* may be made by taking a small globule of pure mercury kept in gum water, and securing it to a black ground, as burnt cork or black paper; or a globule of platina fused by electricity, and attached to a black ground may be employed.

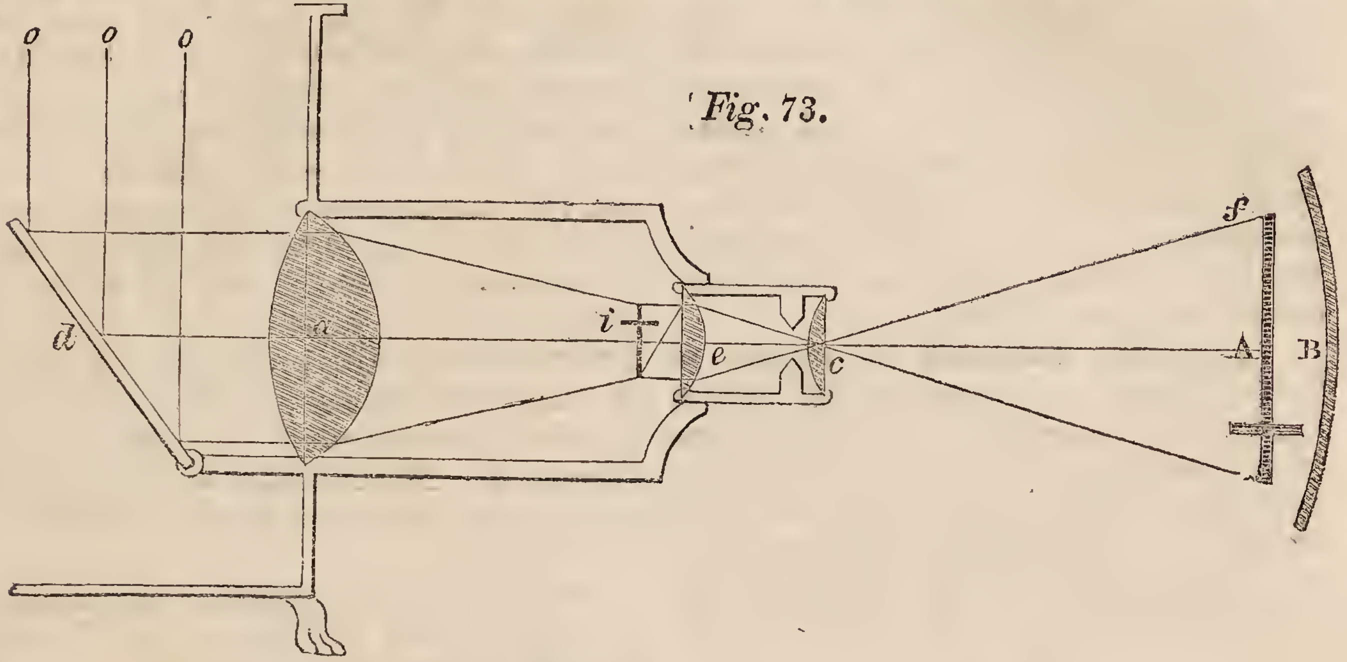
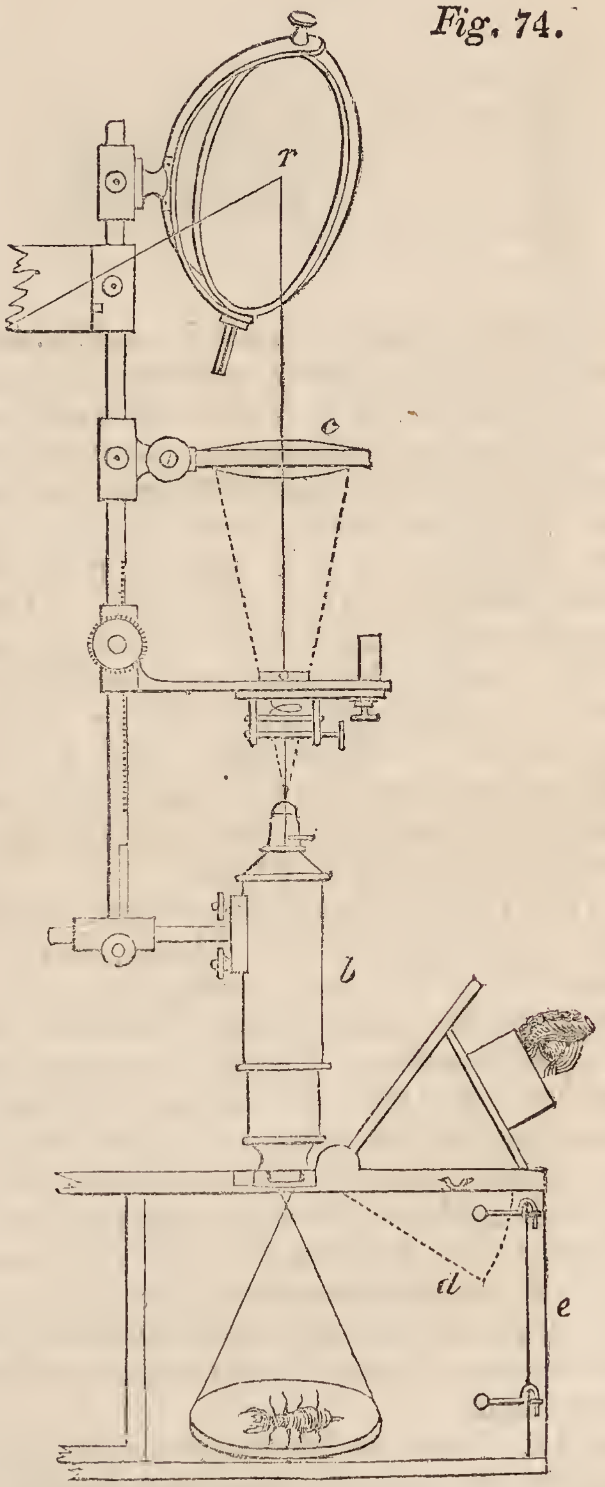


Fig. 73.

its focus; one of these instruments is shown in section *fig.* 73, with a double object-glass *e, c*. The rays from the sun *o, o, o* are received on the looking-glass or reflector *d*, which is capable of being directed in any position, to throw the rays of the sun on the condensing-lens *a;* the object *i* is thus strongly illuminated by the condensed light of the sun. This object is placed in the focus of the microscopic lens *c*, which in the common instrument is the only one employed; but when the combination of two lenses is used the magnified image is improved; this representation, or picture of the object, is received on a wall or screen, and the magnifying power of the instrument will be always as the distances *i c* to *c* A; thus, if the acting focus of the lens *c* is 1 inch when it forms an image on the other side at 5 feet, the linear amplification will be 60 times, or the increase of surface 3600 times. When we decrease the distance of *c* A to about 10 inches, the *decimal* standard, although the magnitude of the image will not be so great yet its distinctness will be improved; and it will be likewise found, that when the image is thrown on a flat surface, the edges and middle are not distinct at the same time, for the rays *c f* are farther from the object-lens than *c* A: to remedy this, the white screen should be concave whose radius is C B, or distance of the lens from the screen; by this all the parts are made equally perfect.

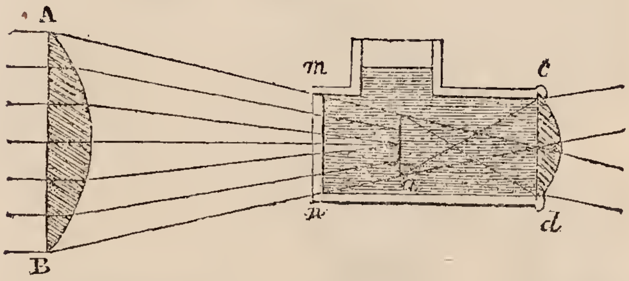
Fig. 74.



(98.) An improved modification of this instrument is shown in *fig.* 74; *r* is the reflector, *c* the condenser, *o* the object, *b* the microscope, which is here the compound instrument with its eye-end fitting into the *darkened* box *d*, having a concave at the bottom to receive the representation of the object. When it is to be used for drawing, the front *e* is let down to admit the hand; this modification of Dr. Goring's, having achromatic object-glasses adapted to it, renders the instrument *truly useful,* from the perfection of the magnified picture. These object-glasses may be used with or without the compound body, at option; but the most perfect object-glass for these purposes is formed by the continuation of a *triple* and *double* achromatic, by which a very large and distinct field is procured, free from colour and aberration.

(99.) The enclosing of objects in fluids, to exhibit them in a more natural form under the compound microscope (described in 89,) may be likewise applied with equal advantage to the solar instrument. This Dr. Brewster proposes to effect in the manner shown in *fig.* 75, where A B is the illuminating lens employed to throw the condensed light of the sun on the object *a*, enclosed in a tubular vessel of the fluid, having an opening in the upper side for the introduction of the object by a pair of forceps; *c d* is the object lens which forms one end of the tube, and *m n* a parallel plate of glass cemented to the other end. "The opacity which arises from a contraction of the parts is thus completely removed, and an additional transparency communicated to them from the fluid." This microscope may be rendered achromatic by using the same fluids and radii as proposed in § (90.)

Fig. 75.



(100.) *The lucernal microscope,* which was contrived by Mr. Adams, is constructed on the same principle as the solar instrument, having however a lamp to illuminate the objects instead of the sun; this lamp is enclosed in a lantern (*fig.* 76) to screen the light from the observers, in the same manner as in a magic lantern: the advantages of employing this instrument over the solar is, that we can command the light at any time, and in any situation, while in every other respect it is similar to that instrument.

(101.) *The magic lantern* is constructed similarly to the two former instruments, but having the object and field glasses of larger diameters and longer foci, to admit more extensive objects; these objects are usually painted representations of familiar or grotesque subjects on glass sliders, having the parts not occupied with the design blackened to obstruct the passage of light. These sliders are introduced by an opening cut in each side of the tube, in the same manner as in *fig.* 73; the diameter of the lenses *c* and *e* are nearly equal to that of the condenser or bull's eye *a*, and are made to slide within the outer tube to adjust the image on the wall at different distances. A part of the lantern is shown by the broken lines attached to the end of the tube at *a*. The reflector *d* is removed, and a lamp put in its stead to illuminate the lens *a*.

(102.) *Phantasmagoria.*—An exhibition some years ago was brought before the public under this appellation. It consists of a magic lantern constructed on a large scale, and having the object-sliders painted in the same manner; but instead of being exhibited on an opaque surface, the figures are thrown on a transparent substance. The manner in which this spectacle was exhibited at the Lyceum, has been described by Mr. Nicholson; but the kind of machinery then employed is not exactly known, though Dr. Young has given various ingenious methods of producing it.

Chapter XVI. — *Camera Obscura; Portable Box; Revolving; Periscopic.— Camera Lucida; Periscopic; Amici's improved.— Teinoscope; Dr. Brewster's; Amici's.— Multiplying Glass.—Kaleidoscope; Telescopic.*

(103.) *Camera Obscura,** or *darkened chamber,* is an optical apparatus for the representation of all surrounding objects under the same angle which they subtend to the unassisted eye, and which are exhibited in their proper colour and shape, so as to enable a person to delineate or trace both near and remote objects, without an acquaintance with the rules of perspective, when they are thrown on the paper.

In the construction of this instrument a convex lens and plane mirror are its principal parts; these are arranged differently, according as it is required to be *portable* or *stationary.* The first of these, called the *portable box camera obscura,* is shown in *fig.* 76, where the square box A has a

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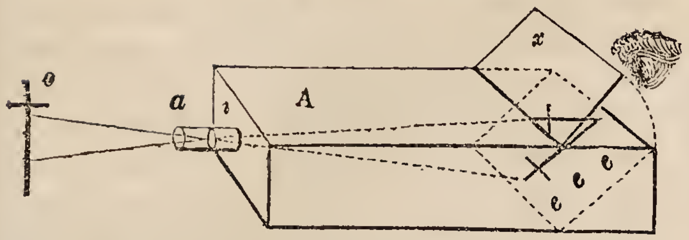
* Invented by Friar Bacon about the 13th century, but attributed by some to Baptista Porta, who published a work with an account of it, in 1560, at Antwerp.

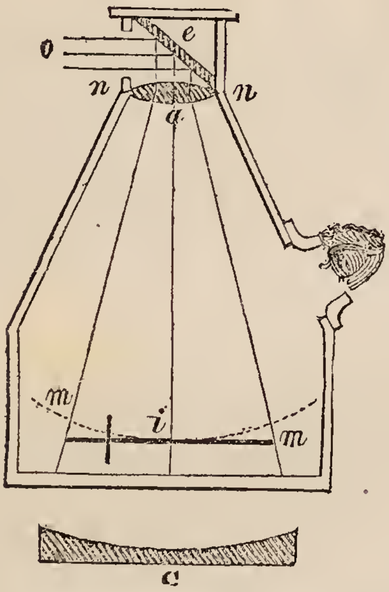
Fig. 76.



circular aperture in the front at *a*. Into this a short tube is inserted, having a convex lens at its outer end: this tube is made to slide out for adjustment; thus for near objects it requires to be drawn out more than for distant ones. Now the rays proceeding from the object *o* pass through the convex glass *a*, and form an inverted image *i* in the posterior focus of the lens; this image is received on a reflector or looking-glass *e*, *e*, *e*, inclined at an angle of 45°, by which it is reflected upwards on to a plate of ground glass at the top of the box; the lid *x* has two side wings to exclude the light when in use. The size of the image *i* to the object *o* will be as the distance *i a* to *o a;* and as the latter distance is usually the greatest, the image is less than the object.

(104.) *The revolving camera obscura* is shown at *fig.* 77, where *e* is the inclined reflector capable of being turned round with its case *n n*, which has an opening in the front at *o* to admit the rays from external objects; these rays are reflected to the convex lens *a*, which forms an image of the object at *i*, in the same manner as in the box camera: there is an opening in the side of the box to view the image as shown by the eye.

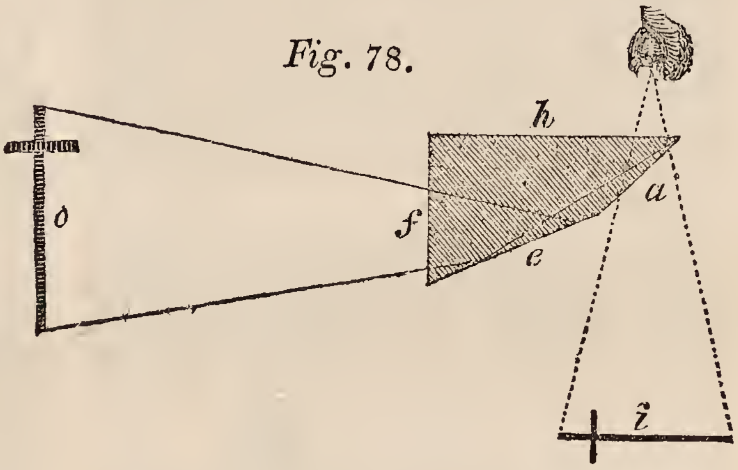
Fig. 77.



(105.) Although the modifications of this apparatus are numerous, they all depend on the same principle, which admits of the following improvement: the lens *a* being double convex converges the rays that pass through it to the same distance from every part of its surface, but as the image is received on a flat plane, the rays *a m* will have to diverge farther than those in the centre *ai*, which are nearer the lens; hence the image will be a distorted representation of the object: to remedy this, when not required for tracing, the image should be formed on a concave surface, like that shown in section at *c*. When, however, the instrument is wanted for the delineation of objects, Dr. Wollaston has proposed to make it *periscopic*, by having the lens *a* formed with such curves that the marginal rays *a m* are rendered longer than the central ones *a i*. The shape of the lens necessary to be employed for this purpose is meniscus, having the concave side next the object, and the radii of the two surfaces as 1 to 2, by which, he informs us, an aperture of 4 inches can be employed, with a lens of 22 inches focal distance; the light and brilliancy of the picture is thus greatly increased.*

(106.) The *camera lucida* was invented by Dr. Wollaston, in 1807, for the purpose of delineating distant objects, and for copying or reducing drawings. This instrument consists of a quadrangular glass prism, by which the rays from an object are twice reflected: its form is shown in *fig.* 78. The object *o* to be traced is opposite the perpendicular surface of the prism *f*, and the rays proceeding from *o* pass through this surface, and fall on the inclined plane *e*, making an angle with *f* of 67½°: from this they are reflected at an equal angle to the plane *a*, making an angle of 135° with *e*, and are again reflected to the eye above the horizontal plane, which makes an angle of 67½° with the last reflection. The rays of light from the objects proceeding upwards from *a* towards the eye of the observer, the observer will be led to imagine the image at *i*, and by placing the paper below in this place,

Fig. 78.



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* *Phil. Trans.*

the image may be traced with a pencil. In order to increase or diminish the size of the picture, the prism is mounted in a brass frame supported by brass tubes, capable of being drawn out or shortened at pleasure. The picture always bears the same relation in size to the object as the distance from the eye to the image or paper is to the distance from the object to the eye; hence by lengthening the tubes the drawing is increased in size: it should be remarked, that by this prism no real image is formed, but it always appears as far below the prism as the object is before it. The brass frame of the prism has usually two lenses, one concave and the other convex, the former to be used in front at *f* for short-sighted persons, the latter above at *h* for long-sights. The periscopic principle of Dr. Wollaston has been applied to this instrument, by forming the upper surface at *h* slightly concave.

(107.) Various modifications and contrivances have been adopted to improve or construct different kinds of camera lucidas for tracing or delineating objects. A very simple one may be made with a plane reflector, either of speculum metal or plate glass, having its face inclined at an angle of 45°; but in this form, as there is only one reflection, the drawing or tracing will have those objects on the left side that in the original object are on the right side, in the same manner as an engraved plate for printing is the reverse of the impression taken from it.

(108.) The most ingenious camera lucida, in which the tracings are similar to the original, is shown in *fig.* 79, which consists of a parallel piece of plate glass *d, e,* C, connected to a reflecting speculum A. In this instrument, the invention of S. Amici, the rays from the object R are thrown on to the speculum A (inclined at an angle of 135°,) and reflected by it on to the plate glass at P, and from thence to the eye above. The instrument is mounted in brass, and has a rectangular opening at the top, whereby the eye is prevented from seeing a reverse image of the objects produced by the metallic mirror. Another construction, which Amici esteems the best, is shown in *fig.* 80, where the rays are made to pass through the plate glass before they impinge on the mirror A, (as shown by the dotted lines,) they are then reflected to *n*, and from thence to the eye, one half of the pupil being directed through the glass *d, e,* C to the paper below, and the other half receiving the reflected rays.

Fig. 79.

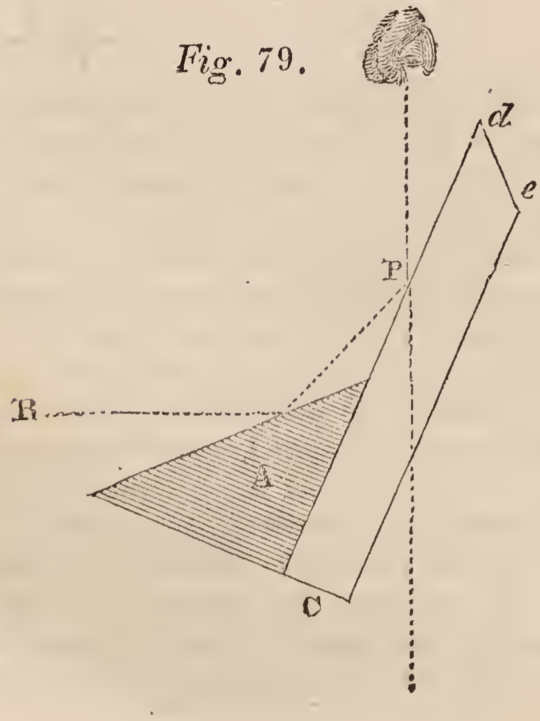
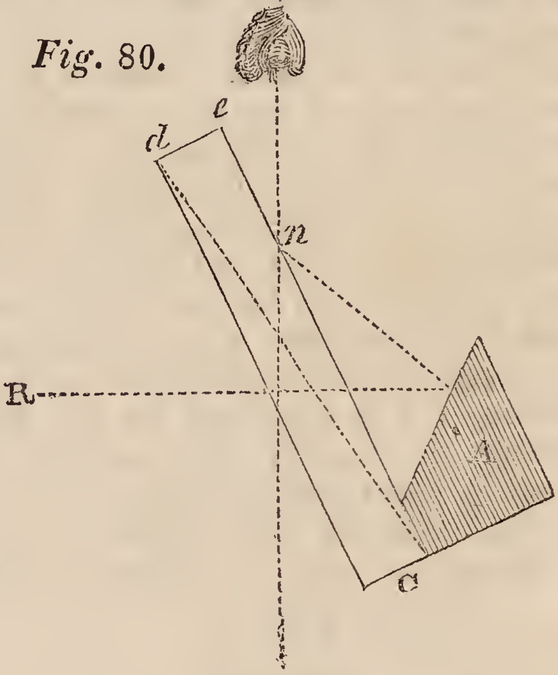


Fig. 80.



(109.) *Teinoscope.*—We stated when treating of the achromatic telescope, that by combining a concave lens of a substance having a higher dispersive power than the convex object-glass, its colour might be corrected without destroying the whole of the refraction produced by it. From numerous experiments made by Dr. Brewster, to determine the irrationality or inequality of the lengths of the different coloured spaces in spectra of equal lengths, he found that the least and most refrangible colours might be destroyed, and *refraction* produced when he employed two prisms of the same substance, provided the prism with the least refracting angle was inclined so that the incident rays entered obliquely, by which means the dispersion is increased in a greater ratio than the refraction:—thus, if a flint-glass prism B (*fig.* 81,) with an

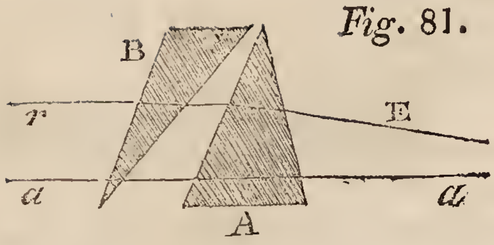


Fig. 81.

angle of 41° 11′, is corrected by another prism of flint-glass A having an angle of 60° 2′, the first being inclined to increase the dispersion; it will be found that the dispersion by this arrangement will be increased, so that the pencil *r* will be considerably refracted towards the axis *a a* by the prism A, and yet emerge colourless at E. From this very curious circumstance of procuring refraction without colour by two prisms of the *same substance*, Dr. Brewster has proposed the construction of object-glasses on the same principle, by which he states that the spherical aberration of the two might more accurately be corrected by employing the form shown in section, *fig.* 82, where the lens A corresponds with the prism A in *fig.* 81, and the convex meniscus B with the prism B. The lens A is plano-convex, but probably the best form would be a convex meniscus, with the convex side turned towards the eye.*

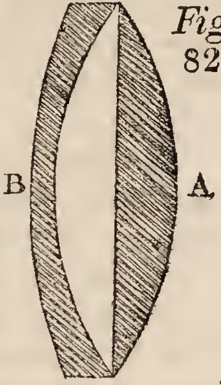


Fig. 82.

(110.) Signor Amici has very recenlty constructed a combination of prisms of the same glass on this principle, in which the chromatic aberration is corrected, and a power of about 3 times obtained. This plan is peculiarly well suited for *opera-glasses*, as the cluster of prisms requires no adjustment of focus for the different distance of the objects, a remote and near one being seen at the same time with equal distinctness; indeed, it does not possess a focus properly so called, a desideratum of great importance in this class of instruments; it is, therefore, superior to them in this respect.

Amici's teinoscope consists of four right angular prisms, having their refractive angles different and connected by pairs; the two pairs being similar, those next the eye or the first *pair* are vertical, and the second pair horizontal, so that equal refraction is produced in every direction. The distance between each pair is about an inch and a half.†

(111.) *The Kaleidoscope** is an instrument invented by Dr. Brewster while investigating the polarization of light by successive reflections between plates of glass, in the year 1814. The patent he obtained for it described it as a new optical instrument "for creating and exhibiting beautiful forms:" this is effected by two reflecting plates inclined to each other at any angle that is an aliquot part of a circle (or of 360°.) These plates are placed between the eye and certain objects to form the intended picture, as shown in *fig.* 83, where *a b* are the two reflecting planes inclined at an angle of 60°, or the sixth of a circle; at the end *c* are placed the objects, and the eye is stationed at the other end: these two plates are usually enclosed in a tube, and the objects, consisting of pieces of coloured glass, beads, &c., are loosely confined between two circular pieces of common glass, the outer of which is usually *greyed*, to make the light uniform. In order to give the picture varied outlines, threads of coloured glass spun or twisted may be mixed with the pieces, being first formed into circles, ellipses, looped curves like the figure 8, curves like 3, or spirals like the letter S. On looking down the tube, through a small hole placed near the meeting of the plates, a beautiful circular figure will be seen having six angles, the plates being inclined the $\frac{1}{6}$th of a circle; if inclined the $\frac{1}{12}$, $\frac{1}{20}$, &c. twelve or twenty will be seen: these beautiful forms, by slightly turning the tube, will be changed, by which an almost infinite variety of patterns may be produced.†

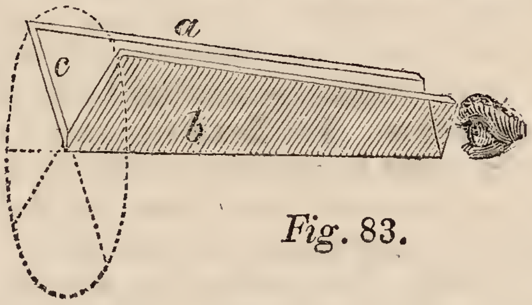


Fig. 83.

(112.) In order to make this instrument capable of taking in distant objects, in the same manner as a telescope, a convex lens or object-glass fixed to a tube sliding for adjustment on the inner tube containing the reflectors should be annexed; by this means an inverted image of any distant object may be formed at the ends of the reflectors, as shown in *fig.* 84, where *o*

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* See *Encyclopedia Edin.* art. Optics, p. 650.

† We have been informed that one of these instruments was made by Dr. Blair previous to that constructed by S. Amici, and that it is now in the possession of Dr. Brewster.

* This name is derived from three Greek words, καλος, *beautiful*, ειδος, *a form*, and σκοπεω, *to see.*

† For the theory of this instrument, see our *Treatise on Optics*, p. 19, fig. 23.

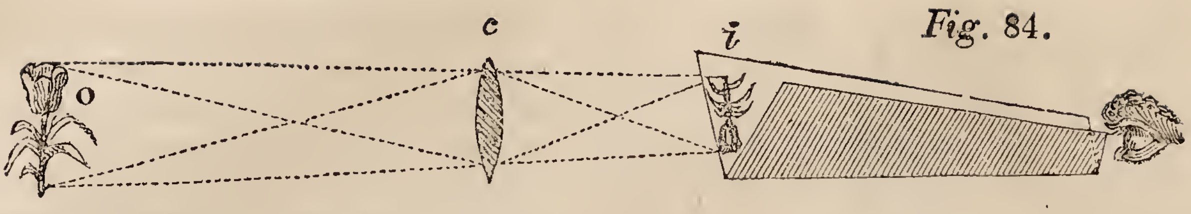


Fig. 84.

is the object, *c* the lens, and *i* the image formed with it. In this manner it has been employed for producing beautiful combinations of *flowers, trees, animals,* &c. When the lengths of the reflecting planes are less than the distance of distinct vision, a convex lens whose focus is the length of the plates should be attached to the eye-end of the tube. In this way polished speculums may be used, by which the brilliancy of the picture is increased, as less light will be lost by reflection. Dr. Brewster has found, that in order to produce perfectly beautiful and symmetrical forms, the following three conditions are necessary.

1. That the reflectors should be placed at an angle which is an *even* or an *odd* aliquot part of a circle, when the object was regular, and similarly situated with respect to both the mirrors, or an *even* aliquot part of a circle, when the object was irregular.

2. That out of an infinite number of positions for the object both within and without the reflectors, there was *only one* position where perfect symmetry could be obtained, namely, by placing the object in contact with the ends of the reflectors.

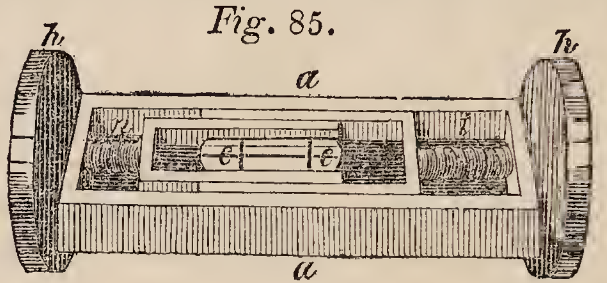
3. That out of an infinite number of positions for the situation of the eye there was *only one* where the symmetry was perfect; namely, as near as possible to the angular point, so that the whole of the circular field could be distinctly seen; and that this point was the *only one* out of an infinite number at which the *uniformity* of the reflected light was a maximum.

Chapter XVII.—*Micrometers.*

(113.) The micrometer is an instrument usually applied to telescopes and microscopes, for the purpose of measuring minute bodies or small angles subtended by bodies at a remote distance, by which their real magnitude is obtained. By the modern introduction of this instrument for the use of the astronomer, and the improvement of the telescope, may be attributed our accurate and extensive acquaintance with the universe of matter; while from the perfection to which the microscope has recently been brought, an equal acquaintance with the minute organization of bodies may be expected. By the application of the micrometer to this latter instrument the power of the naturalist is materially extended; while the micrometer is of the utmost value, for trigonometrical surveys, and in military or naval operations.

(114.) *The common wire micrometer,* usually attached to the eye-pieces of telescopes, is shown at *fig.* 85: it consists of a brass rectangular box *a a*, the upper and lower plate having an opening in the centre, (but in the figure both are removed;) this box is made to slide along an opening cut in the tube of the eye-piece of the telescope at right angles to its axis, so that the wires *c e* may be in the field of view; these wires are fixed to the forks *i n*, moveable in each other by the screws *i* and *n*, connected to the micrometer heads *h h:* there is also another fixed wire at right angles to the two former from *c* to *e*. To measure any small angular distance with this instrument, as the diameter of a planet, the two parallel wires are made to approach or recede from each other by turning the screw-heads *h h* till the body to be measured is exactly enclosed by them, while the longitudinal wire crosses the centre of the body. Having thus accurately measured the planet by the two cross wires, we must next ascertain their distance asunder, in the manner following: suppose there are 50 threads of the screw to an inch, and that the heads are divided each into 100 equal parts; now one of the screws is to be turned till one of the wires is brought into accurate contact with the other, when the number of turns and divisions requisite to effect this purpose will give the diameter of the planet, each division being equal to $\frac{1}{500}$th of an inch. But if we are desirous of determining in seconds, or parts of a degree, it is found by previously measuring a known base, or by ascertaining the time an equatorial star takes in passing from one wire to the other, and from thence deducing the seconds or parts of a second agreeing with each revolution of the screw-head.

Fig. 85.



The essential requisite in this micrometer is, that the wires be perfectly parallel, and that there be no shake in the forks that carry them.

(115.) *The micrometrical telescope* of Dr. Brewster, (§ 40) for measuring distances, may be employed for determining the diameters of bodies by having two parallel wires fixed across the field of view in the focus of the eye-glass; these wires being immoveable will not be liable to any inaccuracies in the screws, or from the uncertainty of obtaining a correct zero. The manner of using it is thus: suppose the inner moveable object-glass to be in the focal point of the principal one, and that the wires, by experiment, exactly take in an object of known magnitude, this will be the *minimum* angle capable of being measured with it. But when we have a larger object than can be enclosed within the wires, the inner object-glass must be brought nearer the principal one, thus reducing the power of the telescope by shortening its focus, so that the angle between the wire will be increased to admit the object to be measured; and as by the laws of optics it is known that when the two object-glasses are in contact the focus is shortest, so the angle between the wires will then be a *maximum*. Hence, any angle between these two points ascertained by experiment, may be determined by divisions registered along the tube.

(116.) The preceding principle of a micrometer may be applied to the Gregorian or Cassagranian telescope, without any additional apparatus; for the magnifying powers of either of these instruments may be varied by altering the distance between the large metal and the eye-piece, and then adjusting for distinct vision by the little metal. In this way Dr. Brewster proposed to determine the angle subtended by any object, having experimentally formed the scale for adjustment.

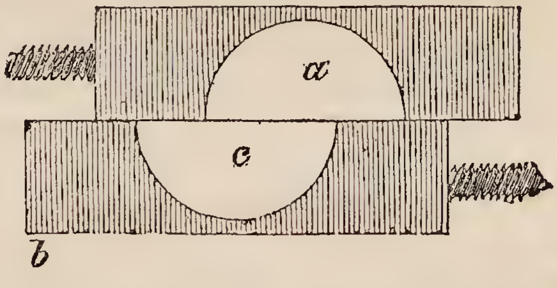
(117.) *Fibres for micrometers.*—After the contact of brass plates, employed by Huygens, for micrometers was discontinued, *silver wire*, *hairs*, and *spiders' webs* were introduced; the former, however, till latterly, could not be produced finer than $\frac{1}{650}$th of an inch diameter in this country, which, consequently, led to the choice of the other fibres. But, by the ingenuity of Dr. Wollaston, wire can now be obtained of only the $\frac{1}{20000}$th of an inch in diameter: this he effects by having a fine platina wire thickly coated with silver, which is drawn out as fine as possible in the usual manner, through steel or jewelled holes, and then, by finally immersing this wire in an acid that will dissolve the silver and not the platina, he obtains a perfect wire of any fineness that may be required.

(118.) The spider's web was first successfully employed for micrometers by Mr. E. Troughton, who used the stretcher, or the long line which supports the web, for this purpose, ths others being too weak. He found thie thread to possess the valuable properties of fineness, opacity, and elasticity.* But the difficulty of procuring this particular thread has led to other contrivances in its stead.

(119.) The micrometer threads of Dr. Goring, which have been termed *artificial cobwebs*, were introduced by him to obviate the easy destruction of the natural ones when kept for any length of time, and from the difficulty of procuring those of the proper kind. These threads are formed from a thick solution of gum caoutchouc in oil of turpentine, and by not employing in their formation a heat greater than that of the human body; after the threads are drawn out, the essential oil evaporates, and leaves the *Indian rubber* in the same state as at first.† The cobwebs made in this manner are not liable to injury by keeping, like the ordinary ones, while they possess the essential properties of opacity, fineness, parallelism, and elasticity, and are far superior to them in strength.

(120.) *The divided object-glass micrometer* is composed of two semilenses *a*, *c*, (*fig.* 86;) these act as two distinct object-glasses, each producing an image of the same object, and in order that their foci shall be of the same length, they are made by dividing a circular

Fig. 86.



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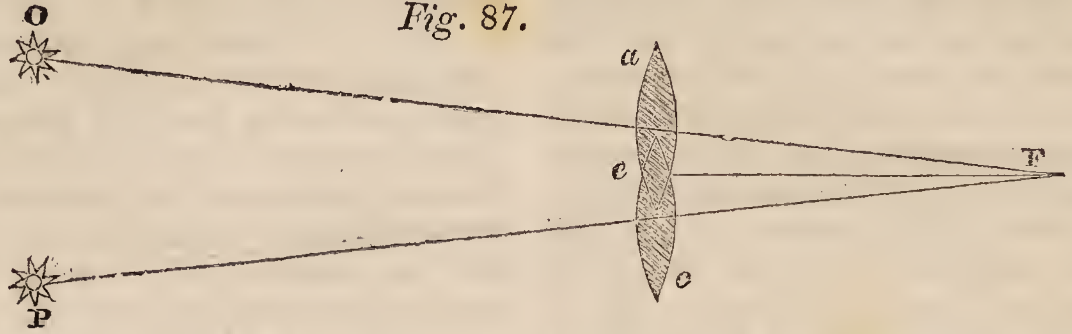
* This quality is lost by keeping.

† It should be observed, that the Indian rubber solution must not be kept in any vessel from which the air is entirely excluded, whereby the slow evaporation of the essential oil is prevented, as this consequence will decompose or change its nature, so that when drawn out into threads it remains in a clammy state and will not dry. This remark equally applies to all its solutions for making *water-proof* articles. The solution may be covered with a piece of wash leather.

lens across its centre. The centres of these semilenses are made to separate more or less, by means of a screw attached to the plates in which the lenses are mounted, as *b b;* and the distance of their centres is measured by a scale and vernier.

Fig. 87.



When it is required to measure the angle subtended by two objects, as O P, *(fig* 87,*)* the semilenses are separated till the images of both objects coincide at their foci F. This being accomplished, the angular distance of the two lenses, reckoning the vertex at E, their focal distance will be equal to the real angle subtended by the two objects at F, or at the place of the object-glasses, (the real distance, of *a* F or *c* F being very small in comparison to the distance of the objects O P from the object-glasses :) to find, therefore, the angle of the semilenses, we have the sides *a c* (*i. e.* the distance of the two centres) and *e* F their focal distance, which are all the data necessary to determine it trigonometrically. But as the angle in practice is usually very small, it may with little error be considered simply as the subtense *a c*, having experimentally determined the distance of the semilenses corresponding to two objects making a known angle with each other, when by simple proportion the angle for any other distance may be found.

(121.) The improvements that have been made on this divided object-glass micrometer by Dr. Brewster, consist in having the centres of the two semilenses at a fixed distance from each other, and by employing another object-glass in the ordinary manner : to produce the requisite variation of angle, the fixed semilenses are made to traverse along the axis of the telescope between the other object-glass and the eye-piece, thus producing a change in the magnifying power of the instrument; and by means of a divided scale along the tube, showing the distance of the semilenses from the principal object-glass, the angle is ascertained.

(122.) The principle of the divided object-glass micrometer has been applied to the microscope, for the purpose of measuring the diameters of various fibres, and from thence determining the quality and value of the material for the manufacture of different articles; its principal application has been in the measurement of wool, from which it is called *an eriometer*. The instrument consists of a compound microscope, either refracting or reflecting, having two semi-concave lenses capable of adjustment by screws and a vernier's scale; this apparatus is fixed between the object and object-glass or metal, and the measurement of the fibre, by the contact of its two images, is effected in the same manner as in a telescope.

(123.) *The mother-of-pearl micrometer*, invented by Mr. T. Cavallo, and described by him in the *Phil. Trans.* for 1791, has, from its simplicity, been very extensively employed in practical astronomy, and is indeed admirably suited for measuring any small angle with expedition. The strip of pearl used for this purpose is minutely divided, and stretched across the diaphragm, or stop, usually placed in the anterior focus of the eye-glass, either of a telescope or compound microscope; so that the divisions may be distinctly seen by the eye at the same time as the object. When we are desirous to measure an object with the former instrument, any given number of equal divisions on the pearl corresponding to a known angle is first determined by experiment; then, on looking through the telescope at the object to be measured, and counting the number of divisions the diameter of the object occupies, the angle it subtends is determined from the proportion of that number to the number answering to the known angle. When this micrometer is applied to the compound microscope, in order to ascertain the magnitude of any minute object, the strip of pearl is stretched across the field-bar of the instrument; it is then brought in the direction of the length of the object to be measured, by turning the tube of the eye-piece till they coincide. Then, if we suppose the number of divisions on the pearl dynameter to be 100 in the space of an inch, and the
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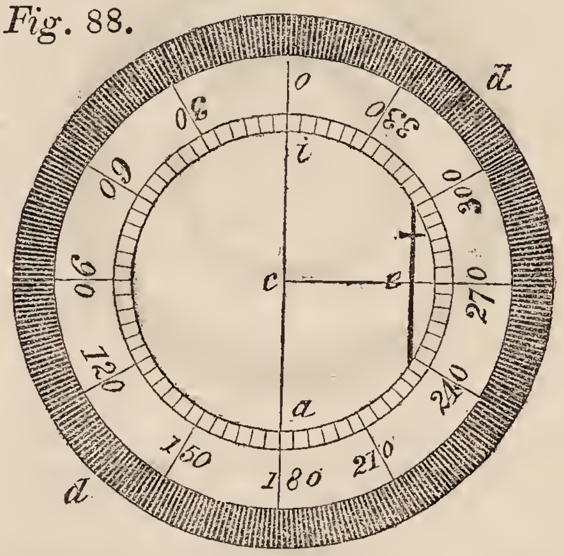
part to be measured is found to occupy two of these divisions, to determine its real size we must ascertain how much the object has been amplified by the object and field glasses. But if no field lens is employed, the power will be as the distance of the object-glass from the image, divided by the distance of the object from that glass. As inaccuracies are however liable to occur in these measures, the best practical manner (which is equally simple, let the number of glasses be soever numerous) is to determine the increase in magnitude by using another scale of very fine divisions in place of an object; knowing how many there are in an inch, and ascertaining the number of divisions on the one employed as an object that are equal to any number of the equal divisions on the pearl dynameter across the field-bar, by dividing the one number by the other, the amplifying power will be obtained. Example. Suppose the divisions on the scale (used as an object) be 1000 in the space of an inch, and one of such divisions is magnified so as exactly to cover one of the divisions on the pearl, which are $\frac{1}{100}$th of an inch, it is evident the scale, or an object placed in its stead, is magnified 10 times. Hence we know that as the object we proposed to measure occupies *two* divisions, it is $\frac{2}{1000}$ths of an inch long; each division on the pearl being equal to the $\frac{1}{1000}$th of an inch on an object placed in the focus of the object-glass. In using the micrometer, the power of the eye-glass is not required to be known, as the divisions on the dynameter are magnified in the same ratio by it as the object.

(124.) *Circular micrometers.* The micrometer last described, as stretched across the field-bar for many astronomical purposes, is found objectionable from the three following circumstances. First, the central rays from the object are obstructed by the pearl, which likewise divides it into two portions. Secondly, the situation of the central portion of the pearl being nearer the eye-lens than the other parts, the various portions will be unequally magnified, although the pearl is really divided into equal parts; and therefore the measurements are inaccurate, unless taken from one particular part. Thirdly, the edge of the micrometer always requires to be in the direction of the parts to be measured: for this purpose, it is necessary always to turn the eye-piece to bring it in the proper direction; this, undoubtedly, is a loss of time, besides the liability of disarranging the instrument for distinct vision. Having thus stated the defects of that micrometer, it is only necessary to mention that the circular micrometer is entirely free from these evils; for the central space being clear, the rays are not obstructed, while the divisions are all equally magnified, and the object can be measured with equal facility and accuracy in any direction: indeed, the advantage of employing this instrument is so great, that we need only mention that the orbits of three out of the four new minor planets were determined by a circular micrometer alone.

(125.) *The circular pearl micrometer* was invented by Dr. Brewster, and consists of an annular portion of mother-of-pearl, *a, i, (fig.* 88,) fixed on its outer edge to the diaphragm *d, d,* at the end of a piece of brass tube, which is capable of being adjusted exactly to the anterior focus of the eye-glass of the telescope or microscope; the inner circumference of the pearl is divided into 360 equal parts or degrees; and when this micrometer is thus adjusted for use, the maximum angle must be determined experimentally: this angle will be subtended by the inner diameter of the pearl *a, c, i.* Now, if we suppose this angle, by observation, to be *two* degrees, we shall be able to find the value of the angle any other object subtends in any direction less than 2°; thus, let the object be represented by the line *e*, which, by inspection, is found to occupy sixty divisions; bisect the angle the object *e* subtends, and it will be equal to the sine of half the angle *a c i:* or the angle which this object subtends is equal to twice the sine of half the angle, *a c* being the radius; so that the object *e* subtends half the maximum angle, or *one degree.* In this manner, trigonometrically, might be found the angle answering to every

Fig. 88.



division, which should be formed into a *table;* and thus it would be given by inspection.

(126.) *The circular suspended micrometer* is an instrument, simple in its construction, and admitting of extreme accuracy in its execution; it is less subject to injury, while it possesses many advantages over others in its application to astronomical observations. This micrometer was invented by the late M. Fraunhofer, and consists of a circular disc of parallel plate glass *a*, (*fig.* 89,) having in its centre a small circular hole of about *half an inch* diameter, and turned very true in a lathe; to the inner edge of this circle a narrow ring of steel *c* is securely fastened, when its inner edge is turned perfectly circular, and reduced very thin. The glass plate, with its steel ring, is then mounted in a brass tube or setting *d, d,* by means of which it can be adjusted to the focus of the eye-glass, (similar to the last micrometer.) This micrometer, when viewed in the telescope, appears like a narrow ring *suspended* in the heavens, from whence it derives its name. The chief advantage in this instrument is the accuracy by which the moment of *ingress* and *egress* of a planet or star is determined; for the body being seen in the field of view through the glass plate, before it comes to the inner edge of the steel ring, allows the precise moment of contact to be very readily observed. The angle subtended by the ring must be found in the same manner as with the other micrometers; or by noting the time an equatorial star passes when near the meridian, and deducing therefrom the angle which the inner edge of the ring subtends. The velocity of a planet may be determined in the same manner while near the meridian; or the difference of the time occupied by a star and the planet passing together, would determine the motion of the latter, making proper allowance when crossing above or below the centre.

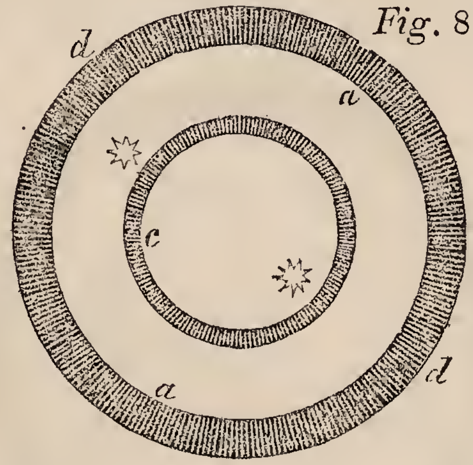


Fig. 89.

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BREWSTER, Sir David (2)

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6. The animal kingdom also contributes its aid in support of the same theory. The choroid coat of the dog and other animals, which produces the *blue, green,* and *red* reflexions from the eye of the living animal, retains the same faculty after death. When the choroid coat dries, it becomes black, and the colours disappear. We have found, however, that after remaining dry for nearly ten years, their colours could still be developed by moisture. The *black* passed instantly into a *brilliant blue,* the *blue* into *green,* and the *green* into *greenish yellow.*

5. *Colours of the atmosphere.*—As the earth is surrounded with an atmosphere varying in density from the surface of the globe, where it is a maximum, to the height of about 45 miles, where it is extremely rare, and just able to reflect the rays of the setting sun, the rays of the sun, moon, and stars are refracted into curve lines, unless when they are incident upon it perpendicularly. Hence the apparent altitude of the celestial bodies is always greater than their real altitude, and they appear above the horizon when they are actually below it.

But while the solar rays traverse the earth's atmosphere, they suffer another change from the resisting medium which they encounter. When the sun, or any of the heavenly bodies, are considerably elevated above the horizon, their light is transmitted to the earth without any perceptible change; but when these bodies are near the horizon, their light must pass through a long tract of air, and is considerably modified before it reaches the eye of the observer. The momentum of the red, or greatest refrangible rays, being greater than the momentum of the violet, or least refrangible rays, the former will force their way through the resisting medium, while the latter will be either reflected or absorbed. A white beam of light, therefore, will be deprived of a portion of its blue rays by its horizontal passage through the atmosphere, and the resulting colour will be either orange or red, according to the quantity of the least refrangible rays that have been stopt in their course. Hence the rich and brilliant hue with which nature is gilded by the setting sun; hence the glowing red which tinges the morning and evening clouds; and hence the sober purple of twilight which they assume when their ruddy glare is tempered by the reflected azure of the sky.

We have already seen that the red rays penetrate through the atmosphere, while the blue rays, less able to surmount the resistance which they meet, are reflected or absorbed in their passage. It is to this cause that we must ascribe the blue colour of the sky, and the bright azure which tinges the mountains of the distant landscape.

As we ascend in the atmosphere, the deepness of the blue tinge gradually dies away; and to the aeronaut who has soared above the denser strata, or to the traveller who has ascended the Alps or the Andes, the sky appears of a deep black, while the blue rays find a ready passage through the attenuated strata of the atmosphere. It is owing to the same cause, that the diver at the bottom of the sea is surrounded with the red light which has pierced through the superincumbent fluid, and that the blue rays are reflected from the surface of the ocean. Were it not for the reflecting power of the air, and of the clouds which float in the lower regions of the atmosphere, we should be involved in total darkness by the setting of the sun, and all the objects around us would suffer a total eclipse by every cloud that passed over his disk. It is to the multiplied reflections which the light of the sun suffers in the atmosphere, that we are indebted for the light of day, when the earth is enveloped with impenetrable clouds.

From the same cause arises the sober hue of the morning and evening twilight, which increases as we recede from the equator, till it blesses with perpetual day the inhabitants of the polar regions.

The cause which we have assigned for the blue light of the sky, and which was, we believe, first given by Bouguer, though a very probable one, still required the evidence of demonstration. In examining this light, Dr. Brewster found that a great portion of it was polarized; and hence it follows, that it has suffered reflexion. M. Saussure found that the intensity of the blue colour increased with the height of the observer above the sea; and it has been observed by others, that the intensity diminishes as the quantity of aqueous vapour is increased. In order to measure this intensity, M. Saussure contrived an instrument called a *Cyanometer.** A circular band of thick paper or pasteboard is divided into 51 parts, each of which is painted with a different shade of blue, decreasing gradually from the deepest blue, formed by a mixture of black, to the lightest, formed by a mixture of white. This coloured zone is held in the hand of the observer, who notices the particular tint which corresponds to the colour of the sky. The number of this tint, reckoned from the greatest, is the intensity at the time of observation. Saussure, Humboldt, Depons, and other travellers, have made observations with this instrument. The following are some of their results:

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* From two Greek words signifying a *measure* and *blueness.*

General intensity in Europe	14°
General intensity in the Caraccas ...	18
General intensity at Cumana	24

6. *Coloured Shadows.*—The shadows of bodies placed only in one light, and at a distance from all other bodies capable of reflecting light, must necessarily be black. In a summer morning or evening, however, the shadows of bodies formed either by the light of the sun, or by that of a candle, have been observed to be *blue;* this obviously arises from the shadows being illuminated with the light of the blue sky. The colours thus produced vary in different countries, and at different seasons of the year, from a pale blue to a violet black; and when there are yellow vapours in the horizon, or yellow light reflected from the lower part of the sky, either at sunrise or sunset, the shadows have a tinge of green arising from the union of these accidental rays with the blue tint of the shadow.

If the light of the sun, or of the candle, be faint, then the shadow of the body formed by the light of the sky will be visible also, and the two shades will be the one *blue* and the other a *pale yellow,* two colours which are complementary to each other. This fact has been ascribed to the circumstance of the light of the candle, and that of the rising and setting sun, being of a yellowish tinge; but though this will increase the effect it is not the main cause of it, as one of the shadows would be yellow, even if the light of the sun and the candle had been perfectly white.

The phenomena of coloured shadows are sometimes finely seen in the interior of a room; the source of one of the colours being sometimes the blue sky, and the other the green window blinds, the painted walls, or the coloured furniture.

The best method of observing and studying this class of phenomena is to use two candles, and to hold before one of them a piece of coloured glass, taking care to remove to a greater distance the candle before which the coloured glass is not placed, in order to equalize the darkness of the two shadows. If we use a piece of green glass, one of the shadows will be *green,* and the other a fine *red;* if we use blue glass, one of the shadows will be *blue,* and the other a pale *yellow,* and so on; the one colour being always complementary to the other, as explained in page 46. The light from the candle with the green glass obviously illuminates the shadow formed by the other candle, and hence it is easy to understand why that shadow is green; but as the other shadow is illuminated only by the common light of the candle which is not red, it appears difficult to discover the origin of the red light. The explanation of this must be sought not among optical, but among physiological principles. We have already seen, when treating of accidental colours, that when a portion of the retina was strongly impressed with any one colour, such as red, that same portion tinged *green* the images of white objects that fell upon it. In like manner, when nearly the whole retina is impressed with any one colour, such as red, a portion of it not impressed with that colour will tinge white objects green,—or, to speak more generally, every excitation of the retina by one colour is accompanied by an excitation of its accidental colour, just as in Acoustics every fundamental sound is actually accompanied by its harmonic sound. Hence, when we see red we at the same time see green, but its impression is less forcible, and the tendency of this double vision of colours is to weaken the original impression, *viz.* the red; because the union of complementary colours produces whiteness. This may be proved by looking for a considerable time at a red wafer, which will appear less and less red the longer we view it; because the green which the retina is seeing at the same time, produces a whiteness which dilutes the red. This we conceive to be the true theory of accidental colours. Its application to coloured shadows is very obvious: When the eye is impressed with the green colour of the light transmitted through the green glass, it at the same time sees red, which, of course, appears only on the shadow upon which a green light falls.

7. *Converging and diverging beams.*—When the sun is descending in the west, through masses of open clouds, the divergency of his beams, rendered visible by their passage through numerous openings, forms frequently a very beautiful phenomenon. It is sometimes accompanied, however, with one of an opposite kind, *viz. the convergency of beams to a point in the eastern horizon opposite to the sun, and as far beneath the horizon as the sun is above it,* as if another sun, throwing out divergent beams, were about to rise in the east. This phenomenon is rarely seen in perfection. Dr. Smith, who observes that he *once* saw this phenomenon on Lincoln heath, describes it as 'an apparent convergence of long whitish beams towards a point diametrically opposite to the sun, and, as nearly as he could estimate, as much below the horizon as the sun was then elevated above the opposite point of it.'

On the 9th of October, 1824, we had the satisfaction of seeing this curious appearance in unusual splendour. The sun was considerably elevated, and was throwing out his diverging beams in great beauty through the interstices of the broken masses of clouds which floated in the west. The eastern portion of the horizon, where the converging lines were seen, was occupied with a black cloud, which seems necessary as a ground for rendering visible, by its contrast, such feeble radiations. The converging beams were very much fainter than the diverging ones, and their point of convergence was as far below the horizon as the sun was above it. About ten minutes after the phenomenon was first seen, the convergent lines were *black*, or very dark. This arose from the real beams having become broad, and of unequal intensity, so that the eye took up, as it were, the spaces between the beams more readily than the beams themselves.*

In order to explain this phenomenon, which is a case of perspective, let us suppose a line to join the eye of the observer and the sun. Let beams issue from the sun in all possible directions, and let us suppose that planes pass through these beams, and through the line joining the eye of the observer and the sun, which will be their common intersection, like the axis of an orange, or the axis of the earth, through which there pass all the septa of the former, and all the planes passing through the meridians of the latter. An eye, therefore, situated in this line, or common intersection of all the planes, will, when looking at a concave sky, apparently spherical, see them diverging from the sun on one side, and converging towards the opposite point, just as an eye in the axis of a large globe would perceive all the planes passing through the meridians diverging on one side, and converging on another.*

Chapter XIX.—*Partial Reflexion of Light—Absorption of Light—Light reflected at different angles from Water — Glass — Metals — White opaque Bodies—from both surfaces of Glass—from a number of Glass Plates.*

From the phenomena described in the preceding chapters, the reader must have observed, that when light falls upon the most transparent bodies, such as water, glass, &c. a certain portion of it is reflected from their surfaces. When we measure the quantity *reflected* and the quantity *transmitted,* we invariably find that the sum of these quantities is less than the light which falls upon the body. Hence it follows, and the fact is a very important one to remember, that *light is always lost in passing through the most transparent bodies.* This light is lost in two ways: a portion of it is absorbed or stopped by the body and forms heat, and another portion is scattered in all directions by irregular reflexion. When light falls on metallic bodies, such as polished silver, or speculum metal, about one half of it is reflected, and the other half lost. The part lost consists, as in the former case, of two portions; one of which, and by far the largest, being absorbed, and the other scattered by irregular reflexion.

No complete set of experiments has yet been made from which the laws of these phenomena can be determined. The principal facts which have been ascertained, we owe to the ingenuity of M. Bouguer and M. Lambert, and these we shall now lay before the reader :—

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* This disposition of the eye is a very curious one, and has, we believe, never been observed. When we look steadily at a carpet having figures of one colour, green for example, upon a ground of another colour, suppose red, we shall, sometimes, see the whole of the green pattern, as if the red one were obliterated; and, at other times, we shall see the whole of the red pattern, as if the green one were obliterated. The former effect takes place when the eye is steadily fixed on the green part, and the latter, when it is steadily fixed on the red portion. It is easy to conceive that when the retina is in a state of irritation or excitation with red light, it will more easily take up, as it were, the vision of a red object than of any other.

† See Smith's *Optics*, vol. ii. Remarks, p. 57, 58; and *Edinburgh Journal of Science*, No. iii. p. 136.

Angles of Incidence.	Number of Rays reflected out of 1000.					
	Water.	Glass.	Quick-silver.	Silver.	Plaster.	Dutch Paper.
89½°	721	..	721	..	..	..
89	692	..	..	..	..	..
88½	669	..	..	..	..	..
88	639	..	..	..	..	..
87½	614	584	..	..	..	..
85	501	543	..	..	..	..
82½	409	474	..	..	..	..
80	333	412	..	..	..	..
77½	271	356	..	..	..	..
75	211	299	..	209	194	203
72½	178	222	..	..	..	..
70	145	210	..	..	..	..
65	97	156	..	..	..	..
60	65	112	..	319	352	332
50	34	57	..	..	..	..
45		41	..	455	529	507
40	22	34	704	..	..	..
30	19	27	..	640	640	..
20	18	25	..	..	..	..
15	18	25	..	802	762	971
10	18	25	..	..	..	..
0	18	25	666	1000	1000	1000

From these results we may draw the following conclusions :—

1. That in *fluids*, *transparent solids* and *metals*, the quantity of light reflected increases with the angle of incidence reckoned from the perpendicular ; whereas in white opaque bodies the quantity of light reflected decreases with the angle of incidence.
2. That at great angles of incidence water reflects more light than even both the surfaces of plate glass.

The following very accurate results were obtained by M. Lambert, who measured the quantity of light reflected both at the first and the second surfaces :

Angles of Incidence.	No. of Rays out of 1000 reflected at *first* surface.	No. of Rays out of 1000 reflected at *second* surface.
70°	158	320
60	77	165
50	47	105
40	34	71
30	26	59
20	22	50
10	20	45
0	20	45

Lambert does not seem to have observed the law of these results. Upon carefully comparing them, we have found that the quantity of light reflected at either surface is inversely as the square of the cosine of the angles of incidence. It is a most curious fact which Lambert has established, that the light reflected at the second surface of a plate of glass is at all angles of incidence more than double of the quantity reflected by the first surface.

M. Lambert likewise obtained the following results for different numbers of very transparent plates of glass at a perpendicular incidence.

No. of Plates of Glass.	No. of Rays reflected out of 1000.	Light transmitted.
1	62	938
2	117	883
3	165	835
4	209	791
5	248	752
6	284	715
7	316	684
8	345	654

When the glass which he used was less transparent, he obtained the following results, the quantity of light lost being given in the last column.

No. of Glass Plates.	No. of Rays reflected out of 1000.	No. of Rays transmitted out of 1000.	No. of Rays lost out of 1000.
1	52	811	137
2	86	660	255
3	108	537	355
4	123	438	449
8	147	195	659
16	152	39	809
32	153	2	846

From the preceding facts it is obvious, that in all the various operations by which we distribute or concentrate light for economical purposes, a considerable portion of it is lost. This portion is much greater in metals than in glass ; and hence, when other circumstances are the same, *lenses* are preferable to *mirrors* or *specula*, either for concentrating the solar rays for the purposes of combustion, or for producing in light-houses an intensely brilliant column of light capable of reaching to a great distance, and penetrating the fogs of the ocean. For the same reason a *Refracting* telescope gives far more light than a *Reflecting* one of the same aperture; and if we could manufacture glass as easily as we can cast metallic specula, the Reflecting telescope would disappear from among optical instruments in actual use.

(3)

LARDNER, Dionysius

A POPULAR ACCOUNT
OF
NEWTON'S OPTICS.

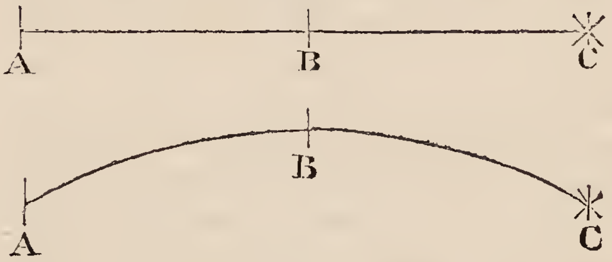
Chapter I.

Of the State of Optical Science before the time of Newton.

(1.) The splendid phenomena of optics must have been among the first natural appearances to attract the attention of mankind. Of all the objects in nature, light is perhaps the most pleasurable. Vision, at once the most perfect and useful of the senses, wholly depends on it. By its agency the sphere of our observation and experience is indefinitely enlarged. It brings us sure and immediate intelligence of existences and events, whose places are remote, and thus gives us a certain degree of omnipresence. Setting aside all the beautiful variety of form and figure, and the gorgeous phenomena of colours, which it is the means of disclosing, *light itself* is a delightful perception. Nature supplies it so continually and so abundantly, that we are apt to forget its value; but, in cases where habit has not blunted the sense of pleasure, it seems to produce singular enjoyment. The infant eagerly directs its gaze to the window or the lamp, and stretches forth its hand as if to grasp an object so agreeable. Persons blind from infancy, but whose organs are not absolutely opaque, derive exquisite pleasure from the perception of the cloudy light which the imperfectly transparent humors allow them.

The property of light soonest noticed was most probably its *rectilinear propagation*, by far the most important of its qualities, and one with which all the others are intimately connected. It was impossible to observe the effects of opaque bodies on light, the confines of their shadows and their effects on the sense of sight, without at once discovering this important law. An opaque body B (*fig.* 1.), placed in a right line A C joining another opaque body A and a luminous point C, deprived that other opaque body A of the light it received from the point C; but if the same opaque body B was placed in any curved or crooked line ABC, joining the luminous point C and illuminated body A, no such obscuration was produced. Again, if a straight line be drawn from the eye to a luminous point, and also any curved or crooked line drawn from the eye to the same point, an opaque body placed any where in the straight line will deprive the eye of the perception of light; but if the same opaque body be placed any where in the curved or crooked line, the perception of light continues. Facts like these must have occurred so constantly at all times, and in all places, the inference from them is so evident and immediate, that it is impossible to suppose that the rectilinear propagation of light was not known even in the most rude and savage state.

Fig. 1.



(2.) Some of the phenomena of *reflection* must also have attracted attention, and excited inquiry at a very early period. The inverted picture of a landscape in the placid water of the lake or river must have been viewed with astonishment. Polished surfaces, natural and artificial, presented themselves in sufficient abundance to furnish numerous experiments on reflection, and thus from the rectilinear propagation of light the step to the *law of reflection* was not very difficult. The equality of the angles of incidence and reflection was taught in the Platonic school, and probably was known considerably prior to that date.

Aristotle devoted considerable attention to the phenomena of rainbows, parhelia, halos, &c. He observed these with accuracy, and ascribed the rainbow to the imperfect reflection of the sun's rays by drops of rain. Some of the properties of concave and convex spherical mirrors appear to have been taught in the Alexandrine school, and are explained in a treatise on optics contained in the works of Euclid.

(3.) The phenomena of refraction not being of so striking a kind, or of such frequent occurrence as those of reflection, do not seem to have been noticed until a later period. The power of refractors to collect the sun's rays to a focus, so as to burn any substance exposed to their influence, was, however, long known. Burning refractors are very distinctly described in Aristophanes' comedy of *The Clouds*, and Aristotle observed the broken appearance of a stick held obliquely in water, and attempted to account for it. In the first century of the Christian era several optical phenomena were investigated by Seneca, and among others he observed that writing viewed through a glass bottle filled with water was magnified. This was, probably, the first discovery of the magnifying power of a refracting medium, bounded by convex surfaces. Seneca also noticed the colours produced by a prism or angular piece of glass, and observed that they were similar to those of the rainbow.

(4.) The most distinguished among the ancients for discovery in optical science was *Claudius Ptolemy*, the celebrated astronomer of Alexandria, who flourished in the second century of the Christian era. This philosopher was the first who observed, with any degree of scientific precision, the phenomena of refraction. He appears to have systematized, improved, and imparted a greater degree of accuracy to all that was previously known concerning the reflection of light at plane and curved surfaces, but he must justly be considered as the first and exclusive discoverer of the principal phenomena of refraction or dioptrics.

Ptolemy observed that if a visible object, a piece of money for example, be laid on the bottom of a vessel, and the eye so placed that the edge of the vessel just intercepts the view of the object, upon filling the vessel with water, the object will seem to be gradually raised, until at length it comes distinctly into view. This fact led him directly to the true nature of refraction, and shewed that the visual ray in passing out of the water at the surface, was bent towards the eye of the spectator, so as to make the object on the bottom of the vessel appear higher in the water than its real position. He invented an instrument to measure the deflection of the ray in passing from the water into air. This instrument consisted of a circle, carrying two *sights* on its graduated rim, and a third sight at the centre. The circle was immersed in the water, with its plane perpendicular to the surface, and so that the surface of the water coincided with one of its diameters, and that one of the sights on the rim was above and the other below the surface. The eye being placed at the sight above the surface, the sight below the surface was moved upon the rim, until the three sights appeared to lie in the same straight line. The distances of the sights on the rim, from the highest and lowest points of the circle, then shewed the angles of refraction and incidence.

With this instrument Ptolemy observed and calculated the refractions corresponding to every ten degrees of incidence in the quadrant, the refraction being made between air and water. He also measured and calculated the refractions between *air* and *glass* and *water* and *glass*, by cutting the glass into a semi-cylinder of the same diameter as the graduated circle, and applying the semicircle to the end of the semi-cylinder.

The results of these experiments showed that the deflection of the ray between glass and water was less than in either of the cases between water or glass and air. From this he was led to conclude that the difference of the densities of the media was the cause of refraction, since water and glass differed less in density than air and glass, or air and water. This fortunate observation suggested the probability that light from celestial objects, in passing into our atmosphere from the surrounding medium, whatever it be, might suffer a deflection, the consequence of which would be that our view of the whole face of the heavens must be distorted; objects appearing variously removed from their true places, according to their various positions with respect to the highest point or the *zenith*. The only point which would not be removed from its true place under such circumstances, would be the zenith itself; for the ray passing from that point to the eye, must enter the atmosphere perpendicularly; and he observed that in his experiments, whenever the ray met the surface of the water or glass perpendicularly, there was no deflection.

Thus he perceived that the zenith was a fixed point, with reference to which he should be enabled to ascertain this interesting fact. The test to which he submitted this is a remarkable instance of philosophical acuteness. In his experiments he had observed that the more obliquely the ray met the refracting surface, the greater was the deflection of the refracted from the incident ray. Hence he supposed that those objects which were more remote from the zenith, and the light of which met the atmosphere more obliquely, were more removed from their proper places. In other words, that the distortion of the firmament by the refraction of the atmosphere was greater near the horizon than near the zenith. Accordingly, he observed the positions of the same star in different parts of its diurnal path in the heavens, and found that it appeared not to move in a *lesser circle* parallel to the celestial equator, but that it continually deviated in a slight degree from such a circle; that this deviation was greater, the more distant the star was from the zenith or highest point, and that the deviation always brought the star nearer to the zenith.

Such were the phenomena he observed, and they were precisely what his experiments suggested. The last mentioned circumstance, of the deviation being always towards the zenith, showed that the refracted ray was bent towards the perpendicular, which proved that the density of the atmosphere must be greater than that of the fluid, if such there be, which pervades the region beyond it.

(5.) A long interval elapsed after the age of Ptolemy, before the science of optics made any advance. About the beginning of the twelfth century, some steps were made towards improvement in the theory of vision. In the thirteenth century Roger Bacon devoted considerable attention to the study of optics; and although he cannot be said to have extended the bounds of the science by positive discoveries, yet he has so plainly described the effects on vision produced by lenses and their combinations, that we cannot, with any regard to justice, deny him a share in the honour of the invention of spectacles, telescopes, and microscopes. In describing the effects of convex lenses of glass, he states that "they are useful to old men, and to those that have weak eyes, for they may see the smallest letters sufficiently magnified." Respecting the effects of combinations of lenses, he says—"We shall see the object near at hand, or at a distance, and under any angle we please. And thus from an incredible distance we may read the smallest letters, and may number the smallest particles of dust and sand, by reason of the greatness of the angle under which we see them; and on the contrary, we may not be able to see the greatest bodies just by us, by reason of the smallness of the angles under which they appear; thus a boy may be as big as a giant, and a man as big as a mountain, forasmuch as we may see the man under as great an angle as the mountain, and as near as we please. Thus, also, the sun, moon, and stars, may be made to descend hither in appearance, and to appear over the heads of our enemies, and many things of a like sort, which would astonish unskilful persons."*

It will be perceived that Bacon not only describes the effects of telescopes, but also distinctly alludes to their causes. It is very difficult to conceive that he could have written thus, without having actually constructed the instruments, and witnessed the effects which he describes.

(6.) In the sixteenth century, Maurolycus explained, with great exactness, the structure and functions of the eye, more especially of the crystalline humor. He showed that those defects which are called long-sightedness, and short-sightedness, proceeded from too small or too great a refracting power in the eye, and showed how and why these defects were removed by the use of convex and concave lenses. Maurolycus failed to discover the formation of the picture on the retina, and the functions of that coat, from the difficulty of reconciling an inverted image with our perception of erect objects.

About this time Baptista Porta, a Neapolitan philosopher, invented the *camera obscura*. He observed that if a small hole be made in the window-shutter of a darkened chamber, the images of external objects will appear depicted in their proper colours on the opposite wall. He then tried the effect of a convex lens fixed in the aperture, and found that the images became much more perfect and distinct. In this way the features of persons outside could be discerned, and known from the image. The pictures, however, thus produced, were inverted. To remedy this, he proposed that, instead of being projected on the wall, the rays from the lens should be received upon a convex mirror properly placed and adapted to the lens.

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* See the Edinburgh Encyclopædia—Optics.

Observing that the images thus formed by the convex lens were magnified, and seen with more distinctness than with the naked eye, it occurred to him that if a convex lens were presented to an object so as to form an image at its focus, an eye placed a short distance behind that image would see the object magnified. Such an arrangement was, in fact, a telescope without an eye-glass.

(7.) The phenomenon of the rainbow could not have failed to attract attention at a very early period. A work has been brought to light by Venturi, written, in 1311, by Theodoric of Saxony, a Dominican friar, in which a rational explanation of the double bow is given. Extensive extracts from this work, with the figures of the single and double reflection within the drops, may be seen in the sixth volume of the *Annales de Physique et de Chimie.*

The art of printing not having been then invented, it is probable that the work of Theodoric did not gain publicity, and was not generally known. Accordingly, we find several eminent philosophers, at the end of the sixteenth and commencement of the seventeenth centuries, engaged in attempts to solve the problem of the rainbow. Some conceived the exterior bow to be a reflected image of the interior one, and thus accounted for the inversion of the colours. Fleschier, of Breslau, attributed the production of the colours of the rainbow to two refractions by the drop, but conceived that a reflection took place at another drop before the light reached the eye. It is curious to observe how slowly and gradually the laws of nature are discovered.

Soon after the year 1600, Antonio de Dominis, archbishop of Spalatro, reduced the phenomenon of the rainbow to actual experiment, and proved that one reflection only, with two refractions, is sufficient to produce the effect. He filled a hollow globe of glass with water, and having placed it in a proper position with respect to a beam of solar light, found that when viewed in the same direction, and under the same circumstances, as the drops of rain which form the bow, the same colours were produced, and were disposed in the same order. He conceived that a solar ray entering at the upper part of the drop, was refracted to the back of it, where it suffered a reflection by the inner surface; passing out again at the lower surface, that it reached the eye of a spectator properly placed, and produced a perception of the colours of the bow. He accounted for the colours in the following manner. The red rays issued from the nearest part of the inner surface of the drop, and having traversed a less quantity of water, preserved the greater degree of intensity; for the red colour was always considered to be produced by the most intense and active portion of the light. The green and blue rings, on the contrary, were those which were reflected from that part of the posterior surface of the drop which was most distant from the point of final emergence, and having, therefore, traversed a greater quantity of water, were more faint. The other colours of the bow were conceived to be formed by these three mixed in various proportions.

If a straight line be drawn from the sun to the centre of the drop, and continued through the centre, it will meet the posterior surface, at a certain point. De Dominis conceived that the rays which produced the same colour were similarly situate with respect to this point, and therefore that such rays ought to form with the line drawn from the sun to the eye of the spectator equal angles. Hence he inferred that each colour should appear in a circular band or in the surface of a cone of which the eye is the vertex, and the line from the eye to the sun the axis. Upon these principles he accounted for the shape of the bow and the order of the colours, and confirmed his theory by corresponding experiments with the glass globe.

It has been considered extraordinary that it should be reserved for De Dominis to make the first important step towards an explication of the most singular and beautiful phenomenon in nature. Montucla declares his work to be obscure and confused, and to betray an unusual ignorance even of so much of optical science as was generally known in that day. Some difference of opinion exists, however, as to the extent of his claims to a share in the merit of the explanation of this phenomenon. Montucla states, that he cannot concede to him greater merit than that of having had a glimmering of the true explanation of the interior bow. From the manner in which he explains the course of the rays, it seems doubtful whether he was even aware of the second refraction which the rays suffered on their emergence from the drop. Montucla, however, denies him any participation in the discovery of the cause of the exterior bow, and states that he had not the most remote idea of the double reflection which constitutes the character of this phenomenon. Dr. Brewster, on the other hand, an high authority on this subject, says, that although his attempt at explaining the interior bow is sufficiently absurd, yet that it is impossible for any philosopher, free from the influence of national partiality, to deny that the Italian prelate has given such an explanation of the general phenomena of the exterior bow, that any other philosopher of more optical knowledge, and of inferior acuteness, could not fail, without any stretch of intellect, to give precision and perfection to the explanation. Newton's own opinion appears to have been similar to that last quoted.

About the period at which De Dominis instituted his experiments, Kepler proposed an explanation of the rainbow, in a letter to Harriot. He conceives that the solar ray which touches a drop of rain is refracted, and penetrating the drop meets the posterior surface, from which it undergoes a partial reflection; that again penetrating the drop, it emerges, and at its emergence undergoes a second refraction, after which it reaches the eye of the spectator. Kepler here approximated very closely to the true solution of the problem. If, however, the ray which finally reached the eye were a tangent to the drop when incident on it, the bow would be much smaller than it is known to be. Harriot did not, on this occasion, give to the subject that attention to which its importance entitled it, but excused himself to Kepler by business and indisposition, promising that he would one day develop the mystery. It is evident that he agreed with Kepler in the necessity of a reflection intermediate between the two refractions, which, indeed, considering the relative positions of the sun, the drops, and the spectator, was sufficiently obvious. But he said nothing which can guide us to a knowledge of how far he was acquainted with the true theory of the phenomenon.

(8.) The accumulated facts and experiments furnished by various philosophers, and the numerous suggestions of optical writers, on the use and applications of lenses, and their combinations, among which those of Roger Bacon and Baptista Porta are more especially entitled to notice, had now prepared the way for the construction, we will not say invention, of telescopes and microscopes. The approach to the construction of the telescope, like the passage from darkness through twilight to broad day, is so gradual, that it is almost impossible, if we deny the invention to Bacon, to assign it to any other. Descartes assigns the discovery to James Metius, a Dutchman, and a citizen of Alkmaer. He commences his Dioptrics with this humiliating confession:—"To the shame of science, this admirable invention was the fortuitous result of experience. About thirty years ago, a person named James Metius, who had never studied, although his father and brother had devoted themselves to the profession of mathematics, but who took great pleasure in making mirrors and burning glasses, having occasion for glasses of different forms, happened to look through two of them, of which one was convex and the other concave. He applied them to the ends of a tube, and thus happily formed the first telescope."

Not satisfied with this origin of the telescope, some authors have sought for one still more humiliating to science, and to the pride of human intellect. It is said that the children of a spectacle-maker at Middleburg, happening to play in their father's shop, were amusing themselves with looking at a weather-cock with two glasses, the one convex and the other concave; and happening to place them in a fit position, they beheld the object magnified, and brought close to them. They communicated their astonishment to their father, who, to make the experiment more convenient, fixed the glasses in a proper manner upon a board. Presently another person adjusted the lenses at the ends of a tube, so as to exclude the lateral light, which disturbed the vision, and thus made the objects appear more brilliant and distinct. The next improvement, which trod upon the heels of the last, was to use tubes which moved one within another, so as to admit of any adjustment of the lenses which might be found necessary.

Without insisting further on this account of the invention, which, besides being unsupported by evidence, is attended with circumstances which give it small probability, we shall notice the better substantiated claim of Zacharias Jansen, urged by Borelli, and attested by legal witnesses, who were regularly sworn by the consular magistrates of Middleburg in the year 1655. Zacharias Jansen appears to have been a spectacle-maker at Middleburg, and the witnesses were his children, a son and daughter. The son assigns the invention to the year 1590, and the daughter to the year 1610. Both, however, agree in the fact of the invention; and the difference in the dates assigned to it, instead of invalidating their testimony, ought to be considered favourable to its truth, since it shows that no conspiracy existed between them. Other witnesses give the honour of the invention to Jean Lapprey, a spectacle-maker in the same place.

All these circumstances, considered with reference to the general state of optical knowledge at the time, render it probable that telescopes were constructed by several persons nearly at the same period, each being ignorant of what had been done by the others. Effects so singular and so brilliant as those produced by the telescope could not be long confined to one country. It will easily be believed, that such an instrument could not continue a mere toy of amusement, or matter of curious observation to philosophers. Among those who applied it to the great ends of science, the name of Galileo stands foremost. If we could credit his own account, it would not be more than justice even to assign to this philosopher a share in the honour of the invention, although we cannot concede to him a priority. He states that he was at Venice when a report of the wonderful effects of this discovery was spread abroad. Doubtful at first to what degree of faith statements apparently so incredible were entitled, he awaited a confirmation of the intelligence, which he received in letters from Paris. Being credibly assured of the reality of the powers ascribed to the new instrument, but uninformed of the particulars of its construction, he applied himself to the investigation of those particulars, by the aid of the established theory of refraction, and completely succeeded in discovering them. He forthwith applied a convex object-glass, and concave eye-glass, to the extremities of a tube, and directing it to distant objects, found that it rendered them three times as large as they appeared to unassisted vision. The success of this first attempt stimulated him to further exertion, and he constructed a second telescope which magnified about eight times. Finally, sparing neither labour nor expense on a subject which seemed to promise results so important, he produced a telescope which magnified *thirty times*, and with this he discovered the satellites of Jupiter, the solar spots, and other phenomena.

This account of Galileo's proceedings is given upon his own authority. It does not seem, however, likely that Galileo could have remained in total ignorance of the means which produced the wonderful effects which had excited such general attention. Besides, he acknowledges that he did not trust to mere public report, but received a letter expressly on the subject from "the noble James Badovere at Paris." Of this letter, which was written to inform Galileo on the subject, he does not give the particulars. Is it likely that in such a communication, made to such a man, no allusion would be made to the means of producing the effects described, nor that lenses were not distinctly mentioned? Add to this, that the general problem of magnifying distant objects by the modification of the rays of light by refraction or reflection, or both, which we must suppose to be that which Galileo asserts that he solved, is very indeterminate, and such as would be extremely unlikely to be deduced from the general theory of optics as then known. Still less probable is it that of the infinite variety of solutions, which so indeterminate a problem admits, he would have chanced to be led to that particular one which had been practised in the north of Europe.

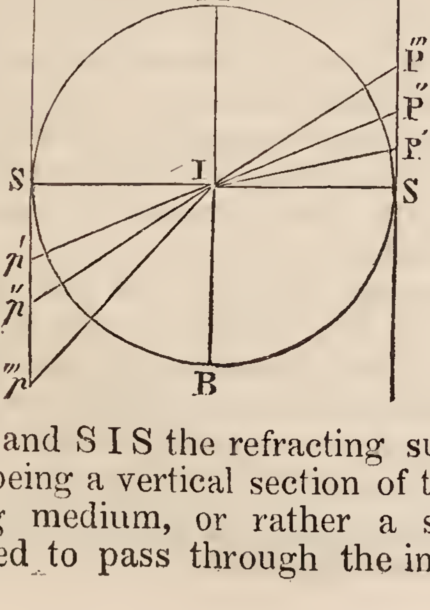
(9.) The honour of the invention of the astronomical telescope belongs indisputably and exclusively to Kepler. In his work on Dioptrics, he distinctly suggests the substitution of a convex eye-glass for the concave one previously used. He shows that in this case the image will necessarily be inverted; but the advantage of an enlarged field of view, in which this instrument excels the Galilean telescope, seems to have escaped his notice. To this we may, perhaps, impute the circumstance of Kepler's never having actually constructed telescopes upon the principle which he suggested, considering, probably, that as they presented inverted images, their use for terrestrial objects would be awkward and inconvenient, and that they possessed no advantage in astronomical observations over those used by Galileo and others. Kepler was, however, aware of the method of correcting the position of the image, by the use of additional eye-glasses; but probably considered that the complexity of the instrument, and the loss of light, would render it inferior to the Galilean telescope.

The optical discoveries of Kepler were not confined to the telescope. He made experiments on refraction by lenses, and succeeded in establishing some of the properties of their foci. He also explained the formation of the inverted image on the retina. He attributed erect vision from an inverted image to an operation of the mind, by which it refers the lower part of the image to the upper side of the eye, but considered it beyond our power to determine the manner in which the mind perceives the images of objects upon the retina. He investigated the power of the eye to accommodate itself to different distances, and attributed it to the contracting power of the ciliary processes.

(10.) At the period to which we now refer, the beginning of the seventeenth century, the most important discovery in the theory of refraction since the time of Ptolemy, was made by Willebrord Snellius, professor of mathematics at Leyden. This philosopher, by a careful comparison of numerous refractions at different incidences, found that if a sphere were described round the point of incidence, and a cylinder circumscribed this sphere, having its axis perpendicular to the refracting surface, the parts of the incident and refracted rays, between the centre of the sphere and the cylinder, were in a constant ratio, so long as the refracting medium remained the same.

To explain this important law more fully, let I, *fig*. 2, be the point of incidence, and S I S the refracting surface, S B S being a vertical section of the refracting medium, or rather a section supposed to pass through the incident ray, and perpendicular to the surface. With I as centre, and any distance I S, describe a circle in the plane of the section, and draw tangents at S S, perpendicular to the refracting surface. Let the incident rays meet the upper tangent at P′, P″, P‴, &c.; and let the corresponding refracted ray meet the lower tangent at p', p'', p''', &c. It was found by Snellius, that the ratios $\frac{P'I}{p'I}, \frac{P''I}{p''I}, \frac{P'''I}{p'''I}$, &c. were equal so long as the media S A S and S B S remained the same. Taking the radius I A or I B as the unit, the lines P′I, P″I, P‴I, &c. are the cosecants of the angles of incidence; and the lines p'I, p''I, p'''I, &c. are the cosecants of the angles of refraction. These quantities are, therefore, in a fixed proportion. The cosecants of angles being the reciprocals of their sines, it follows that when the media on each side of the refracting surface are given, the sine of the angle of incidence bears to the sine of the angle of refraction an invariable ratio.

Fig. 2.



(11.) The discovery of the law of refraction has been sometimes erroneously ascribed to Descartes. With a degree of disingenuousness and want of candour, or rather of common justice, which not unfrequently characterised the conduct of that great philosopher, he has announced in his Dioptrics, published eleven years after the death of Snellius, the law of refraction as the result of his own inquiries, without taking the slightest notice of the previous discovery of Snellius, although there is no doubt that he was acquainted with it; and there is even strong reason to believe that Descartes had the manuscripts of Snellius in his hands, and availed himself of the full and unrestricted use of them.

About the time of the death of Snellius, which happened in 1626, at the early age of thirty, Descartes applied himself to optical investigations; and, in 1637, published his Treatise on Dioptrics. Guided by the law of refraction, with the recent discovery of which he was made acquainted, he made some important additions to, and improvements in, the science. The fact that spherical lenses were incapable of collecting rays of light into an exact focus, had been known by experience; and the cause of this spherical aberration, as well as its quantity and laws, were easily deduced from the Snellian law. To comprehend the nature of this defect of spherical lenses, let the light incident upon them be conceived to be divided into a number of concentrical rings; the exterior ring, or that which is most distant from the centre of the lens, will be collected by refraction into a focus, at a point on the axis of the lens at a certain distance from the surface. The next ring of light within the last will be also collected into a point, but at a greater distance from the surface; and so on, each ring of light is collected into a focus, the distance of which from the surface increases, as the distance of the ring from the centre of the lens diminishes. To illustrate this, let L L be a section of the lens at right angles to its axis; and suppose its surface divided into rings as represented in *fig.* 3, and let the order of the rings be reckoned from the edge of the lens towards the centre, calling the external ring *the first ring*, the next within that *the second ring*, and so on. Let L L, *fig.* 4, be a section of the lens by a plane through its axis; and suppose that the whole lens, except the first ring, be covered by an opaque circular cover, and light be incident on its surface, this light will be refracted to a certain point f_1 in the axis; and this point will, therefore, be the focus of the *first ring*. Again, removing the cover from the lens, let another be substituted, which will leave the second ring alone exposed to the light. The rays will now be collected in the point f_2. The same process being continued, and the third, fourth, and other rings being successively exposed to the light, their foci will be found at the points f_3, f_4, &c. the rings nearest to the centre of the lens having their foci most distant from its surface. It was easily deduced from the law of refraction, that the foci of the rings near the centre of the lens were much closer together than those near its surface.

Fig. 3.

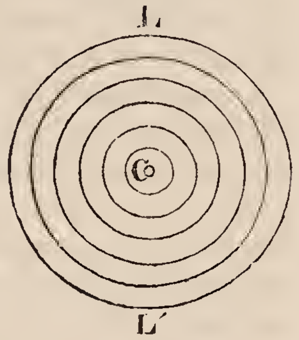
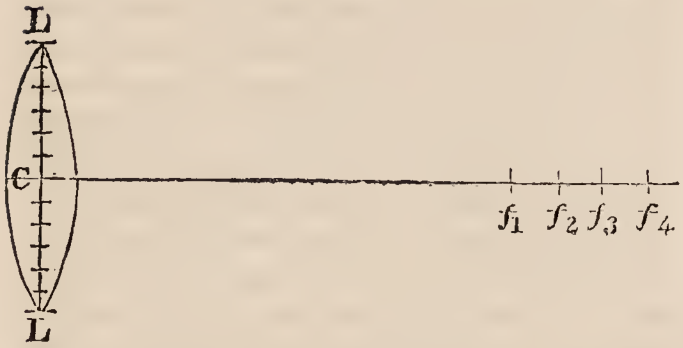


Fig. 4.



Since the images of objects were known to be formed by collecting the rays emerging from them into the focus of the lens, it followed from these considerations that each of the rings, into which we have supposed the surface of the lens to be divided, must produce a separate image; and thus an indefinite number of images of the same object would be found at different distances from the lens, and arranged in regular succession along the axis. This effect, called *spherical aberration*, caused a confusion in the appearance of the image, which confusion was increased with the magnitude and curvature of the lens.

Seeing that this defect was essential to the very nature of spherical lenses, Descartes proposed to investigate the figure of a lens which should be free from this defect; and such that each ring, into which its surface would be divided, might collect the rays to a focus at the same distance from the lens. The high analytical acquirements of this mathematician, united with the knowledge of Snellius's law, rendered the solution of this problem a matter of no great difficulty. He accordingly found a class of curves, since called the *Cartesian ovals*, which possessed the required property. When the densities of the medium of incidence and the medium of refraction are given, the figure of the surface, which will collect into an exact focus rays emerging from any given point, will always be determined by one of these ovals. When the incident rays are parallel, the oval becomes a conic section *.

Ignorant of the cause of the principal defect of lenses, which was subsequently discovered by Newton, Descartes expected much greater results from this discovery than it was capable of producing. He invented machines, and engaged skilful artists to grind spheroidal lenses, according to the figures suggested by his theory. After the expenditure of much labour and ingenuity, no adequate advantage was obtained, and at this day, when practical science has attained such an extraordinary degree of perfection, the spherical lenses are still universally used.

(12.) In unfolding the theory of the rainbow Descartes has been singularly happy, and certainly has brought the explanation of this phenomenon as near to perfection as could be done by one who was ignorant of the different refrangibility of light. Here, as in the case of the *law of refraction*, this geometer has fixed an indelible stain upon his fame, by a similar act of unprincipled selfishness and injustice towards Antonio de Dominis, to whom he never even alludes in his treatise. Descartes explained why the interior bow has a diameter of 42°; this was an easy inference from the theory of De Dominis and the law of Snellius. But the genius of Descartes was very conspicuous in his solution of the problem of the exterior bow. He shewed that the sun's rays entering the inferior part of the drop, emerged with a second refraction from the superior, after undergoing two reflections within. The double reflection accounted for the faintness of the exterior bow, and the inversion of its colours. Thus we see that ignorance of the different refrangibility of light alone prevented this philosopher from bringing the theory of the rainbow to absolute perfection.

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* For an account of these curves, see Lardner's *Algebraic Geometry*, p. 452.

(13.) In the middle of the seventeenth century a discovery was made by Erasmus Bartolinus, a Danish mathematician, which formed the first of the most brilliant train of experiments, and may be considered the basis of the most splendid speculations which ever adorned the annals of science; speculations which regard not merely the phenomena of light and vision, but which seem to furnish man with new sensibilities, which are to *touch* what the microscope is to *sight*, and disclose to his view wonders of nature which would refuse to unveil themselves to any other power. In this extensive field, more than in any other, has the philosophic genius of our own times shone forth.

Bartolinus received from some Danish merchants, who frequented Iceland, specimens of the crystal, of extraordinary transparency and dimensions, since known by the name of *Iceland spar*, which is *carbonate of lime*, in a crystallized form. While making optical experiments with pieces of this crystal, he discovered that it exhibited a double image of objects seen through it, and consequently inferred that each ray of light in passing through it was cloven into two rays, which were refracted at different angles, while the angle of incidence of the whole ray of which they were the component parts remained the same. Slight observation was sufficient to prove that both of these refracted rays did not obey the Snellian law, for if that were the case, both images of an object seen through the crystal would maintain the same position with respect to a perpendicular to the refracting surface, while the crystal was turned round that perpendicular as an axis. Such, however, he found not to be the fact. By the revolution of the crystal, one of the images only was observed to obey the above-mentioned law. Hence he inferred that one of the two refractions was performed according to the common law, but that the other obeyed an extraordinary law, not before noticed by opticians. The results of these experiments were published by Bartolinus, in a work, entitled *Experimenta crystalli Islandici dis-diaclastici quibus mira et insolita refractio detegitur*. Copenhagen, 1699.

The publication of this work soon drew the attention of the celebrated Huygens to the subject of double refraction. Previously to this he had published a theory of refraction and reflection, founded on the hypothesis, that light, like sound, was propagated by the undulations of a subtle and elastic medium, which he supposed to pervade all space permeable to light. Others held that light was corporeal, and composed of infinitely small and subtle corpuscles, which were emitted from every luminous body, and entering the eye impinged upon the retina, and produced sensation. This corpuscular theory, which is as old as Pythagoras, was adopted by Newton for the explication of the phenomena of optics; but it is proper to add, that he is careful not to intermingle with the reasoning from his experiments any assumptions from this theory, which could at all affect the validity of his results, all of which are entirely independent of any hypothesis, and such that any theory must account for before it can be admitted as a true one.

Huygens took up the subject of *double refraction*, in order to obviate any objections to his undulatory theory, which might arise from it. He accordingly explains distinctly the law which regulates the refraction of the extraordinary ray, and attempts to shew how the phenomenon may be accounted for by spheroidal undulations, while ordinary refraction is ascribed to spherical undulations. As, however, this part of the subject is not intimately connected with the discoveries of Newton, we shall not here enter into further detail upon it, but refer the reader to our history of optics, in which this and many other matters entirely omitted here will be treated at large.

While examining the two rays into which the incident ray is cleft by the Iceland spar, Huygens discovered one of the facts from which Newton subsequently deduced the polarisation of light. He found that if these two rays entered a second prism of the same crystal similarly placed with respect to the ray, each of the two rays were again refracted, according to the same law which regulated their refraction in the first crystal; but he found that if the second prism were placed at right angles to the first, the laws by which the rays were refracted were interchanged. In every intermediate position of the second crystal, each of the two rays from the first crystal were divided into two, so that the whole primitive ray was resolved into four. These complicated phenomena Huygens confessed himself unable to solve, and it remained for the genius of Newton to explain the curious and beautiful property of polarisation.

(14.) Newton was born in 1642, and in 1663 James Gregory published his invention of the reflecting telescope, consisting of two concave mirrors, the larger of which, receiving rays from the object, was perforated at its axis, and opposite to the perforation and facing the larger, the smaller was placed so as to receive the rays reflected from the larger, and reflecting them to bring them to a focus in a tube placed in the perforation. The image thus formed was viewed with an eye-glass as in the astronomical telescope.

Three years after this Newton invented his reflecting telescope, in which he dispensed with the perforation in the principal speculum, thereby preserving the central rays which were most essential to the formation of a perfect image. The rays reflected from the great speculum in Newton's telescope were again reflected to an aperture in the side of the tube by a plane reflector placed at an angle of 45° with the axis of the tube. At this aperture they were viewed in the usual way through an eye-glass.

(15.) In 1665 the work of Francis Maria Grimaldi, an Italian Jesuit, was published, containing his discovery of the *inflexion* or *diffraction* of light. He admitted a ray of light through a small hole into a darkened chamber, and observed that it was diffused into the form of a cone, and that the shadows of bodies placed in this light were larger than they would have been had the light passed without interruption or deflexion at their edges. Upon more minute and careful inspection, he observed that these shadows were skirted with three coloured fringes, which grew narrower as they receded from the body. He also observed, that when the light was strong there were similar streaks of colour within the shadow, which increased in number in the same shadow when it was received at a greater distance from the body. From these phenomena he concluded that light, in passing the boundaries of an opaque body, was turned from its rectilinear course, an effect which has been called *inflexion* or *diffraction*.

Another experiment instituted by this philosopher indicated a still more extraordinary property of light. Through two small apertures he admitted two cones of light into a darkened chamber, the apertures being so placed that the cones did not penetrate one another until they reached a considerable distance from them. Beyond this distance the light was received upon a screen, and it was observed that the part of the screen illuminated by both cones was the least bright, and that its degree of illumination was increased by depriving it of the light of one of the cones, by stopping one of the apertures. Thus it appeared that a body may actually become more obscure by increasing the quantity of light which shines upon it.

We have in this chapter attempted to give a short sketch of the principal discoveries in optics before the time of Newton. In doing this we shall perhaps be accused of having given an unequal or disproportionate attention to particular topics, and of having touched too slightly upon others. It must however be remembered that our design was not to write a history of optics, but to shew the reader how far the way had been paved for Newton, and what advances had been already made in some of those theories which he has the merit of completing; and on the other hand to shew that on other subjects, such as the unequal refrangibility of light, absolutely nothing was known, nor any advance made, even in the shape of conjecture.

We shall proceed in the next chapter

to give the reader, in a popular form, an account of Newton's Optics, a work which communicated to the world a series of discoveries of transcendent beauty, and which would themselves have been sufficient to have conferred immortality on the name of Newton, even had he never written his PRINCIPIA.

CHAPTER II.

The Compound Nature of Solar Light established by Experiments.

(16.) BEFORE he enters upon the interesting detail of the experiments which led to his great optical discoveries, Newton explains in eight axioms the leading principles and established results of the science before his time. We have, in the preceding chapter, enlarged more fully on the subject than he has done in the *axioms*, that the reader may be the better enabled to appreciate the discoveries we have to explain. The propositions respecting the properties of light which Newton gives under the title of *axioms*, are not to be understood as partaking of the nature of mathematical axioms. On the contrary, many of them are results, not only of careful experiment, but of very subtle and complex reasoning. "I have," says Newton, "now given, in axioms and their explications, the sum of what hath been hitherto treated of in optics. For what hath been generally agreed on, I content myself to assume, under the notion of principles, in order to what I have further to write. And this may suffice for an introduction to readers of quick wit and good understanding, not yet versed in optics; although those who are already acquainted with this science, and have handled glasses, will more readily apprehend what followeth."

The following are the propositions assumed by Newton as axioms:—

Axiom I.

The angles of reflection and refraction lie in one and the same plane with the angle of incidence.

Axiom II.

The angle of reflection is equal to the angle of incidence.

Axiom III.

If the refracted ray be turned directly back to the point of incidence, it shall be refracted into the line before described by the incident ray.

Axiom IV.

Refraction out of the rarer medium into the denser, is made towards the perpendicular; that is, so that the angle of refraction be less than the angle of incidence.

Axiom V.

The sine of incidence is either accurately or very nearly in a given ratio to the sine of refraction.

Axiom VI.

Homogeneal rays which flow from several points of any object, and fall perpendicularly, or almost perpendicularly, on any reflecting or refracting plane or spherical surface, shall afterwards diverge from so many other points, or be parallel to so many other lines, or converge to so many other points, either accurately or without any sensible error; and the same thing will happen, if the rays be reflected or refracted successively by two or three or more plane or spherical surfaces.

Axiom VII.

Wherever the rays, which come from all the points of any object, meet again in so many points, after they have been made to converge by reflection or refraction, there they will make a picture of the object upon any white body on which they fall.

Axiom VIII.

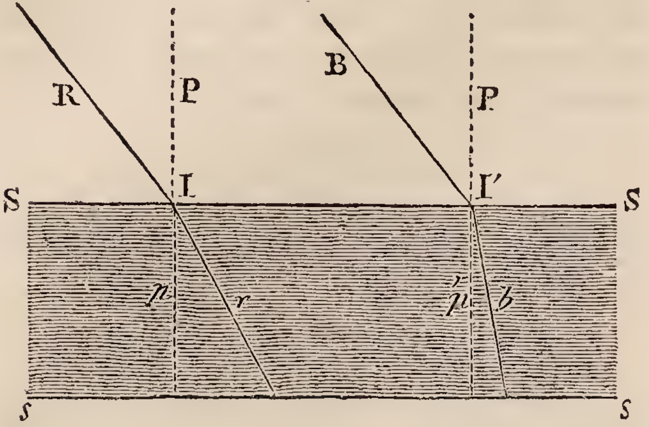
An object seen by reflection or refraction, appears in that place from whence the rays, after their last reflection or refraction, diverge in falling on the spectator's eye.

The work opens with the announcement of the discovery that LIGHTS WHICH DIFFER IN COLOUR, DIFFER ALSO IN REFRANGIBILITY.

For those readers who are not familiar with the science, we shall first explain the meaning of this property, and next describe the nature of the experiments by which Newton established it.

Let S S *s s*, *fig.* 5, be a section of any transparent medium, as water or glass. Let R be a ray of red light incident at I, and supposed to proceed from some red object in the direction I R, or to have been transmitted through red glass. In like manner, let B be a ray of blue light incident at I′, in a direction B I′, parallel to R I, and proceeding as before from a blue object in that direction or transmitted through blue glass. The angles of incidence formed by the rays R and B, with the perpendiculars P to the refracting surface will be equal, because the rays R and B are parallel. On entering the medium they will be both deflected by the law of refraction towards the perpendiculars *p*. Let *r* be the course after refraction of the red ray, and *b* the course after refraction of the blue one. If these rays were equally refrangible they would be equally deflected towards the perpendicular. Such, however, is not the case. The red ray *r* will be less deflected from its original direction than the blue ray *b*, and therefore the angle under the lines *r* and *p* will be greater than the angle under *b* and *p*. Hence, red light is said to be less refrangible than blue light.

Fig. 5.

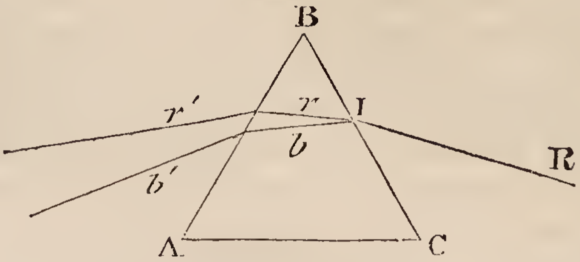


We might expect that this effect would be easily reduced to experiment by colouring two sticks, one red, and the other blue, and placing them parallel to each other, obliquely in water. In this case, the broken appearance which the sticks should exhibit would be different, the angle formed by the parts in the water with those in the air being more obtuse in the red than in the blue stick. It happens, however, that the difference between their deflections is so small, that, in this way, it would be very difficult to render it perceptible.

To obviate this inconvenience, and render the difference of refractions so great as to be easily perceptible, Newton contrived that the different lights should each be twice refracted, so that being originally parallel, they should thus acquire the sum of the divergencies which such refraction alone would give them. This he accomplished by using a glass prism, in the manner which we shall now describe.*

(17.) Let A B C, *fig*. 6, represent a section of a triangular glass prism at right angles to its axis (an end view of it.) A ray of red light entering at I in the direction R I will, by the refraction of the glass, be deflected in the direction *r*, *towards* the perpendicular to the surface B C, on which it is incident. On meeting the second surface B A, it will emerge, suffering a second refraction on passing into the air. But here, as it passes from glass into air, it will be deflected *from* the perpendicular to the surface B A in the direction *r*′, which increases still more its deviation from its original course R. In like manner, a blue ray incident in the same direction at I will suffer two deflections at the surfaces of the prism; but in each case will be more deflected from its original direction R, than the red ray. The deviation of the blue from the red ray, on emerging from the surface B A, will evidently be found, by adding together the two deviations at the surfaces B C and B A. Thus, although either deviation alone might be too small to be perceptible, yet the sum of both will produce a sensible effect.

Fig. 6.



(18.) To reduce this to absolute experiment, Newton states that he took a black oblong stiff paper, terminated by parallel sides, with a line drawn from side to side, dividing it into two equal parts. One of these was coloured with an intense *red*, and the other with an intense *blue*. He then covered the wall and shutters, surrounding the window of the room with black cloth, to prevent the light reflected from them from interfering with his experiment. On a table before the window, also covered with black, he placed the coloured paper, the line dividing the colours being perpendicular to the plane of the window. Things being thus arranged, he held the prism, with its angle B, *fig*. 7, upwards, so that the light coming from the paper R B should be twice refracted before it reached the eye placed behind the prism, as represented in the cut. Upon viewing the appearance through the prism, he found that the blue half of the paper appeared a little more elevated than the red half, which plainly proved that the rays from the blue half were more deflected from their original direction than those of the red half. Had the rays been equally refrangible, the parts of the paper would have maintained the same relative position, although the place of the entire paper would appear to be elevated.

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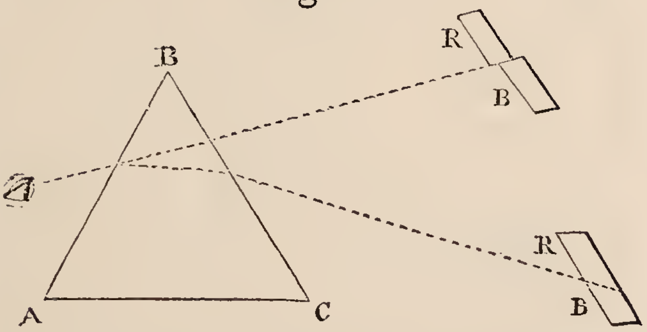
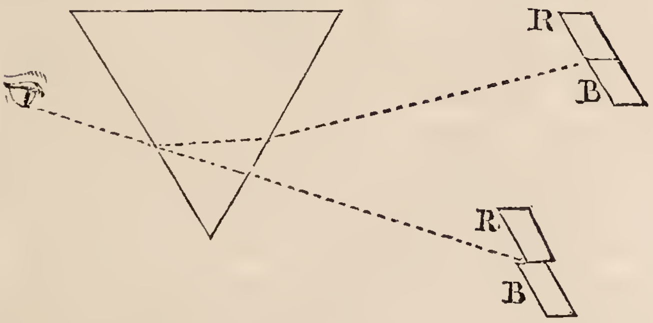
* We advise every reader who has not seen the prismatic spectrum, before he proceeds further, to procure a prism, or a common angular piece of glass with plane sides, and to transmit through it a beam of the sun's light admitted through an aperture in the window-shutter of a dark room. The facility with which he will comprehend Newton's interesting experiments will thus be much increased,

Fig. 7.



In order to confirm the conclusion deduced from this experiment, Newton viewed the same paper with the refracting angle B, *fig.* 8, of the prism presented downwards. He found, as he expected, that the blue half of the paper was lower than the red, being more depressed, owing to the greater refraction of the rays.

Fig. 8.



(19.) He now determined to submit the question to a different test. It was previously known that the position of the focus, in which rays from any luminous object are collected by a lens, depended on the refracting power of the glass upon the light. The greater that refracting power, other things being the same, the nearer to the lens will be the focus into which the rays are collected. It was also well known that the place of the focus might always be detected by the presence of an image of an object placed before the lens. We have already stated (p. 8), that a certain degree of indistinctness will attend this image, when the lens is spherical; but still a certain point of greatest distinctness will always be found, which may be regarded as the place of the focus, were all spherical aberration removed. Now, if it be true that light of different colours is differently refrangible, and that according to the result of the experiments already described, *red light* is less refrangible than *blue light*, it would follow that the images of red and blue objects, formed by the same lens, ought to be found at different distances from it.

To apply this test to the coloured paper already mentioned, Newton wrapped round it an extremely fine thread of black silk, so as to form fine black lines upon the red and blue ground, as represented in *fig.* 9; where, for distinction, the black lines are parallel on the red, and cross each other on the blue. He covered the wall of a dark chamber with black cloth, and attached to it the coloured paper, with the silk thread wound round it, as in the figure. Immediately under this he placed a light, so as to illuminate the paper thoroughly, the flame not rising above its lower termination; and, therefore, not intercepting any of the light reflected from it. He then placed a double convex lens M N, *fig.* 10, of about four inches and a quarter diameter, at about six feet two inches from the paper. The image of the paper was received upon a white screen, at the

Fig. 9.

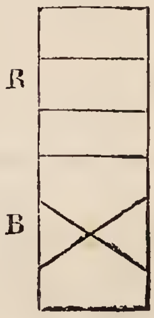
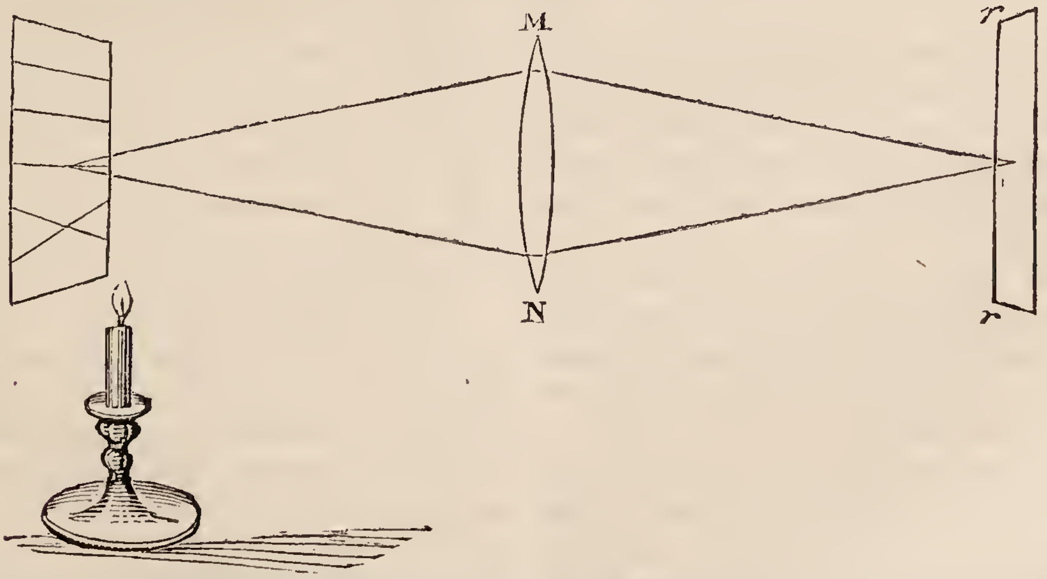


Fig. 10.



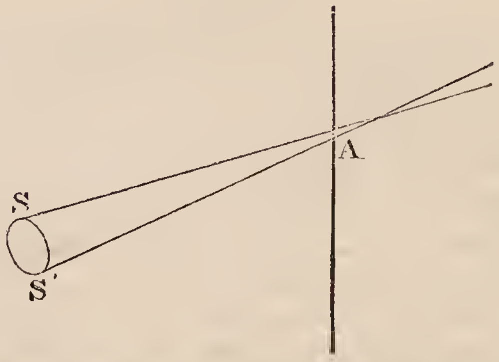
same distance behind the lens. This screen was slowly moved to and from the lens, until that position *rr* was obtained, in which the black lines upon red were seen with the greatest distinctness depicted in the image on the screen. This was evidently the focus of the red rays, because the black lines became distinctly visible only when the red boundary was precisely defined. In this position the black lines on the blue part of the image were faint, confused, and scarcely distinguishable. The screen was then moved slowly towards the lens. As it moved the lines on the red part became faint and confused, while those on the blue part became clear and distinct, being found to be as distinct when the screen was removed an inch and a half nearer to the lens, as the lines on the red had been in the first position; while, on the other hand, the lines upon the red part were scarcely observable. The conclusion was irresistible; the same lens placed at the same distance from red and blue surfaces illuminated by the same candle, brought the rays from the one surface to a focus nearer than those from the other; and, therefore, had a greater refracting power on the blue rays than on the red; which conclusion harmonized exactly with the results of the prismatic experiments previously instituted.

(20.) Notwithstanding the very conclusive nature of these experiments, Newton reasons from them with a degree of caution and circumspection truly philosophical. "From these experiments," he says, "it follows not that all the light of the blue is more refrangible than all the light of the red; for both lights are mixed with rays differently refrangible, so that in the red there are some rays not less refrangible than in the blue, and in the blue there are some rays not more refrangible than in the red; but these rays, in proportion to the whole light, are but few, and serve to diminish the event of the experiment, but are not able to destroy it: for if the red and the blue colours were more dilute and weak, the distance of the images would be less than an inch and an half; and if they were more intense and full, that image would be greater, as will appear hereafter."

These experiments were conclusive respecting light which proceeded from the colours of natural bodies. But it still remained to analyze the direct solar light, and to determine the nature of the beams of white light, which are the means of rendering all coloured objects visible, and of causing them to transmit the coloured light which emerges from them.

(21.) If a ray of light, direct from the sun, be admitted through a small aperture A, *fig.* 11, in the window-shutter of a dark chamber, a circular image of the sun will be formed by the rays admitted through the hole, and may be received upon a paper screen, as at S S′. We shall, for the present, suppose the hole to be so small, that its diameter may be neglected, and it may be regarded as a physical point. The rays which proceed from the several points of the sun's disc entering the aperture A, cross each other, and that from the highest point proceeds to S′ the lowest point of the image, while that from the lowest proceeds to S the highest point of the image. In like manner the ray from the right-hand side of the sun proceeds to the left-hand side of the image, and that from the left-hand side to the right-hand side of the image. In the same way every point of the sun's disc is referred to that point of the image diametrically opposite in position. The image is therefore inverted in whatever way it be considered with reference to the sun.

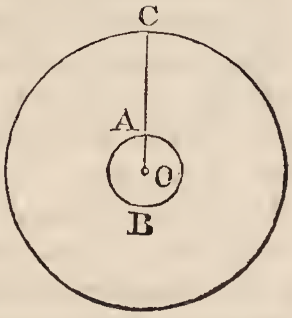
Fig. 11.



The magnitude of the image, or illuminated spot, on the screen, evidently depends on the distance of the screen from the aperture A, increasing as that distance increases; and the diameter of this image subtends at the hole A the same angle as the sun subtends at it, or as the apparent diameter of the sun.

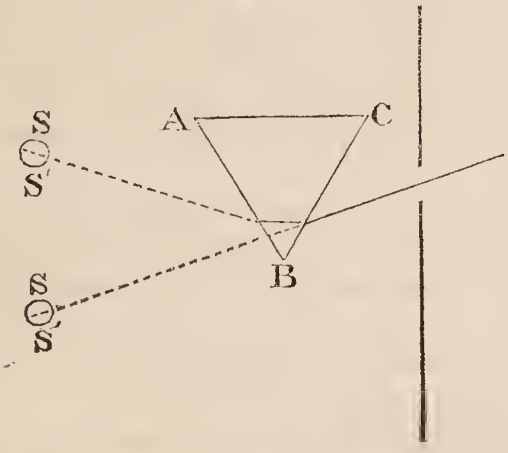
We have here supposed that the hole has no sensible magnitude, or is a physical point. If this be not the case, and, on the other hand, the aperture have a sensible diameter, all that we have above stated will be true of every separate point in it: so that there will be innumerable images of the sun; the centres of which will be diffused over a space upon the screen equal to the size and shape of the aperture. At present we will consider the aperture circular. To find, then, the magnitude of the illuminated spot upon the screen, first describe a circle A B, *fig.* 12, whose diameter is equal to that of the hole. Then, adding to the radius of this circle another line, A C, whose length is such as would subtend at the hole an angle equal to the sun's semidiameter, with the whole distance O C describe a circle. This circle will be the magnitude of the illuminated spot on the screen.

Fig. 12.



(22.) Instead of allowing the beam of light to pass directly from the aperture to the screen, let it be intercepted by a prism A B C near the hole, with its refracting angle B presented downwards. The refraction by the surfaces of this prism might be expected to deflect the beam from its original course, and raise it to an higher position, as represented in *fig.* 13. The part of the screen on which the refracted ray falls, is, accordingly, elevated by the effect of the prism, but this is not the only effect produced. The breadth of this *spectrum*, as it is called, is exactly equal to the diameter of the illuminated spot which would be projected on the screen, if the prism were removed. But, instead of a circular image being projected on the screen, the illuminated part assumes an oblong form, such as R V, *fig.* 14. The sides are straight, and the ends semicircular—the length being perpendicular to the axis of the prism.

Fig. 13.



It appears, therefore, that of the rays which pass through the prism, some are refracted to a higher part of the screen than others, those toward the end V being more elevated by the refraction than those towards the end R. From this Newton inferred, that of the light which passed through the prism, some was more, and some less refracted, those rays which passed towards the highest points, V, being more refracted than those which passed towards the lowest points R.

Fig. 14.

The oblong form given to the illuminated part of the screen, was not the most curious or surprising effect produced by the prism. This image or spectrum exhibited the most beautiful series of colours, each depicted on the screen with a degree of splendour and intensity far exceeding those of the colours of any natural object. Beside these the most brilliant colour which nature presents, or the most refined efforts of art could exhibit, would seem faded and dim. The lower extremity, R, exhibited the most dazzling *red*, and above this, in regular succession, were ranged the colours, *orange*, *yellow*, *green*, *blue*, and *indigo*, the upper termination, V, being *violet*. These colours were not separated by distinct limits, but the tints seem to melt imperceptibly one into another, it being impossible to determine exactly where any one ended and the next began. Thus the red was tinted off insensibly into the orange, the orange into the yellow, and so on.

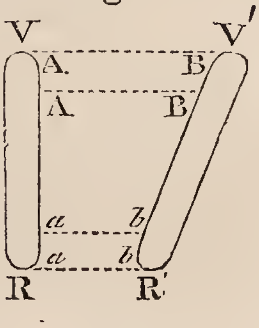
From these observations, it is apparent that the red light, and all that portion of light which partook of this character, being deflected to the lower part of the spectrum, is less refracted than the blue light, and those colours of the same class which are refracted to the upper part. It would therefore seem to be an obvious inference, that the solar beam incident on the prism, was a mixture of different kinds of light; that the prism acting on the component parts refracted some of them in a greater, some in a less degree; those partaking of the blue character being more refracted than those of the red kind. This inference seems also to harmonize with the former experiments made upon coloured bodies (18, 19), whereby it was proved that red light is less refrangible than blue.

(23.) Newton repeated the preceding experiment in another way. The prism being placed as before, at the aperture in the window shutter, he placed his eye behind it, so as to receive the rays emerging after the second refraction from the prism. In this case the eye supplied the place of the screen in the former experiment, the rays which formed the spectrum entering the pupil as they emerged from the prism. He found the same effects to ensue. He beheld the oblong spectrum as before, the order of the colours being in all respects the same.

(24.) It appears by the experiment which we have described that the action of the prism was such as to dilate the ray in a direction at right angles to its length, and thereby to give the spectrum the oblong form. Now if this dilatation was the consequence of the action of the material of the prism, and not of the various refrangibility of the component parts of solar light, it must inevitably follow that a second prism placed with its length vertical, and consequently at right angles to the first, so as to refract the light sideways, would dilate the ray as much in the horizontal direction as the first did in the vertical. The inference from this would necessarily be, that the combined action of two prisms would be to give a square spectrum, as much length being obtained by the action of the one, as breadth by the action of the other.

Accordingly Newton tried this experiment. Let R V, *fig.* 15, be the spectrum produced by the first prism. He placed another prism with its length vertical, and consequently at right angles to the first, and when the rays were intercepted by it, the effect was, that instead of the spectrum, V R, being spread over a square surface, it retained its breadth, but was transferred to the position V′R′.

Fig. 15.



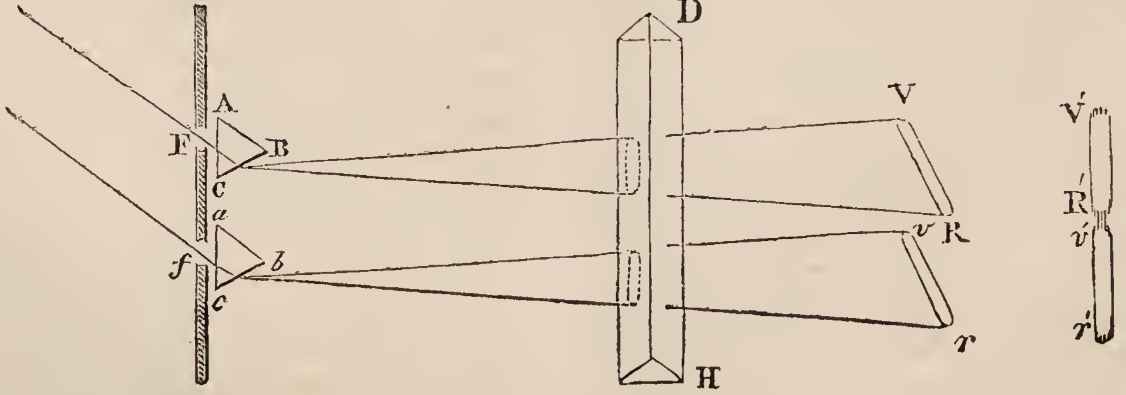
This result furnished a most convincing and beautiful confirmation of the theory of Newton. The rays at the upper end of the spectrum were deflected by the second prism, through the distances A B, from their former position, while the rays at the lower end were only deflected through the smaller spaces *a b*. The breadth of the spectrum remained unaltered, plainly shewing that the second prism had no power to dilate the rays which formed it. The rays which were most refracted by the first prism, were those of the blueish character, which occupied the upper part of the spectrum. These same rays were also most refracted by the second prism, being most removed from their first position (through the spaces A B). Also the rays which were least refracted by the first prism, were those of the reddish character which occupied the lower part of the spectrum; and these also are least refracted by the second prism, being those which are least removed from their places through the distances *a b*.

To put this question even more beyond dispute, Newton received the rays from the second prism on a third, placed with its length parallel to the length of the spectrum, and found the same effect repeated, the third position of the spectrum being inclined to the second in the same manner as the second was inclined to the first, but no dilatation taking place, and the breadth of the spectrum remaining the same. He states that he used a fourth prism with the same result.

It is important to remember that in all these experiments the light is all incident on each prism *at the same angle.* For if its parts fell upon the surface at different angles, different quantities of refraction would be the natural and necessary result.

(25.) Newton contrived a very elegant experiment to shew the regularity with which the prisms determined the magnitude, figure, and position of the spectrum. Before two small apertures F, *f*, *fig.* 16, in the window shutter he placed two similar prisms A B C, *a b c*, the one immediately beneath the other, with their lengths parallel and horizontal. These

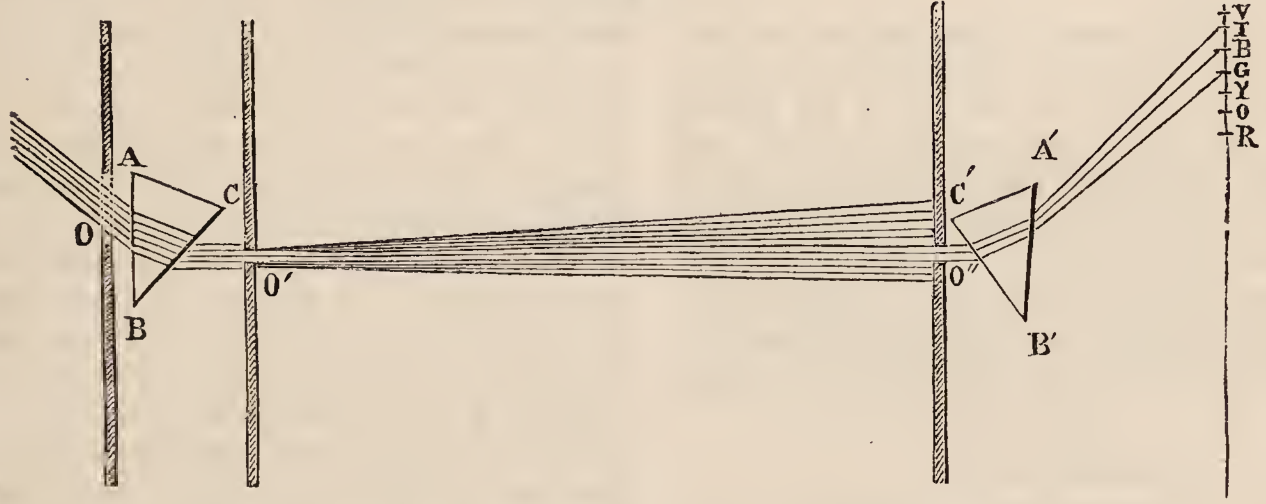
Fig. 16.



cast two spectrums R′ V′, *r′ v′*, on the opposite wall so as to lie in the same right line, and having the lengths at right angles to the floor, the lowest point R′, or red end of one, being contiguous to the highest point *v′*, or violet end of the other. A third prism, DH, was now placed with its length vertical, and of course at right angles to the other two, and so as to receive the rays emerging from them. The two spectrums were immediately translated from their former positions to other positions RV, *rv*, no longer in the same line, but similarly inclined to the former, and therefore parallel to each other.

(26.) The next test to which Newton submitted the problem was even more conclusive and convincing than any of the preceding. Through an aperture, O, *fig.* 17, in the window-shutter he admitted a beam of the sun's light which he received upon a prism A B C, placed before the aperture. The spectrum produced by the refraction of this prism was received upon a screen perforated by a small hole O′. The several coloured lights of the spectrum being diffused over a considerable space upon the screen, and the aperture O′ being small, the light of but one colour passed through it, while the prism A B C remained stationary; but when this prism was slowly turned round its axis, the spectrum moved upwards and downwards on the screen, so as to transmit in succession the lights of the several colours through the aperture O′. At a distance of about twelve feet from this screen another was placed, having, in like manner, a small aperture O″. The beam of coloured light transmitted through the aperture O′ was received upon the second screen, diffusing itself over a space of some magnitude. A small ray of this light passing through the aperture O″ was received upon a prism A′ B′ C′ placed immediately behind it, and by this prism was refracted to a certain point upon the opposite wall.

Fig. 17.

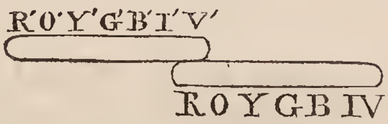


The prism A′B′C′ remaining fixed, the prism ABC was turned until the red end of the spectrum fell upon the hole O′. A ray of red light now passed from O′ to O″, and was refracted by the prism A′ B′ C′ to the point R on the wall. This point was marked. By a slight motion of the prism A B C, the *orange* light was next brought upon O′, and a ray of it passing in the direction O′ O″ fell upon the prism at the same angle as the red light had before been incident. This orange ray was refracted by the prism A′ B′ C′ to a point O on the wall a little above the point R to which the red had been brought. The *yellow*, *green*, *blue*, *indigo*, and *violet* rays were in succession transmitted in the same way to the prism A′ B′ C′, all being incident upon it at the same angle, and they were severally found to be refracted to the points Y, G, B, I, and V. Thus it appeared that the several coloured lights into which the sun-beam was resolved by the prism A B C were, under the same circumstances, differently refracted by the prism A′ B′C′, each light being refracted the more, the nearer its situation to the violet end of the spectrum.

The conclusiveness of this result would have satisfied an ordinary inquirer, and it would have immediately been made the basis of a theory. The ardour of discovery was, however, in Newton tempered by philosophical circumspection, and in the unwearied patience of his research, he left untried nothing which could put his hypothesis to the proof, and overturn it if false. In the records of scientific discovery there is not a more splendid instance of an investigation in which theory and experiment mutually guide and support each other. In the first experiments, Newton found that the coloured lights reflected from the surfaces of natural bodies were differently refrangible; and subsequently he shewed that the colours produced by the refraction of a sunbeam by a prism were also differently refrangible, and that the colours which, reflected from natural bodies, were most refrangible were also the colours most refrangible in the refracted solar light. But in order, as it were, to identify the two experiments on natural colours and coloured light, he instituted the following experiment.

(27.) By means of two prisms, as described in (25), he projected two spectrums on the wall, so as to be placed end to end in the same direction, and so that the violet end of the one joined the red end of the other. At some distance from the wall he placed a slender piece of white paper, with straight and parallel edges, and so arranged that the red light R, *fig.* 18, of the one spectrum should illumine one half of the paper, while the violet light, V′, of the other spectrum illuminated the other half. The paper thus appeared of two colours, red and violet, similar to the painted paper used in the experiment described in (18). The remaining lights of each spectrum passing beyond the paper fell upon the wall, which was hung with black, in order that light reflected from it might not disturb the experiment. It is evident that this arrangement was such as to place the coloured light produced by the refraction of the prism exactly under the same circumstances as the light reflected by the colours of natural bodies in the experiment already alluded to. Accordingly Newton viewed the illuminated paper through a prism held parallel to it, as in that experiment, and found exactly the same result, *viz.* that the violet half was separated from the red by a greater refraction, so that the parts of the paper, instead of forming one straight band, were now separated from one another, but placed in parallel directions. Instead of using a band of paper, he sometimes used a white thread, one half of which he placed in the violet, and the other in the red light, and observed the same effect, the thread appearing to be broken, and one half of it moved out of its place, but parallel to its former position.

Fig. 18.

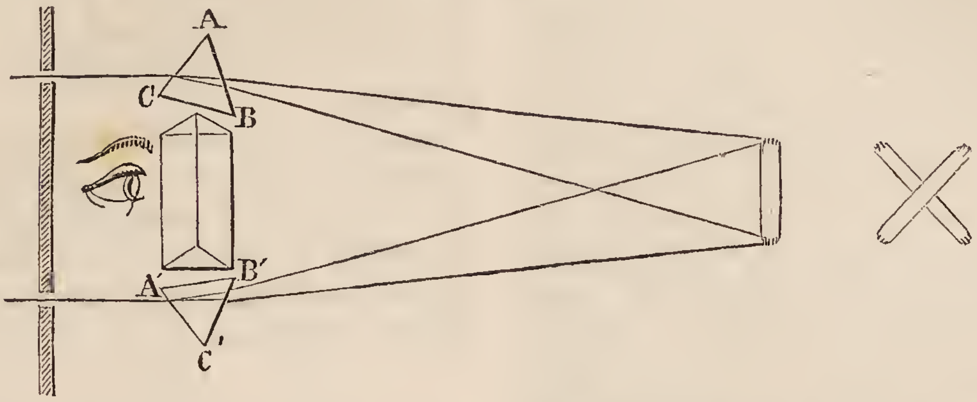


In this experiment, by turning one of the prisms upon its axis, he was enabled to illuminate one half of the thread successively with the *violet*, *indigo*, *blue*, *green*, and the other prismatic colours, while the other prism, maintaining its position constantly illuminated the other half of the thread with *red light*. Upon viewing these phenomena successively with a third prism, he found that in each case the parts of the thread illuminated with lights of different colours were separated, but the separation was greatest between the red and violet, less between the red and indigo, still less between the red and blue, and so on, being very small between the red and orange. But when both parts of the thread were illuminated with the reds of the two spectra, the thread appeared no longer broken. It is scarcely necessary to observe, that all these phenomena were such as must have been easily foreseen from the supposed unequal refrangibility of differently coloured lights. Newton might have carried this experiment further, and probably he did so, although he has not particularly mentioned it. He might have thrown on the thread every possible distinct pair of the prismatic colours, by moving both prisms on their axis, and the result would be that the apparent separation of the parts of the thread, when viewed through the prism, would have been great in proportion to the distance between the two colours in the spectrum.

(28.) All those experiments instituted by Newton, in which the refracted light was received upon a screen, were repeated with the same success, the light being admitted immediately to the eye from the prism, without the use of the screen. Thus the experiment (25) in which the two spectra, lying in the same line, were refracted by the prisms to parallel lines, was repeated thus. The spectra were viewed through a prism without disturbing the screen, and their apparent position, as seen through the prism, was found to be the same as when refracted by the third prism, and received upon the screen.

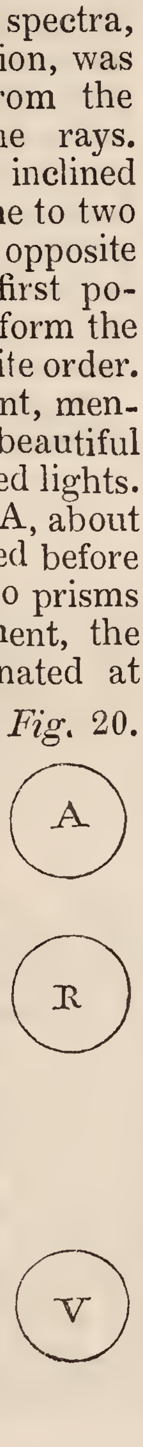
Two prisms ABC, A′B′C′, *fig.* 19, were also placed at apertures in the window-shutter, the refracting angle of one being directed upwards, and that of the other downwards. The spectra produced by these prisms were both in an upright position; but had the colours arranged in an opposite order, the red end being the higher in the one, and the lower in the other. One of the prisms was slowly turned on its axis, until the spectrum produced by it was thrown upon the spectrum produced by the other. The two spectra thus appeared to form one, which, in its colours, differed from either of them, the red of the one being mixed with the violet of the other, the orange with the indigo, and so on. This mixed spectrum was now viewed through a third prism, held as represented in the figure. The effect which ensued was, the separation of the spectra, which assumed the cross position shewn in the figure. This experiment is another variety of that which we last described; the inclined positions assumed by the spectra, with respect to their first position, was explained in (24) to arise from the different refrangibilities of the rays. In the present experiment, this inclined position is given at the same time to two spectra. The inclinations are in opposite directions with respect to the first position, because the lights which form the spectra are disposed in an opposite order.

Fig. 19.



(29.) The following experiment, mentioned by Newton, is another beautiful example of the analysis of mixed lights. A circular piece of white paper A, about one inch in diameter, was placed before a black wall, and using the two prisms mentioned in the last experiment, the paper A, *fig.* 20, was illuminated at the same time with the red light from the one, and a deep violet light from the other. By this mixture the paper assumed a rich purple colour. The circle A was then viewed through a prism at some distance, and the appearance exhibited was two circles, R and V, the circle R, nearer to the paper being *red*, and the more remote one, V, *violet*. The prism in this case refracted the red and violet light, mingled in the circle A, through different angles; the red being least refrangible was removed to R, and the more refrangible violet light carried so far as V.

Fig. 20.



To confirm this, the apertures before the prisms, which cast the red and violet lights on A, were in turns covered, so as alternately to deprive the circle A of these lights. It was accordingly found that the circles R and V alternately vanished, plainly proving that all the light of R came from the prism which cast the red light on A, and that all the light of V came from the prism which cast the violet light on A.

By turning one of the prisms at the window upon its axis, the circle A was successively illuminated with all the prismatic lights, while the other prism, being stationary, constantly projected on it an intense red light. The effect produced to an eye viewing these changes through the third prism was, that the circle V changed its colour according to the change in the light used with the red upon A. But the change of colour was not the only alteration observed in V. Its *position* was also changed. When the blue light was mixed with the red, it appeared nearer to R. Still nearer when green was mixed with red on A. In a word, the circles R and V always exhibited the colours mixed upon A, and their separation from A and from each other always corresponded to the separation of the prismatic spectrum from the direct course of the light, and to the separation of the two lights in the spectrum from each other.

From all these experiments no doubt could remain that lights which differed in colour differed also in refrangibility. One test more however remained, analogous to that which was applied to the colours of natural objects in (19). If the difference of refrangibility be admitted, it will necessarily follow that the same double convex lens will have different foci for differently coloured lights, the focus of the more refrangible light being nearer to the lens than that of the less refrangible. Thus if the lens be exposed to a beam of violet light proceeding from a given object, and collects that light to a focus at a certain point, it should collect red light to a focus at a more distant point, and the lights of intermediate colour to points between these extreme limits.

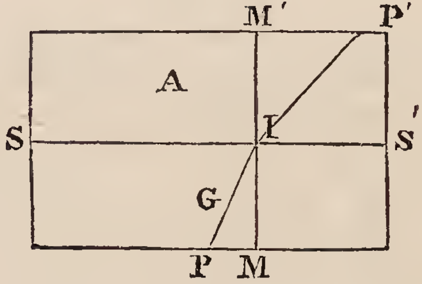
(30.) In order to reduce the doctrine to this test, Newton cast a strong red light, by means of a prism, upon the page of an open book in a dark room. At a certain distance from the book thus illuminated, he placed a double convex lens, so as to give an image of the book at its focus; this image was received upon a sheet of white paper properly placed behind the lens. The book and the lens being fixed in their respective positions, the paper was moved until that situation was found in which the image of the page and its letters were most distinctly depicted on the paper. This position was of course the focus to which the red light reflected from the book was collected by the lens. The prism in this experiment was so placed, that as the sun moved in the heavens, the several coloured lights of the spectrum were successively cast upon the book, without disturbing either its place or those of the prism or the lens. The position of the book was ascertained in which the letters appeared most distinct, and it was found that as the successive prismatic lights, in regular order from *red* to *violet*, passed over the book, the place of greatest distinctness gradually approached the lens, so that the full *violet* light required for distinctness that the book should be two inches and an half nearer to the lens than for the *red* light.

In this experiment it was necessary to render the chamber extremely dark, in order to prevent the pure prismatic light cast upon the book from being diluted by the white light which might be scattered about the room. In proportion as this adventitious light was admitted, it was found that the distance between the extreme foci became less. And this is plainly the reason why the distance between the extreme foci of prismatic light was found to be so much as two inches and an half, while the distance for light reflected from natural bodies was only one inch and an half (19). For the colours of natural bodies never have the extreme vividness, purity, and splendour which are obtained by the prism.

(31.) The doctrine of the different refrangibility of light led to an obvious consequence respecting its reflexibility, which Newton easily foresaw, and from which he derived another beautiful test to establish the truth of his theory. To render this intelligible to those who are not familiar with optical investigations, we must here be permitted a short digression.

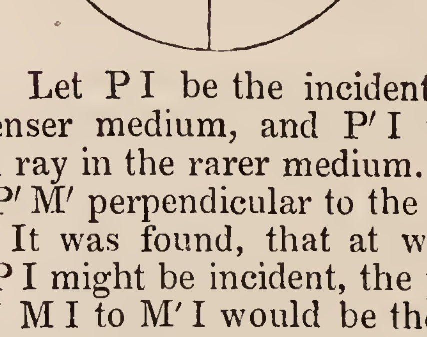
It will be recollected that long before the time of Newton, it was known that the deflection of a ray of light, in passing from a denser into a rarer transparent medium, was *from* the perpendicular. Thus, let A, *fig.* 21, be air, and G glass, and let P I be a ray incident on the surface S S, which separates the air from the glass. Let M M′ be the perpendicular at the point of incidence I. Since air is less dense or heavy than glass, the ray P I, instead of persevering in the direction P I, will take a direction further from the perpendicular I M′, such as I P′. This fact was long known. The law of this deflection, as discovered by Snellius, has been already explained; but it may be explained under another point of view, as follows:—

Fig. 21.



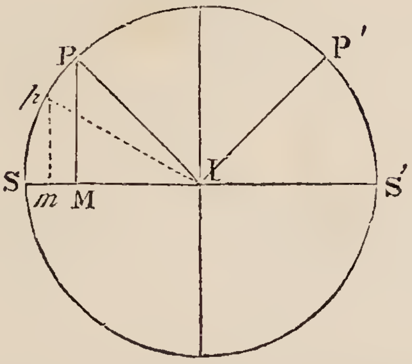
Round the point of incidence I, *fig.* 22, describe a circle in a perpendicular plane. Let P I be the incident ray in the denser medium, and P′ I the refracted ray in the rarer medium. Draw P M, P′ M′ perpendicular to the surface S S′. It was found, that at whatever angle P I might be incident, the proportion of M I to M′ I would be the same, so long as the media on each side of the surface remained unaltered. Thus, suppose M I were two-thirds of M′ I, *r* I being incident at I, its refraction may be thus found: take a distance I *m′* from I, of which I *m* is two-thirds, or such that I *m′* is equal to I *m* and the

Fig. 22.



half of $I m$, and from m' draw $m' p'$ perpendicular to $I m'$; the line $I p'$ will be the direction which the ray $p I$ will take in passing through the denser medium.

Fig. 23.



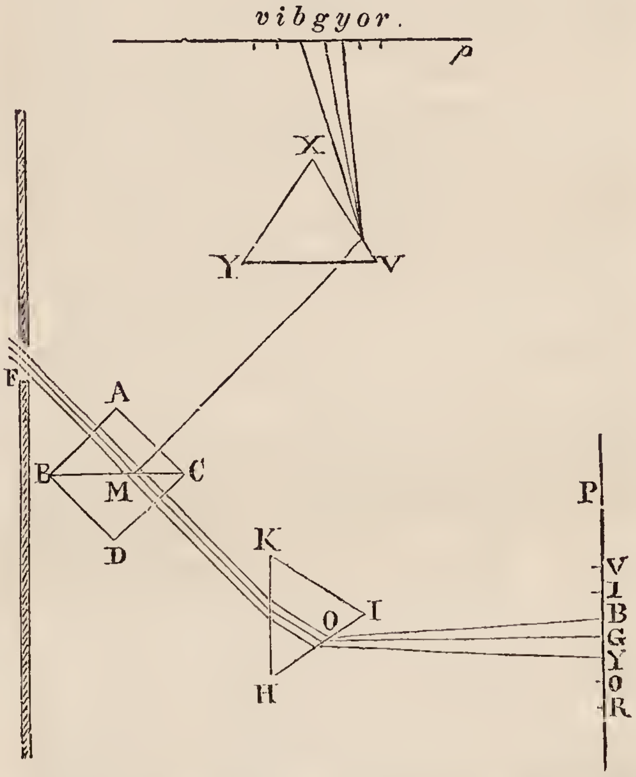
Now suppose that a ray P I, *fig.* 23, were to meet I in such a direction that M I is two-thirds of I S. If in this case, in conformity with the rule just explained, we take a length from I equal to I M, together with the half of I M, that length will be I S′, and S′ would be the point from which the perpendicular ($m' p'$, see last *fig.*) should be drawn to meet the circumference. But this point S′ being itself on the circumference, the perpendicular ($m' p'$) altogether disappears, its length being reduced to absolutely nothing; and the point (p') of the circumference to which the ray is deflected, would be the point S′ itself. Thus the Snellian law would shew that, in this case, the ray incident at I would not pass into the rarer medium. Experience, however, proved that, in this case, the ray P I was *reflected* according to the common law of reflection, in the direction I P′, making the angle P′ I S equal to I P S.

If the incident ray made any angle $p I m$ less than P I M, the law of Snellius likewise became inapplicable. For, in this case, m I being more than two-thirds of I S, the distance from I taken upon I S would be beyond the point S, and therefore outside the circle, so that the perpendicular could never meet the circle; and accordingly the refracted ray could have no direction conformable to this law. In all such cases experiment shewed that the ray was reflected. In this illustration we have supposed the fixed proportion to be two-thirds, but the conclusion may be drawn if any other proportion be adopted.

It appears, therefore, that in passing from a denser medium into a rarer there is a certain degree of obliquity beyond which the ray cannot be refracted; and, on the contrary, will be reflected back into the denser medium, according to the common law of reflection. It further appears, by what has just been explained, that this degree of obliquity, which limits the possibility of refraction, depends on the degree of refraction which the ray suffers in passing from the one medium into the other, and that the limiting obliquity is greater where this refraction is greater.

Aware of this property of refracting media, Newton perceived, that if the doctrine of unequal refrangibility were granted, it would follow that the limiting obliquity, in passing from a denser medium to a rarer, would vary with the refrangibility of the light, being greater for the violet and more refrangible lights than for the red and less refrangible lights. Thus it would follow that those lights which had a greater aptitude for refraction, were also more susceptible of reflexion, and *vice versâ*. He accordingly submitted the doctrine to this test by the following ingenious experiment.

Fig. 24.



(32.) He took two prisms, B A C, *fig.* 24, and C D B, of the same quality of glass, having the angles A and D right, and the angles at B and C, in each 45°, and placing their broadest faces B C together, tied them in this position, so as to form a square prism. This compound prism was placed before an aperture F, which admitted a beam F M of the sun's light, so as to fall perpendicularly on the face A B of the prism. This ray was incident upon the thin plate of air B C between the prisms, which were not brought into absolute contact. Passing through this and the second prism, it emerged from the surface C D parallel to its incidence; for since the refracting angles of the two prisms, A C B and C B D, are turned in opposite directions, and are equal, they neutralise each other's effects, and the light emerges as it entered. The ray M O, emerging from the compound prism, was received upon another prism I H K, and was dilated in the same manner as if it had been received directly from the aperture F, and the spectrum was received in the usual way upon the screen at P.

Another prism X V Y was placed above the compound one, in such a position as to receive a ray reflected from the surface B C; and above it a paper screen, placed to receive the light refracted by it.

Let us suppose that in the position thus given to all the prisms, a vivid spectrum appears on P, and no appearance of light is exhibited on *p*. The compound prism is now slowly turned round its axis in the direction A C D B, so as gradually to increase the obliquity of the ray F M to the surface B C. At a certain position, a strong violet light will appear upon the screen *p* at *v*. Maintaining the prisms, for the present, in this position, let the circumstances of the experiment be examined. The violet part of the spectrum on P will be found to have disappeared. If a screen be interposed between the prism I H K, and the compound prism, it will be found that the light which falls upon it will be of a colour which would result from the mixture of all the colours of the spectrum, except the violet. If a screen be interposed between the prism X V Y and the compound prism, it will be found that the light which falls upon it is the violet. The inference is most obvious. The incident ray F M has obtained the limiting obliquity, corresponding to the most refrangible or violet light. This light is accordingly reflected by the surface B C, and is transmitted through the prism X V Y to the screen *p*, where it appears. The violet light not passing with the other parts of the sun-beam through the prism B C D, has disappeared from the spectrum on P, which, therefore, now terminates with the indigo light. The beam M O, before it is dilated by the prism I H K, is in fact the sun-beam F M deprived of the violet light, which has been reflected at M, and, therefore, it is composed of all the colours of the spectrum except violet, as appears by the spectrum, which is now exhibited on P. Thus it follows that violet light is totally reflexible at an obliquity, which is insufficient to prevent the refraction of lights of other colours.

The prism A C D B was now slowly turned round a little more in the same direction, and the effects observed. The screen *p*, in addition to the violet light *v*, was now illuminated by the indigo *i*, of which the spectrum on P was observed to be deprived. The place of the indigo light on *p* was also next to *v*, but so as to be less refracted. On interposing the screen between the prism I H K and the compound prism, it was found to be illuminated with a colour, which would result from the mixture of all the colours of the spectrum, except *violet* and *indigo;* and on the other hand, on interposing it between the prism X V Y and I H K, it was illuminated with a colour which would be produced by the mixture of violet and indigo.

The inference from these effects is consistent with the former one. The prism A C D B had now attained that position which gave the incident ray F M the limiting obliquity of the indigo light. It was accordingly reflected, together with the more reflexible violet light. The lights of other colours were transmitted, and produced the effects observed on P, and between I H K and A C D B.

This process was continued, the prism A C D B being slowly and gradually turned on its axis in the same direction. The lights, blue, green, yellow, orange, and red, successively disappeared from the spectrum on P, and at the same times appeared in succession in their proper places on *p*. In each case the colour of the light in the beam M O was such as would result from the mixture of the colours on P, and the colour of the light M N was such as would result from the mixture of the colours on *p*.

All these effects are obvious and beautiful consequences, and therefore confirmations of Newton's doctrine. As the obliquity of the incident ray F M to the surface B C is gradually increased, it becomes successively equal to the limiting obliquities of the several coloured lights, and in the same succession reflects them to the screen *p*; the other screen P being in the same order, and at the same instants of time, deprived of them. Thus it appears, not only that lights of different colours are differently refrangible, but also that they are differently reflexible; and that those which are more refrangible are also more reflexible, and *vice versâ*.

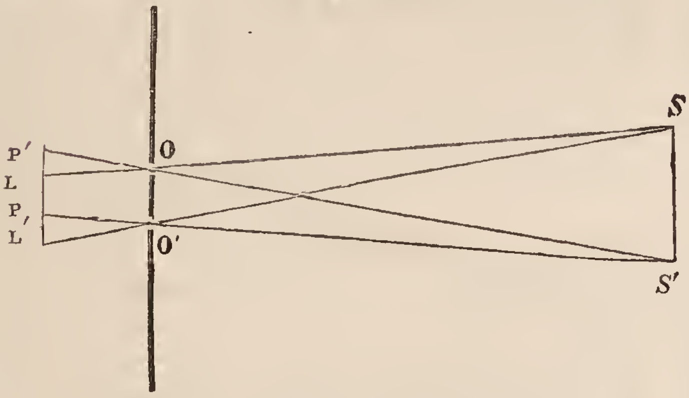
Chapter III.

Of the Methods of obtaining Homogeneous Light.

(33.) By the results of the experimental investigations described in the last chapter, Newton was convinced that solar light was not simple and homogeneous, but was a mixture or composition of many lights, differing from each other in certain respects, but more particularly in *refrangibility*. This quality he adopted as a test for pure unmixed or homogeneous light. A beam, every ray of which was equally refrangible, and which, therefore, did not admit of being dilated by a prism, he considered to be pure homogeneous light. Although the parts into which the solar beam was decomposed were shewn, by the experiment described in p. 17, to be incapable of further dilatation; yet this method did not give all the purity to the light which philosophical exactitude demanded, for the reasons which we shall now explain.

It will be recollected that the aperture in the window-shutter (p. 14) casts a circular illuminated spot on the opposite wall, the diameter of which is equal to the diameter of the hole, together with a line, which being drawn upon the wall would subtend at the hole an angle equal to the apparent diameter of the sun. But this circular spot is not uniformly bright. It is more faintly illuminated at its edges than at its centre, the cause of which will be easily understood. Let O O′, *fig*. 25, be the hole in the window shutter; S S′ the sun's diameter. A ray of light from S′, passing the upper boundary O of the hole, falls upon a screen at P; and a ray from S, passing the lower edge O′ of the hole, falls upon the screen at P′. Now, it is apparent that no part of the sun's disc, except the lowest point S′, can shine upon P, the upper part of the window shutter intercepting the light from all the other points; and that no part of his disc, except the point S, can shine upon P′, the lower part of the window shutter intercepting the light. Hence it appears that the points P P′, and the entire of the edge of the illuminated circle on the screen, will be more faintly illuminated than any of the parts nearer to its centre.

Fig. 25.



The ray from S′ passing above the lower edge of the hole at O′ will illuminate L, and thus the point L will be exposed to light from the entire disc of the sun. The same may be said of L′, and of all intermediate points. The several points from L to P′ will be exposed to light from only a part of the sun's disc, that part being smaller the more distant the point is from L, so that the light becomes gradually more faint from L to P′. The same may be said of the light from L′ to P. From this it appears that the circular illuminated spot on the screen is composed of a small circle whose diameter is L L′, uniformly illuminated, surrounded to the distance L P′ by a ring of light of gradually decreasing brightness, and fading away until it becomes insensible. This ring is called the *penumbra* *.

Now since the effect of the prism, as has been already proved, is to stretch out this luminous circle into an oblong form, the breadth being the same, and the sides and ends being illuminated by the rays which form the penumbral ring surrounding the unrefracted circle, it follows that the sides and ends of the spectrum will be bounded by a penumbral skirt of the breadth of P L′; and such, in fact, was the result of the experiments.

In the experiments which Newton now desired to institute, it was necessary that the light should be obtained of as uniform an intensity as possible, and therefore it was necessary to remove or very much diminish the penumbral fringe which we have just described. But it was still more necessary that light should be obtained which was perfectly pure and homogeneous, and it so happened that the same cause which produced the fringe and varied the intensity of the light, also impaired its purity.

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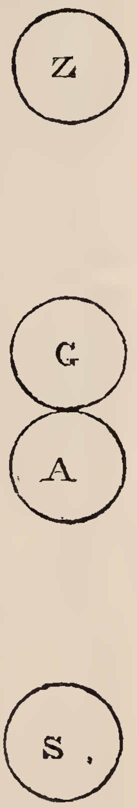
* The hole may be so small, that its apparent diameter at the screen will be less than that of the sun. In this case the part of the figure L L′ will disappear, and the whole spot on the screen will have the character of penumbra, the centre being the most luminous point.

This will be understood by attending to the effects of the prism on the circular spot.

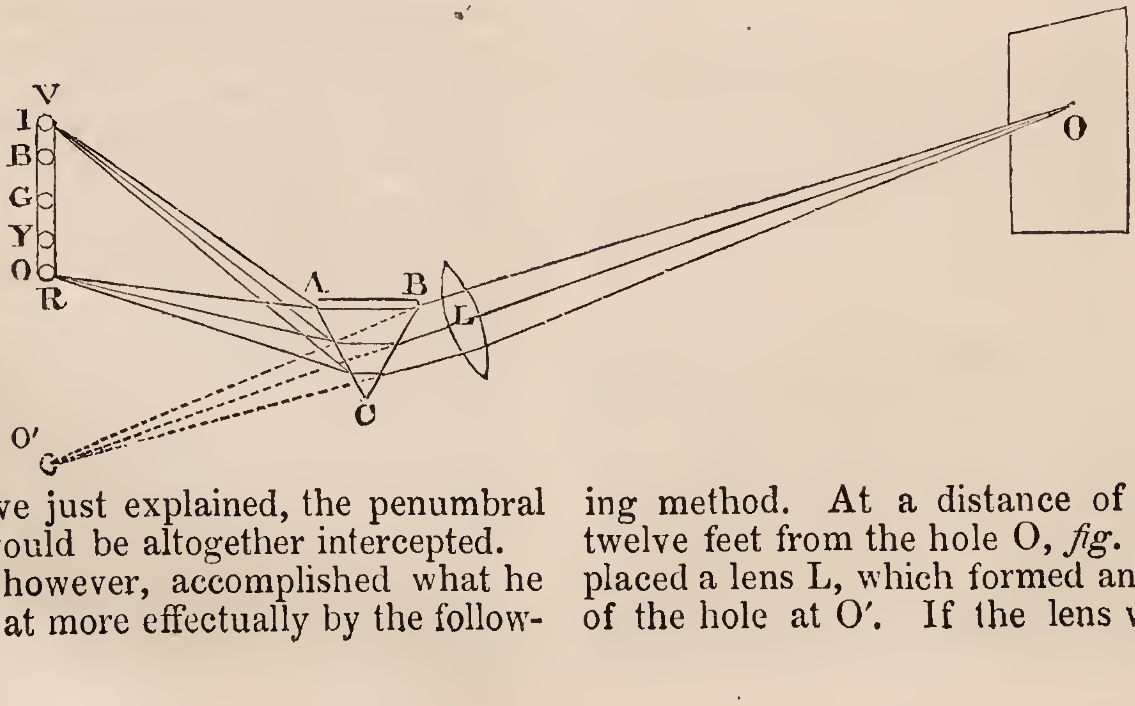
Fig. 26.



Let S, *fig*. 26, be the circular illuminated spot cast upon the screen by the light proceeding directly from the aperture without being intercepted by the prism. Let S A be the distance on the screen to which the least refrangible rays are deflected. These rays, which before fell upon the circle S, will now fall upon the circle A. Let S Z be the distance on the screen to which the most refrangible of the rays incident on S are deflected by the prism. These rays, which before fell upon the circle S, now fall upon the circle Z. The rays of all the intermediate degrees of refrangibility are deflected by the prism, so as to fall upon circles whose centres occupy the entire space from A to Z. From A take a distance A C, equal to twice the radius of the circle A, or, what is the same, to the breadth of the spectrum. A circle described round the centre C, with a diameter equal to A C, will evidently touch the circle A, but neither circle will be within or upon the other. It is evident, however, that every circle of the same diameter, whose centre lies between A and C, must lie partly upon the circle A, and partly upon the circle C. Hence it is evident, that the rays which, deflected from S, illuminate the circle A, and those which illuminate the circle C, are not intermixed. But between the points C and A is a considerable space, and between the degrees of refrangibility which would cause rays to be deflected to these points, are many intermediate degrees, in virtue of which rays would be deflected upon innumerable circles, whose centres lie between C and A. It follows, therefore, that all these lights of intermediate refrangibilities are intermixed with lights upon the circles A and C, neither of which, therefore, shine with pure homogeneous light. Since the lights of different refrangibilities which are thus intermingled are diffused over circles whose centres occupy the space A C, the number of such circles must be proportional to the space A C, or to the breadth of the illuminated circle S, increasing and diminishing as that breadth is increased or diminished. In this proportion, then, lights of different refrangibilities are mixed in the spectrum; and, therefore, every means which can diminish the breadth of the spectrum will proportionally increase the purity of the lights.

The first method which occurred to Newton to accomplish this, and, at the same time, to remove the penumbral fringe already mentioned, was to perforate the screen in the space L L′ (*fig*. 25), and to receive the light transmitted through the perforation on a second screen behind the first. By this means the penumbral ring would be received upon the first screen at L′ P, L P′, and the uniform light of L L′ would be admitted to the second screen through the perforation. In this case, the breadth of the illuminated circle on the second screen, and therefore that of the spectrum, would be nearly equal to that of the perforation. In proportion as the breadth of the spectrum would be thus reduced, the intermixture of heterogeneous lights would be diminished, and, as we have just explained, the penumbral light would be altogether intercepted.

Fig. 27.

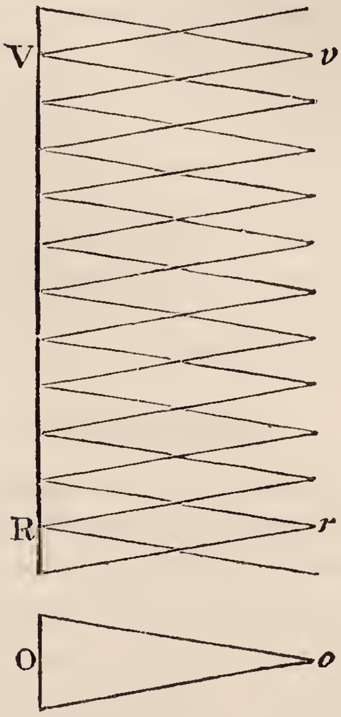


He, however, accomplished what he aimed at more effectually by the following method. At a distance of ten or twelve feet from the hole O, *fig*. 27, he placed a lens L, which formed an image of the hole at O′. If the lens were so placed that O L and O′L should be equal, which was always possible, the magnitude of the image O′ would be exactly equal to that of the hole. This image was therefore considerably less than the illuminated circle cast on the wall or screen without the interposition of the lens. A prism A B C, placed behind the lens, deflected the rays emerging from it, and formed the oblong spectrum R V, the breadth of which was equal to O′, and which was free from any penumbra. The length of the spectrum in these experiments is never changed. In the present experiment he succeeded in reducing the breadth to about one-sixtieth of the length; and therefore diminished the mixture of heterogeneous lights in any part of it proportionally. Although it was impossible by this method, or perhaps by any other, to obtain a beam of absolutely homogeneous light, yet what was thus obtained was sufficiently simple and homogeneous for all the purposes of experiment.

Some other ingenious expedients were resorted to for the simplification of light. The diminished breadth of the spectrum, while it gave a purified homogeneous light, gave a very small quantity of it. To obtain it in greater abundance, it occurred to Newton to form a great number of small holes in the shutter, in the same horizontal row, so as to obtain several of these spectra placed parallel to each other, and thus form one broad one, in which the lights should be as homogeneous as in a single one of small breadth. Or what was equivalent, and still more effectual, instead of a row of holes, he formed one narrow slit in the shutter extending in an horizontal direction, so as to admit a thin sheet of light. By this means a spectrum of any required breadth may be formed, in which the light is as pure and homogeneous as in a spectrum formed by light admitted at a round hole, whose diameter is equal to the breadth of the slit.

Newton suggests another very ingenious means of obtaining a spectrum, in which lights would be supplied of different degrees of purity and intensity. Let a narrow triangular hole be cut in the shutter in an horizontal direction, as *fig.* 28, O *o*. The sunbeam admitted through this hole will have an edge like a knife. The broadest part O of the ray will form a spectrum RV, in which the intermixture will be in proportion to the base O of the triangle; but as the ray diminishes in breadth towards *o*, the corresponding parts of the spectrum, towards R V, are increased in the purity of the light, heterogeneous rays being less intermixed as the breadth of the triangle is diminished, as is evident from the figure. Having a spectrum of this kind, experiments may be tried either in its stronger, though less simple light, on the side R V, or in the weaker and more simple light on the side *r v*, as may seem more suitable to the objects of the investigation.

Fig. 28.



In all experiments on homogeneous light, Newton states that he found it necessary to use great precaution in order to be secure of success. The chamber should be carefully darkened to avoid the disturbance arising from rays of white light scattered casually about. The glass, both of the prisms and lens, should be free from veins, striæ, air bubbles, and other inequalities. He found it so difficult to procure good prisms, that he frequently used transparent liquids inclosed in hollow prisms, formed of pieces of plate glass fixed together at proper angles.

With light simplified by the methods we have now explained, he tried the experiment explained in (26), and found that a pure homogeneous ray admitted of no dilatation by the prism, and therefore concluded that the light of which it was composed was all equally refrangible. With a view of establishing the same principle, he took two small circular pieces of white paper, and illuminated one with light direct from the sun, and the other with homogeneous light obtained by one of the methods which we have just explained. Viewing these circles thus illuminated through a prism, he found that the white circle was dilated into an oblong spectrum; but that the other still appeared as perfectly circular as when viewed without the intervention of the prism. He now placed minute objects in the homogeneous light, and viewing them through the prism, found them distinct; but when the same objects in the sun's direct unrefracted beam were similarly viewed, they became confused and indistinct.

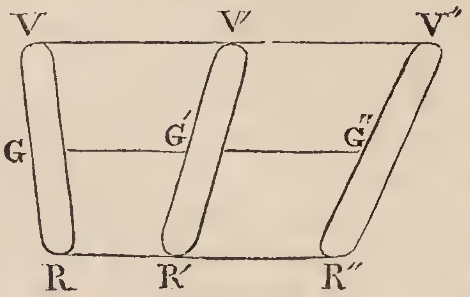
Chapter IV.

Law of refraction of homogeneous light —imperfection of refracting telescopes —Newton's reflecting telescope.

(34.) Newton having now succeeded in establishing the unequal refrangibility of the rays which compose solar light, his next step was to determine the law by which each species of light was refracted. It was about the year 1665, being then in his 25th year, that he appears to have commenced his investigations respecting the composition of light; and on the 8th of February, 1672, he communicated his discovery to the Royal Society, of which he had just been elected a Fellow. About fifty years prior to this period, Snellius discovered the *law of refraction*, which being afterwards adopted by Descartes, was well known, and generally received at the period to which we now allude. The philosophers who observed this law were not aware that all the rays of light were not equally refrangible; and Newton concluded that they adapted their measures to the mean rays, or those which lie in the middle of the spectrum, and which therefore have an intermediate refrangibility between those of the red and violet, the least and most refrangible rays. Hence he concludes from the experiments of his predecessors, that the green light of the prismatic spectrum is refracted according to the Snellian law.

Newton showed, by the experiment which we shall now describe, that when the sun's rays were all incident on the prism at the same angle, the sines of the refractions of the several component rays always have to each other the same proportion. It follows, therefore, that they must always have the same proportions to the sine of the angle of incidence, for since the green ray obeys this law, and they all have fixed proportions with respect to it, they must all obey a similar law. The experiment by which this was determined was as follows.

Fig. 29.



Let R V (*fig.* 29) be the spectrum produced by a prism in the usual way. Let another prism be placed at right angles to this, as described in (24), and let R′ V′ be the new position which the spectrum assumes. The distances R R′, V V′ measure upon the screen the deflections of the red and violet rays, and lines parallel to these, as G G′, measure the deflection of the intermediate rays. From the proportion of these lines, that of the sines of the refractions of the several kinds of rays was determined by mathematical reasoning. The prism placed at right angles to the first was now removed, and another with a different refracting angle was substituted for it, which removed the spectrum to the position R″V″. The proportion of the sines of the refractions was now deduced from measurement and computation as before, and was found to be unaltered. A third prism, with a different refracting angle, was used with the same result. Thus it appeared that the sine of refraction of each light bears a fixed proportion to that of the green light, while the sine of refraction of the green light bears a fixed proportion to the sine of incidence, by the law of Snellius; from whence it follows that in each kind of light the sine of the angle of refraction bears a fixed proportion to that of the angle of incidence, but that this proportion is different in light of different kinds.

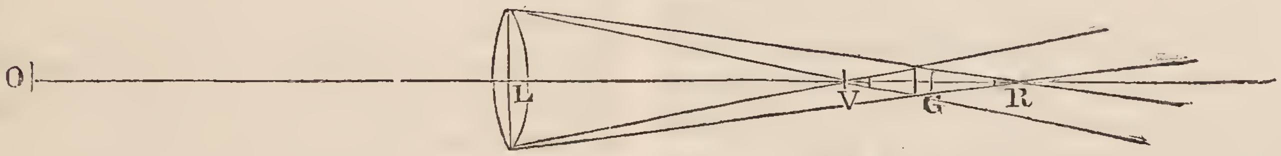
Newton gives an investigation, by which he shows that this law of refraction may be deduced independently of experiment, by mathematical reasoning, from the supposition that bodies refract light by acting upon its rays in directions at right angles to their surfaces. We shall not give here the details of this investigation, which also furnishes an explanation of the fact we have already stated, namely, the total reflection of the light at the limiting obliquities. It is, however, in the Principia that this latter consequence is deduced from it.

(35.) Having determined the proportion of the sine of incidence and refraction for the red, green, and violet, supposed to pass from air into glass, to be as follows:—

Red Sin . inc. : Sin . ref. : : 77 : 50
Green Sin . inc. : Sin . ref. : : $77\frac{1}{2}$: 50
Violet Sin . inc. : Sin . ref. : : 78 : 50

he proceeded to investigate the effect which this difference in the proportion produces on the images of objects formed by the object-glasses of refracting telescopes. When the curvature of the object-glass is not great, compared with its diameter, the angles of incidence of rays proceeding from a point at any considerable distance are very small, the rays being evidently very nearly perpendicular to the surface of the glass. By a well known principle of mathematics, the sines of very small angles are in the same proportion as the angles themselves: and, therefore, in the case to which we now allude, the angles of incidence will be to those of refraction as the above numbers, and the deviations of the rays above mentioned will be as the differences of those numbers. That is to say, the deviations of the *red*, *green*, and *violet*, will be as 27, $27\frac{1}{2}$, and 28.

Fig. 30.



Let L be a lens presented to a distant object from which the rays may be considered parallel. Let V be the focus to which the *violet*, or most refrangible rays are collected; and R the point to which the *red*, or least refrangible rays are collected; and let G be the focus of the medium or *green* rays. The places of the points, V, G, R, may be determined by geometrical reasoning, if the proportions of the sines of incidence and refraction as above given, and the curvature of the lens, be known. The result is, that if twice the distance, L G, be divided into 55 equal parts, the space V R is equal to one of these parts.

It may also be proved that if O be a lucid point, and V, G, R, the foci of the *violet*, *green* and *red* light from it, and as before twice the distance L G be divided into 55 equal parts, V R will have the same proportion to one of those parts as O G has to O L.

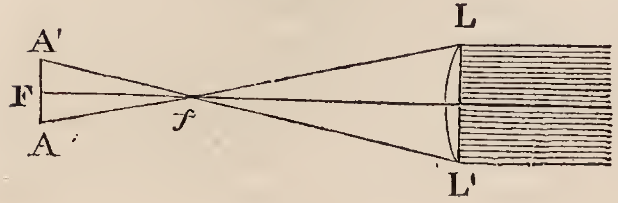
It will be observed that the violet rays diverging from V meet the red rays converging towards R at a certain point between V and R, and that at this distance all the rays which are refracted by the lens are collected into the smallest possible circle. The diameter of this circle being computed when the incident rays are parallel, was found to be about the 55th part of the diameter of the lens.

(36.) To verify these inferences, Newton repeated again the experiment described in (19), but adopted the method mentioned in p. 24, of rendering the prismatic light homogeneous. In the course of these experiments he encountered many practical difficulties arising from imperfections, such as veins, air bubbles, &c. in the glass of which his prisms were formed, of which, as well as of his efforts to avoid or remove them, he gives a very detailed and interesting account. He also found considerable difficulty in determining the exact foci of the lights of blueish character at the upper end of the spectrum, owing to their extreme faintness. On the whole, however, he succeeded in satisfying himself that the foci of the rays of different colours were at those points to which the computation made on their supposed unequal refrangibility assigned them. Thus it appeared that the object glass of a refracting telescope formed as many distinct images of an object placed before it, as there are lights of different degrees of refrangibility: that these images differed in colour, the blueish ones being nearest to the object glass, and the reddish most remote from it, and that between these were included images of a greenish and yellowish hue; that these images extended over a space along the axis of the telescope, equal to about 2-55ths of the focal length of that glass; and that the smallest space into which the innumerable images of the same point in the object can be collected on a plane at right angles to the axis of the telescope, is a circle, whose diameter amounts to about a 55th part of the diameter of the object glass. "So that it is a wonder," says Newton, "that telescopes represent objects so distinct as they do. But were all the rays of light equally refrangible, the error arising only from the sphericalness of the figures of glasses would be many hundred times less."

The effect called spherical aberration,

and its cause, have been already indicated in (11.) Let L L′, *fig.* 31, be the section of a plano-convex object-glass made by a plane passing through its axis, and let parallel rays of pure homogeneous light be supposed to fall on the plane side, perpendicular to the surface. If the surface of the lens be conceived to be divided into a number of concentrical rings, as described in (11), the foci of each ring will be more distant from the lens, the nearer the ring is to the edge of the lens. Let *f* be the focus of the marginal ring, and F that of the central rays. The foci of all the intermediate rings will lie between F and *f*. The rays diverging from all the foci between *f* and F are collected in a circle having the line A A′ for its diameter, and this is evidently the smallest space within which *all* these rays are collected. The diameter of this circle, therefore, measures the *lateral aberration* which parallel rays would sustain from the sphericity of the lens; and Newton calculated this, in order to compare the imperfection of telescopes, arising from this cause, with that imperfection which arises from the unequal refrangibility of light.

Fig. 31.



By geometrical reasoning, the details of which we cannot properly introduce here, it is proved that the square of the radius of the spherical surface of the lens, multiplied by the square of the sine of refraction, has to the square of half the breadth of the lens L L′ multiplied by the square of the sine of the angle of incidence the same proportion as half that breadth bears to the aberration A A′.

Newton then proceeds to show that if the object-glass were a plano-convex lens, having its plane side turned towards the object, having the radius of its convex surface 100 feet or 1200 inches, and the diameter of the lens four inches, the diameter of the smallest circle into which equally refrangible rays would be collected, would be about $\frac{1}{80000}$th of an inch. The calculation is as follows, the proportion of the sine of incidence to that of refraction being supposed to be 3 to 2.

Square of the radius of the spherical surface of the lens, (radius being 1200 inches)	1440000
Square of the sine (2) of refraction	4
Their product . .	5760000
Square of half the breadth of the lens, (the breadth being 4) . .	4
Square of the sine (3) of the angle of incidence	9
Their product	36

The proportion of these products is that of 160,000 to 1; and such is the proportion of half the breadth of the lens (*i. e.* two inches) to the aberration, which is, therefore, the 160,000th part of two inches, or the 80,000th part of an inch.

The diameter of the lateral aberration arising from unequal refrangibility of light, would, in the case of the lens just described, be the fifty-fifth part of four inches, or four fifty-fiths of an inch. The lateral aberration produced by the spherical form of the lens has, therefore, to that produced by the unequal refrangibility of light, so small a proportion as 1 to 5800.*

It follows, therefore, that the imperfection of telescopes, which arises from the spherical form of lenses, bears an exceedingly small proportion to that which is caused by the unequal refrangibility of light. But even the small error arising from the spherical form may be almost removed, as Newton suggests, by a compound object-glass, formed by two glass lenses with water between them. So that thus all the labours of Descartes, and others who devoted themselves to the formation of spheroidal lenses were fruitless, since even had they succeeded in producing lenses absolutely free from spherical aberration, the effect would not have been perceptible.

(37.) Reasoning thus, Newton did not hesitate to pronounce *the improvement of refracting telescopes desperate,* a conclusion which forms a striking exception to the almost superhuman sagacity which characterised all the philosophical researches of this extraordinary man. What renders this error the more wonderful, is that the property of light, on which the perfection of refracting telescopes has since been found to depend, seems to have pressed itself forward, and even to have courted his attention in the very experiments from which he deduced his erroneous conclusion. (*Fig.* 32.) Let A B C, A′B′C′ be two prisms, formed of different transparent substances. Let S B, S′ B′ be rays of the sun falling on them in parallel directions. Let B V, B′V′ be the most refracted or violet light in each, and B R, B′R′ the least refracted or red light. The deviation of the rays from their original direction, produced by the refraction of the prisms, will be different for each component part of the incident light. Newton supposed that the deviations of the different coloured lights from the common direction when incident, have to each other a certain fixed proportion; so that with the same average refraction or deviation from their common original direction, they would be dilated or separated from each other in the same degree. This may perhaps be more easily comprehended if thus explained. Let A B C, A′ B′ C′ be two prisms of different materials, receiving parallel rays S B, S′ B′, of solar light. Let the lines B M, B′ M′ divide the angles V B R, V′ B′ R′, formed by the extreme red and violet rays into equal parts, or so that V B M shall be equal to R B M, and also V′ B′ M′ to R′ B′ M′. Also, suppose the prisms to have such refracting angles, that the rays B M, B′ M′ shall be parallel. The deviations of these rays, *s* B M, *s*′ B′ M′, from their original directions B *s*, B′ *s*′ must be equal. Under these circumstances Newton concluded that the angles V B R and V′ B′ R′ would be equal, and that the deviation of every ray in the spectrum V R, from its original direction B *s*, would be equal to the deviation of the similar ray in the spectrum V′ R′ from its original direction B′ *s*′. Such is not the fact, and it is almost inconceivable how Newton, who had avowedly examined the spectra produced, not only by prisms of different kinds of glass, but also by liquids contained in hollow glass prisms, could have escaped noticing a fact that would at once have led him to the discovery of achromatic telescopes.

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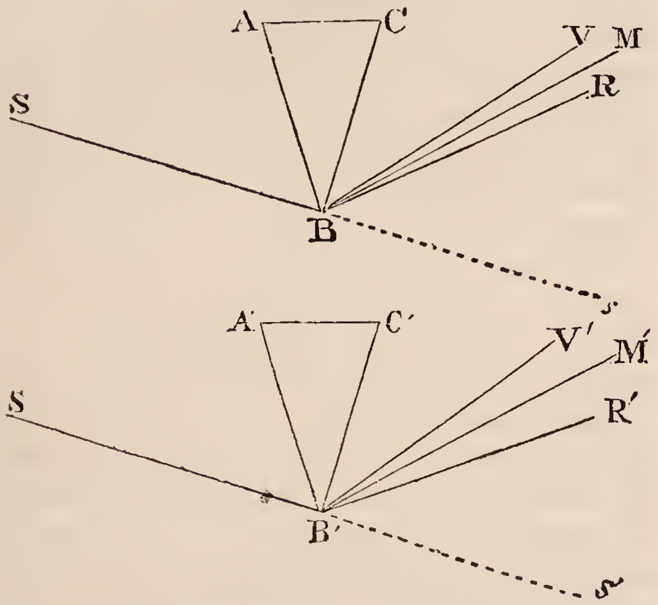
* This proportion is calculated with reference to the green or mean rays. If, however, it be taken with reference to those rays which produce the strongest effect in vision, it will only be as 1 to 1200.

Fig. 32.



In fact the prisms being circumstanced as we have just described, so as to produce equal deflections of the sunbeam from its original direction, the dilatation or *dispersion* of the rays from each other, and which may be measured by the divergence V B R, V′ B′ R′ of the extreme rays, will be different according to the material of which the prism is composed. Newton, on the other hand, concluded that when the deflection of the sun's beam by different prisms was the same, the dispersion would also be the same. Had he thought of measuring the lengths of spectra produced by different prisms, equally deflecting the light, he could not have failed to have found them different, and would have naturally been led to the discovery of achromatic telescopes, as we shall now explain.

Since prisms of different materials, with an equal deflection of the beam, produce spectra of different lengths, and since also the length of a spectrum varies with the position of the prism, or, what is the same, with the deflection of the light, it follows that if two prisms of different materials be exposed to beams of the sun's light, one of them may be turned until such a position be given to it, that the length of the spectrum produced by it shall be equal to the length of the spectrum produced by the other prism.

In this case the deflection of the beam by the two prisms producing equal spectra must be different, for if not, as we have before stated, the spectra would have different lengths. If one of the prisms be inverted with respect to the other, all other things remaining the same, the spectra will still keep the same length, but the colours will be reversed. Now suppose that instead of transmitting different beams of light through the two prisms, the *same beam* be successively transmitted through them, the one being placed behind the other, but the former arrangement being in all other respects preserved, it is quite evident that the dispersion of the one prism will have a tendency to neutralise the dispersion of the other, and that in the beam emerging from the second prism, the prismatic lights will be so mingled as to render the emergent beam nearly colourless. This will appear from considering that the tendency of the one prism to disperse the rays in one way, bringing the violet ray highest and the red lowest, is exactly equal to the tendency of the other to disperse the light in the opposite way, bringing the violet lowest and the red highest. But this mutual compensation will not obtain in the deflection of the light, since the power of the second prism to deflect downwards is not, in its actual position, equal to the power of the first to deflect upwards; so that the prism which has the less deflecting power will destroy so much of the deflecting effect of the other as is equal to its own, but an effective deflection will remain, by which the beam will be turned from its original direction.

Thus we arrive at the important fact, that a beam of light may have its direction changed by refraction, so that the directions of all its component rays shall be equally changed, or nearly so, although they be differently refrangible. What may be done by prisms may also be effected by lenses; and therefore an object-glass of a telescope may be so constructed as to collect all the rays of different refrangibilities nearly to the same focus,* and thus an achromatic telescope may be formed. Such was the discovery that Newton left to adorn a future age, a discovery presented to him by his own experiments, a fact rendered not improbable by his own reasoning, consistent with his own theory, and soliciting investigation and inquiry at almost every step of his own researches, yet which investigation and inquiry he seems, by an unaccountable pertinacity, to have stepped out of his way to avoid.

Newton seems not to have maintained an uniform opinion at all times on this point. The first edition of his Optics was published in 1704, and the second in 1717. In both of these he pronounces the improvement of refracting telescopes to be *desperate*. And yet, in a letter to Mr. Oldenburg, dated July, 1672, three years before his "discourse about light was written at the desire of some gentlemen of the Royal Society," he vindicates himself from a charge of Dr. Hooke, "who reprehended him for laying aside the thoughts of improving optics by refractions," in the following words—"What I said was in respect of telescopes of the ordinary construction, signifying that their improvement is not to be expected from the well figuring of glasses, as opticians have imagined. But I despaired not of their improvement by other constructions, which made me cautious to insert nothing that might intimate the contrary. For although successive refractions which are made all in the same way do necessarily more and more augment the errors of the first refraction; *yet it seemed not impossible for contrary refractions so to correct each other's inequalities, as to make their difference regular, and if that could be conveniently effected, there would be no further difficulty.* Now to this end I examined what may be done not only by glasses alone, but by a complication of divers successive mediums; as by *two or more glasses or crystals, with water or some other fluid between them;* all which may together perform the office of one glass, especially of the object-glass, on whose construction the perfection of the instrument chiefly depends. But what the results in theory or by trials have been, I may possibly find a more proper occasion to declare."

In this passage he hints at the principle on which achromatic telescopes depend, and even the manner of applying that principle in their construction, and yet fifty years of his life after this employed in perfecting his theory, seem only to have confirmed his error.

(38.) Abandoning all further inquiry into the methods of improving refracting telescopes, Newton, at the part of his Optics to which we have now arrived, proceeds to explain his contrivances for the construction of a reflecting telescope. In the end of a tube he placed a concave spherical reflector, which he constructed of metal, and polished with his own hands. The image from this was deflected by another plane reflector placed in the axis of the tube, so as to be received by an eye-glass in the side of the tube at which it was viewed by the observer. He suggests the possibility of constructing a concave reflector of glass, as being in some respects preferable to metal, but does not seem to have carried this into effect. Newton was fully aware of the defects of reflecting telescopes compared with refractors, owing to the much greater loss of light in reflection, and the greater aberration proceeding from their spherical form. These, however, he thought inconsiderable when compared with those defects of the refracting telescope, which proceeded from the unequal refrangibility of light.

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* In strictness, two prisms or lenses will only bring two colours accurately together, the law of dispersion being different throughout the whole spectrum: the rest, however, will be very nearly coincident, and consequently colour very nearly got rid of. By the combination of three prisms or lenses, three colours may be accurately combined, and the rest still more nearly than before; and so in succession.

Chapter V.

The Theory of Colours.

(39.) The colours exhibited by refracted and reflected light were phenomena with which philosophers had been familiar before the time of Newton. These effects were generally ascribed to the action of the reflecting or refracting body, and to the edges of opaque bodies which marked the limits of shadow, in imparting to the light qualities which it did not possess before encountering these bodies. Thus it was thought, that in passing through glass or other transparent substances formed into a prism, the solar beam is endued with a virtue by the action of the medium upon it, by which it reddens, or otherwise colours any body which it afterwards illuminates. In like manner, it was supposed, that in passing the edge of an opaque body a similar effect might be produced.

Before he proceeds to explain and establish his theory of colours, Newton shows that this hypothesis of his predecessors is untenable and inconsistent with facts. The coloured spectrum being produced by the prism in the usual way, the lights of the several colours may be successively intercepted by the interposition of an opaque body, so that any one of the colours may bound its shadow. These colours will remain unaltered by thus passing the edge of the opaque obstacle; and therefore he concludes that the light in passing the body receives no modification which affects its colour.

He further shows, that the same light, refracted in the same manner, passing the same opaque edges, will throw upon the paper which it illuminates different colours, according to the direction in which the paper is placed with respect to the rays. He argues, that if the colorific property were a virtue imparted to the ray by the edges of the aperture through which the light is admitted, or by the refracting medium through which it has passed, this could not happen, inasmuch as the colouring quality would then be independent of the position of the paper.

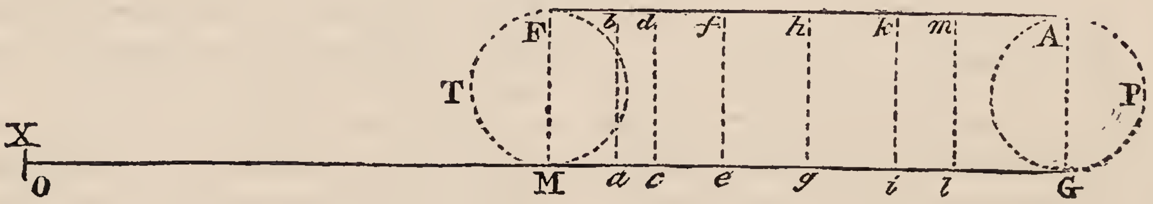
But perhaps the most conclusive argument against this theory is derived from the experiment explained in (32). It appears in that experiment that the confines of shadow produce no effect whatever; for the colour of the whole of the light emerging from the compound prism is always the same, that in the middle of the beam being in nowise different from that at the borders. Neither can the colour proceed in this case merely from the action of the glass, because it changes from white to yellow, orange, red, &c., that action remaining the same. Besides this, the refractions being equal, and in contrary directions, would mutually destroy each other's effects. It may further be argued, that if the light owed its colour to the action of the glass, it would not have the colour before its passage through the prism K I H; yet it was found, in that experiment, that when all the colours in the spectrum P were made to vanish, except the red, the light producing that colour on the screen P was found to produce the same colour on a screen which received it between the compound prism and K I H, before it was refracted by the latter. Thus the light which reddens the screen P would also redden it if unrefracted by the prism K I H, and the same may be said of the lights of other colours.

From these and, indeed, all other experiments which have been described, it abundantly appears, that "all homogeneous light has its proper colour answering to its degree of refrangibility, and that this colour is unalterable, either by refraction or by reflection." When pure homogeneous light of any colour illuminates a body, whatever the natural colour of that body may be, it will appear, when so illuminated, to have the colour of that light only which shines upon it. The apparent colour of the body is that of the light which it reflects; and it can reflect no light but that which shines upon it. Thus if a body whose natural colour is blue be placed in a dark chamber, and illuminated by the red light of the prismatic spectrum, it will appear red; and, on the other hand, a body whose natural colour is red, illuminated in the same way with blue light, will appear blue.

(40.) In the theory derived by Newton from the experiments which have been explained, the white light of the sun is supposed to be compounded of several component lights which have qualities different each from the others. They are all refrangible according to the same law discovered by Snellius (10); but, as we have already shown, they possess this quality in different degrees. This property is accompanied by another intimately connected with it. Any two of the component parts of solar light which differ in refrangibility, differ also in *colour;* and therefore the light of the sun is composed of various species of light of different colours, the mixture of which produces whiteness.

Newton next proceeds to determine the degrees of refrangibility corresponding to the rays of different colours. To determine this by experiment, he delineated, on a paper, the outline of the spectrum, *fig.* 33, F A P G M T, and refracting the sun's light by a prism, as described in p. 15, he held the paper so that the spectrum might exactly fall upon the space marked out upon it. He employed an assistant, whose perception of colours he considered to be better than his own, who drew lines across the paper, marking the confines of the several colours. Thus *a b* divided the *red* from the *orange; c d,* the *orange* from the *yellow; e f,* the *yellow* from the *green; g h,* the *green* from the *blue; i k,* the *blue* from the *indigo; l m,* the *indigo* from the *violet.* This experiment was frequently repeated on the same, as well as on different papers, and the results were found to be generally accordant. Let G M be drawn to X, so that M X shall be equal to G M, the spaces measured from G to the several boundaries of the colours were found to have the following proportion:

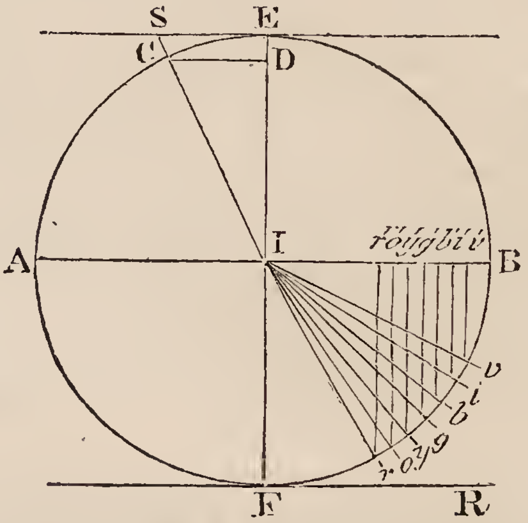
Fig. 33.



X G,	X *l*,	X *i*,	X *g*,	X *e*,	X *c*,	X *a*,	X M,
1,	$\frac{8}{9}$,	$\frac{5}{6}$,	$\frac{3}{4}$,	$\frac{2}{3}$,	$\frac{3}{5}$,	$\frac{9}{16}$,	$\frac{1}{2}$.*

The spaces measured along the spectrum occupied by the lights of the several colours may be considered to measure the differences of the sines of refraction of those rays having one common sine of incidence. But the proportion of the sine of incidence to that of refraction from glass into air has been already ascertained to be 50 to 77 for the least, and 50 to 78 for the most refrangible rays; it follows, therefore, that if 50 be the common sine of incidence, the sines of refraction for the rays at the boundaries of the several colours, beginning from the *red,* will be $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78, which may be familiarly explained thus. Let A B, *fig.* 34, be the surface of glass from which the ray S I passes at the point I into air. Let the ray S I be solar light. Round the point I as centre describe a circle, and through I draw a diameter E F perpendicular to the refracting surface A B. From the point C, where the ray meets this circle, draw C D: this is the sine of incidence. Let it be divided into 50 equal parts. Upon I B from I take a length I *r′* equal to 77 such parts, and draw *r′r* perpendicular to I B; again, take I *o′*, equal to $77\frac{1}{8}$ of those parts, and draw *o′o* perpendicular to I B. In the same manner, take I *y′*, I *g′*, I *b′*, I *i′*, I *v′*, equal to $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78 parts respectively, and draw, as before, *y′y*, *g′g*, *b′b*, *i′i*, *o′o*. From I draw I *r*, I *o*, I *y*, I *g*, I *b*, I *i*, I *v*. These lines will determine the directions of the *red, orange,* and the other rays corresponding

Fig. 34.



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* The analogy observed by Newton between the proportion of these intervals, and the musical intervals must be regarded as merely fanciful.

to the different degrees of refrangibility; the ray I B being after refraction resolved into I *r*, I *o*, &c.

If a ray of light be successively transmitted through several transparent media having different refracting powers, it may so happen that, on its emergence from the last of these media, it shall take a direction parallel to that which it had when incident upon the first of them. In this case the several refractions which the ray suffers in passing through the media, compensate and neutralise each other, so as to produce, on the whole, no deflection of the ray from its original course. Newton observed that, under these circumstances, whenever the incident ray was white, the refracted ray was also white. But he found, on the other hand, that if the refractive powers of the media were not thus related, and that a deflection of the incident ray from its original direction finally took place, a separation of the white ray into its component colours was produced. From these results he inferred that the same succession of media, which mutually neutralised the refractions of any one species of homogeneous light, also neutralised them on all the others, so that if one component part of the solar beam emerged parallel to its incident direction, all the others would emerge with it in the same directions, thus forming an emergent white beam. But, on the other hand, that if on the whole any deflection of the incident beam were finally produced, such deflection would be different for the different component lights; and, therefore, a decomposition or dispersion would ensue.

(41.) From these facts experimentally exhibited, Newton inferred, by mathematical reasoning, the following theorems:

I. The differences between the sines of incidence and refraction, when the ray passes from several different media into the same medium, are to one another in a given proportion.

II. The proportion of the sines of incidence and refraction for any one species of homogeneous light from one medium into another, is composed of the proportions of these sines from the first medium into any third medium, and from that third medium into the second medium.

By the first of these theorems, the refractions of all sorts of rays from any medium into air may be found, if the refraction of any one sort be known. By the latter, the refraction out of one medium into another may be found, if the refractions of both of them into a third medium be known.

"These theorems," says Newton, "being admitted into optics, there would be scope enough of handling that science voluminously after a new manner; not only by teaching those things which tend to the perfection of vision, but also by determining mathematically all kinds of phenomena of colours which could be produced by refractions. For to do this there is nothing else requisite than to find out the separations of heterogeneous rays, and their various mixtures and their proportions in every mixture. By this way of arguing, I invented almost all the phenomena described in these books, besides some others less necessary to the argument; and by the successes I met with in the trials, I dare promise, that to him who shall argue truly, and then try all things with good glasses and sufficient circumspection, the expected event will not be wanting."

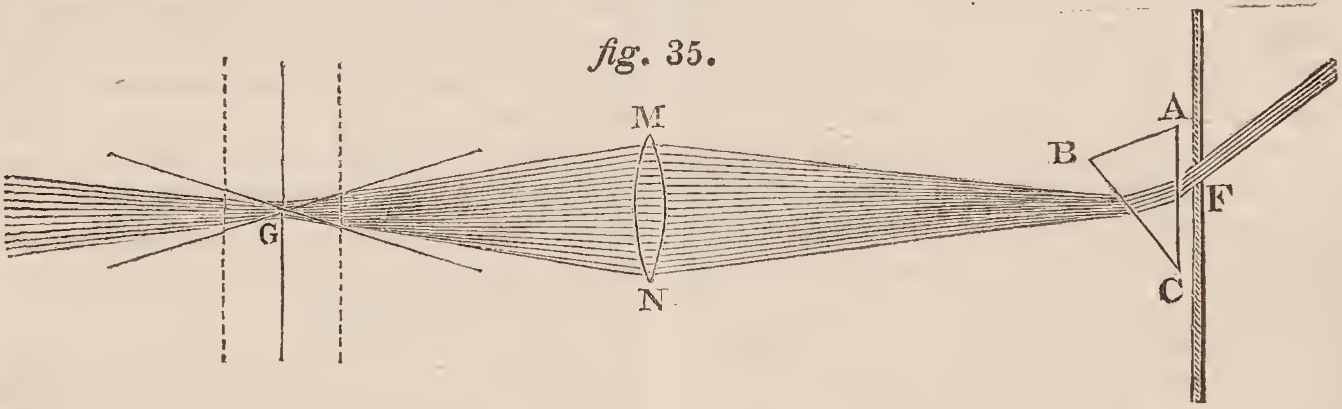
(42.) Although colour is one of the qualities of homogeneous light, it is not, like the degree of refrangibility, a *test* of its purity or homogeneity. For compound lights may be produced, the tints of which will not be distinguishable from those of homogeneous light. If the red and yellow lights produced by a prism be projected on the same white paper, they will give it an orange tint, precisely the same as the pure homogeneous orange light, which lies between the red and yellow lights in the spectrum. If another white paper be illuminated by this pure orange light, it will have exactly the same appearance as to colour as the paper which receives the compound light. But if these two papers thus illuminated be viewed from a distance through a prism, it will be found that no change will take place in the appearance of the paper illuminated by the pure orange light, while that which receives the compound light will be divided into two images of its component colours, red and yellow. In the same manner any two alternate colours in the spectrum will, by their mixture, produce the intermediate tint. Thus blue and yellow will produce green, and so on.

(43.) "Whiteness, and all grey colours between white and black, are formed by mixtures of all the colours; and the whiteness of the sun's light is compounded of all the prismatic colours mixed in a due proportion." The experiments by which Newton verified and established this important proposition, are characterised with such singular elegance and ingenuity, that we shall not apologise for giving the particulars of them at some length.

The prismatic spectrum being projected on a screen, a white paper was held before it, in such a manner as not to intercept the rays from the prism, and so that the paper should be as nearly as possible equally distant from all the colours. Under these circumstances, the paper appeared *white.* The colours which produced this white were evidently the several colours of the spectrum reflected from the screen upon the paper, and consequently reflected in the same proportions as they hold in the spectrum itself; from whence we may infer that the mixture of these colours produces white. If any of the colours of the spectrum be intercepted, the paper will appear to be illuminated with that colour which would be produced by the mixture of those which remain; a circumstance which further confirms the inference, that the white produced, when no light is intercepted, is the consequence of the mixture of all the colours.

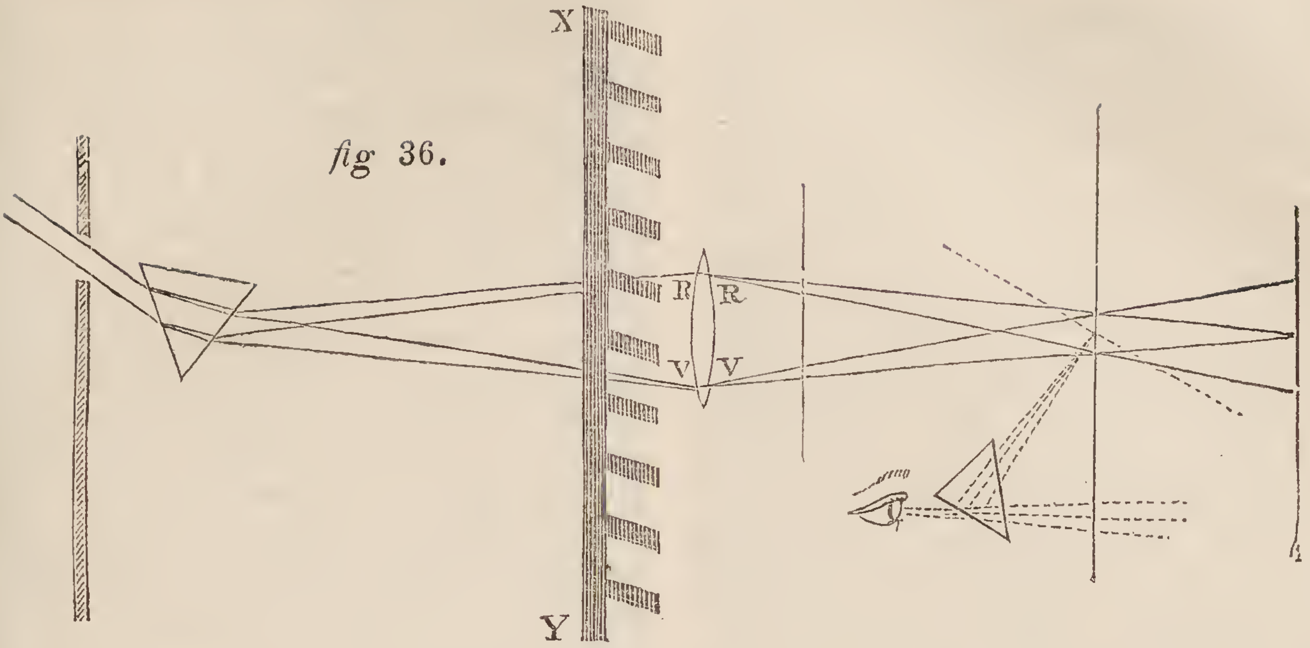
Let the spectrum *fig.* 35, be projected upon a lens M N, which will cause the coloured light to converge to its focus G, and there to fall on white paper. If the paper thus illuminated be moved to and from the lens, it will be found that when near the lens the paper will be intensely coloured. As its distance from the lens is increased, the colours will seem to approach each other, and be collected into a smaller space, until at last, at the focus G, they will be collected and perfectly mixed together: here the illuminated spot on the paper will be white. By removing the paper to a greater distance from the lens, the rays which before converged, having crossed each other at the focus G, will now diverge. The colours also will be inverted, those rays which were above in the former case being now below, and *vice versâ.*

fig. 35.



Let the paper be now placed at the focus, so as to be illuminated with white light free from colour. We are to prove that this whiteness arises from the admixture of all the coloured lights of the spectrum in their due proportions. Let all the colours except the *red* be intercepted by an opaque screen, placed between the prism and the lens. The spot on the paper will now appear *red.* By raising the screen let the orange be admitted with the red through the lens. The spot on the paper will now take a tint which would be produced by a mixture of red and orange. Again, let the yellow light be admitted, and a similar result will be obtained, the colour being one which would be produced by the mixture of red, orange, and yellow. In a word, let any number of the prismatic colours be intercepted between the lens and the prism, and the colour on the paper will be that due to the mixture of those colours which are not intercepted. From which we infer that if no colour be intercepted, the white light on the paper must arise from the mixture of all the colours.

Let X Y, *fig.* 36, be an instrument formed like a comb, with teeth about an inch and a half broad, at intervals of about two inches asunder. By interposing successively the teeth of this instrument between the prism and the lens, a part of the colours was intercepted, while the rest went through between the teeth. The teeth of this instrument being passed before the lens, all the colours are successively thrown upon the paper. Now, when this motion is rapid, so that the colours on the paper succeed each other in very quick succession, the eye loses all sense of colour, and the paper appears white. Yet it is certain that the paper is not at any instant white. In this case the perception of whiteness is produced by the continuance of the impression which each colour makes upon the sense of sight, until all the other colours have *fig* 36.

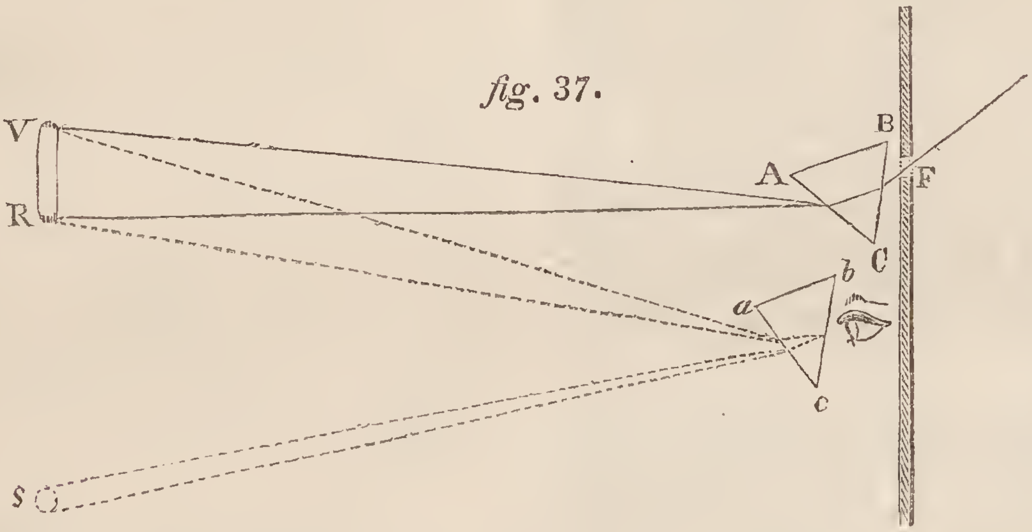


likewise affected the organ. The effect is thus compounded of the influences of the several colours upon the eye, as much as if they all affected it at the same moment.

(44.) In this explication of the phenomenon just described, we assume the fact, that when a visible object affects the eye, it continues to be perceived after it has ceased to be present. Thus, if a light be suddenly extinguished, the light itself and all the objects which it rendered visible continue to be seen for a certain short space of time after the extinction. This curious fact admits of very simple proof. If a burning coal or lighted stick be moved rapidly in a circle, it will be seen in every part of the circle at once, so as to have the appearance of a ring of fire; which proves that the impression which the light in one part of the circle makes upon the eye, continues until it returns again to the same part of the circle, to make another impression.

The colours of the spectrum may be recomposed, so as to form white light, by a second prism, instead of the lens mentioned in the last experiment. Let R V, *fig*. 37, be the spectrum formed by the prism A B C, and let this be viewed through another prism *a b c*, placed in such a manner that the rays which converge from R V will be received as if they emerged from a circular image of the sun at *s*. In this case the rays enter the eye exactly as they would if it were placed before the aperture F, and presented towards it. The colours proceeding from R V are thus mixed on entering the eye, and appear white.

fig. 37.



If any of the colours of the spectrum R V be removed by intercepting a part of the light between R V and the prism A B C, the colour which will be perceived through the prism *a b c*, will be that which would be formed by the mixture of the remaining colours. But if the comb mentioned in the last experiment be quickly moved between R V and A B C, so as to throw the several colours on the screen in rapid succession, the eye will again perceive white, for the reason already explained.

The same result was obtained by various other means, such as projecting several spectra produced by different prisms, on the same part of the same paper, by moving several spectra rapidly up and down, &c. &c. In all these cases the colours submitted to experiment were, however, *prismatic*. To establish his theory more completely, Newton now proceeds to inquire whether the colours of natural bodies were endued with qualities similar in all respects to those of prismatic light. To accomplish this, having procured powders of colours similar to those of the spectrum, he mixed them together as nearly as possible in the proportion which they were found to hold in the spectrum. He found that the mixture was not a pure white, such as that produced by the composition of the prismatic colours, but was a dim, greyish white; such, in fact, as would be produced by mixing a small quantity of black with a pure brilliant white.

(45.) It was not difficult to account for this circumstance, which Newton appears even to have foreseen. The colours of natural bodies arise from a quality, in virtue of which they reflect one component part of the solar beam more copiously than the others, and therefore affect the sense of sight with the colour so reflected. Thus a body which we call *red*, is one which reflects a very large portion of the red light of the solar beam, and absorbs nearly the whole of the other six colours. But it is found that no body reflects the light of its proper colour so copiously as a white body would reflect the same light. If a white and a red object be placed beside each other in a dark room, and both be illuminated with red homogeneous light, by means of a prism, the white object will be more intensely red than the red one.

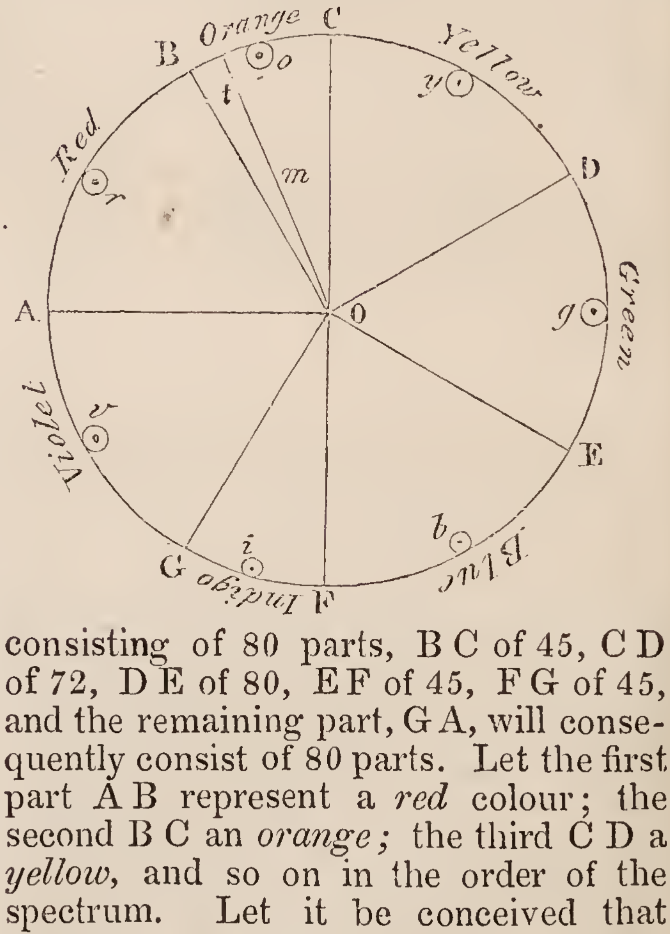
(46.) Since then coloured bodies do not any of them reflect *all* the light of their proper colour, we are not to expect by their mixture to obtain a clear white, but rather such an obscure white as would result from imperfect illumination. That the colour produced by mixing powders in the manner already mentioned is exactly of this kind, Newton proved by the following ingenious experiment.

He placed the mixture of powders on the floor of the chamber, and beside it a piece of white paper. The room being darkened, a beam of light was admitted, so as to illuminate intensely the powder, the white paper remaining near it, but in the shade. Viewing them from a distance, he could perceive no difference, both appearing to have exactly the same whiteness. Another person happened to enter the room during the experiment, and Newton, without informing him of the previous arrangement, asked him, "Which of the two whites were the better, and in what they differed?" After he had deliberately viewed them, he answered, "That both were good whites, that he could not say which was better, nor wherein they differed." Thus it was evident that the colour produced by the mixture of the powders was a true white, but only deficient in the *degree* of whiteness; just as twilight is as true a light as broad sunshine, differing from it only in quantity.

(47.) Having established the important fact, that *white* must result from the mixture of all the colours of the spectrum in the proper proportions, Newton proceeds to the consideration of the more general question as to the colour which would result from the composition of any given colours in any assigned proportion. For this problem he gives the following very ingenious solution.

With the centre O, *fig*. 38, and a radius O D, describe a circle A D F, and let the circumference of this circle be divided into 447 equal parts. Take A B, consisting of 80 parts, B C of 45, C D of 72, D E of 80, E F of 45, F G of 45, and the remaining part, G A, will consequently consist of 80 parts. Let the first part A B represent a *red* colour; the second B C an *orange;* the third C D a *yellow*, and so on in the order of the spectrum. Let it be conceived that these are all the colours of uncompounded light gradually passing one into

Fig. 38.



another as they appear in the spectrum, so that, in effect, the circumference of the circle will exhibit, as it were, a *round* prismatic spectrum. By the principles of mechanics, let the centres of gravity of the arcs A B, B C, &c. be respectively found, and let these points be *r*, *o*, *y*, *g*, *b*, *i*, and *v*. Now, suppose that it be required to determine the tint which would result from the mixture of red, green, and blue, in certain given proportions. Let circles be described round the points *r*, *g*, and *b*, the magnitudes of which are to be made proportional to the quantities of the three colours in the proposed mixture. Let the common centre of gravity of these circles be found, and let it be *m;* and from the centre O through *m* draw O*m*, to meet the circle at *t*. The colour at the point *t* will be the tint sought, and the line O*m* will represent "its fullness, or intensity, that is, its distance from whiteness." Thus, if *t* should fall exactly in the middle of any of the arcs, A B, B C, &c. the tint will be the purest of the corresponding colour; but if it be distant from the middle point, it will partake of the colour which occupies the next arc, towards which it lies. Again, if *m* fall on the centre O, the colour will be, as it were, infinitely diluted, and will be a perfect white; but, on the other hand, the nearer *m* is to the circumference, the more intense and florid the tint will be.

Newton conceived this method to be sufficiently accurate for practice, although not mathematically true. This is a subject, however, in which much improvement has been introduced in later times. It would not be to our purpose here to enter upon it, our design being merely to present to the reader in a popular form a sketch of the labours of Newton in the *science of light*. Those who desire a short account of the modern discoveries, will find one in the admirable article on Light, by Mr. Herschel, in the Encyclopædia Metropolitana.

In applying his theory of light to explain the phenomena of the colours of natural bodies, Newton assumes, "that every body reflects the rays of its own colour more copiously than the rest, and derives its colour from their excess, or predominance, in the reflected light." When a beam of solar light falls upon a violet, a decomposition immediately ensues. The red rays, and those of the less refrangible character, are either transmitted through the body, or absorbed and stifled; those of the bluish, or violet hue, and of the more refrangible species, are copiously reflected, and produce in the spectator the effect which in ordinary language is denominated the violet colour of the object.

Several ingenious experiments support this reasoning. A natural object, whatever be its colour, will, if placed in homogeneous light, take for the time the colour of that light, proving thereby its capability of reflecting, *in some degree*, lights of all colours. But when it is placed in homogeneous light of its own colour, it will appear much more resplendent than in light of any other colour. Hence we infer that it possesses a capability of reflecting light of its own colour more abundantly than light of any other colour. Thus cinnabar, a red substance, placed in homogeneous red light, exhibits a splendid red; let it, however, be illuminated with green or blue light, and it will assume these colours, but with great faintness.

The colours of transparent liquors vary with their thickness. If a red liquor be poured into a glass of conical or tapering shape, and held between the light and the eye, it will appear of a pale dilute yellow at the narrowest part of the glass; a little higher, where the glass is wider, it becomes orange; higher still it becomes red; and, finally, in the widest part, exhibits a deep dark red. We must, therefore, infer that a small quantity, or thickness of the liquor, intercepts a portion of the violet and indigo rays, so that the remaining rays which it transmits form a pale yellow. A greater quantity of the liquor, besides stopping the violet and indigo, also arrests the blue rays, and a part of the green, transmitting the other component parts of light, the mixture of which produces an orange. A still greater quantity of the fluid will intercept all the green, and a great part of the yellow, so that the transmitted light approaches to a red, becoming a deep dark red, when the quantity of the fluid is so great as to absorb the whole of the orange light.

We have in this description assumed several distinct effects, but the changes of colour are not sudden, but take place by an imperceptible gradation, an obvious consequence of the tapering form of the glass. If the glass were formed of a number of cylinders rising one above another, the diameter of each exceeding that below it by a certain magnitude, the changes of colour would be sudden and distinct; and the liquid in each cylinder would present, in the vertical direction, an uniform colour.

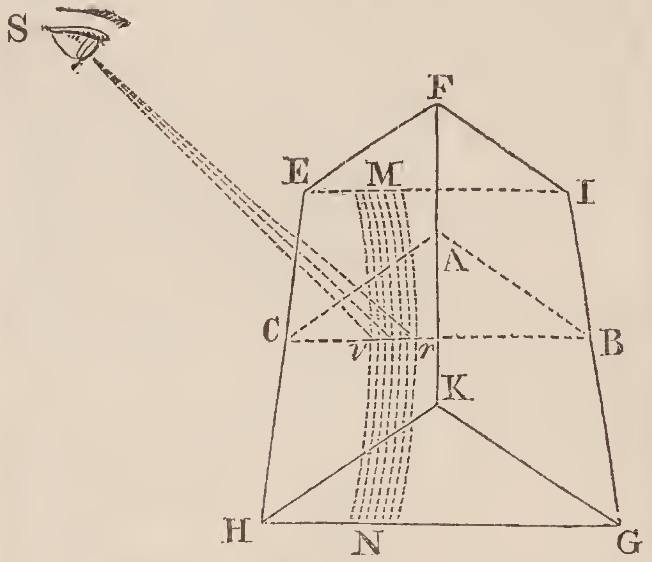
(48.) Connected with the power of transparent liquids to reflect and transmit the different component parts of solar light, Newton mentions two very remarkable facts noticed by Halley and Hooke, but which these eminent philosophers were unable to explain. Halley, having descended in a diving bell to the depth of several fathoms in the sea, observed, upon holding his hand in the sun's light, which penetrated the water, and shone into the bell through a small glass window in the top, that the light upon his hand was *red*. Whereupon he examined the lower part of his hand illuminated by light reflected from the water below, and found it *green*. This circumstance is thus accounted for by Newton. The sea-water reflects back the violet and blue rays most easily, and transmits most copiously the red. In the sun's light transmitted to considerable depths, the red rays therefore predominating, objects illuminated by them assume a red hue. At depths to which the violet rays cannot penetrate, the reflection of the blue, green, and yellow light separated from the red, which is transmitted, must compound a green.

Two liquids may be obtained, one of which transmits the rays of the red character, and the other those of the blue, the former intercepting the bluish light, and the latter the red. If both liquids be placed between a spectator and the light, they will be found perfectly opaque, although either alone is transparent. This is evident, since all the rays which can be transmitted by either are intercepted by the other. Hooke casually, and without anticipating or expecting the result, actually tried this experiment. He filled two hollow glass wedges, one with a red, and the other with a blue liquor. On placing the wedges together, and looking through them at the light, he found them absolutely opaque.

(49.) We have explained, according to the Newtonian theory, the most striking phenomena of coloured lights produced by prisms. The explication of others will be found in every elementary treatise on optics. One very singular prismatic phenomenon, however, still remains to be noticed, and is entitled to attention, as well for the strong confirmation of Newton's theory which it furnishes, as from the ingenious manner in which that theory is shown to account for it.

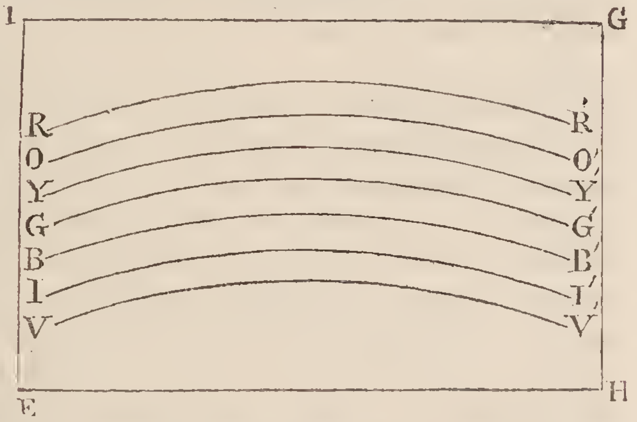
Let HKG, *fig*. 39, be a prism placed before an open window, with its base H E I G horizontal, the face F K G I presented to the light of the clouds, and let the base be viewed through the face F K H E by an eye at S. The base H E I G will now be observed to be separated into two parts by a beautiful iridescent arch, formed of colours of violet and bluish tints. This arch is concave towards the eye, and that part of the base which is towards the edge I G, or above the arch, exhibits a most vivid reflection of the firmament, not yielding in splendour to the direct view of the heavens. On the other hand, the lower division of the base next the edge E H, appears nearly dark, reflecting but a very small portion of the light incident upon it. The arch next this sombre space is fringed with a violet colour, which is gradually tinted off into a vivid blue towards the convex edge, which bounds the bright part of the base.

Fig. 39.



To account for this curious phenomenon, it must be remembered that the different parts of solar light are differently reflexible; also, that when rays of light are incident on the base of a prism, having previously passed through its side, there are certain angles of obliquity at which it will be impossible for the rays to pass through the base, and they will then be reflected. The limit of obliquity at which they will cease to penetrate the base, and will be reflected, depends on their degree of refrangibility. The most refrangible, and consequently the most reflexible rays, are the violet, next to these the indigo, then the blue, and so on through the other colours of the spectrum, the red being least reflexible. Let H E I G be the base of the prism. From the eye, let lines be supposed to be drawn to the base, inclined to it at that angle which limits the reflexion of the violet light. These lines being all equally inclined to the base, must meet it at points which lie in the arc of a circle. Let this arch be V V′, *fig*. 40. Again, let lines be drawn at the limiting angle of the indigo rays. This angle being less than that for the violet, the corresponding arc I I′ will be beyond V V′. In the same manner the limiting arcs B B′, G G′, Y Y′, O O′, R R′, corresponding to the other prismatic lights, *blue*, *green*, &c. may be drawn.

Fig.40.



It follows then, that all the violet rays in the solar light will be reflected from the part of the base of the prism whose boundary is V V′ G I; all the indigo from I I′ G I; all the blue from B B′ G I; all the green from G G′ G I; all the yellow from Y Y′ G I; all the orange from O O′ G I, and all the red from R R′ G I. Hence it appears that the space between the arcs, V V′ and I I′, is illuminated with a pure violet light only; that between I I′ and B B′ is illuminated by both violet and indigo mixed; between B B′ and G G′ there is a mixture of violet, indigo, and blue; between G G′ and Y Y′ is a mixture of the former colours, with the addition of green; from Y Y′ to O O′, yellow is added to the compound; the next arched band introduces orange, and the last the red. Now the last mixture constitutes a pure white. The former also a white, but one which, being deprived of the pure red rays, takes a faint tint approaching a bluish colour, but which is not distinguishable from a perfect white. In the next space the red and orange being removed, the mixture produces a greenish blue, which rapidly deepens, and becomes a strong blue, when the yellow rays are removed. The arc towards its inner termination is a pure violet.

Newton next applies his theory to explain the phenomena of rainbows. As this subject has been already fully discussed in our treatises on Optics according to the same principles, and in exactly the same manner as it is treated by Newton, it is not necessary to repeat it here.

CHAPTER VI.

On the phenomena exhibited by thin transparent plates—the theory of the fits of easy reflexion and transmission deduced from these phenomena.

(50.) THE first book of Newton's Optics contains the discussions which have been detailed in the last four chapters. In these investigations a ray of light upon its impact on the surface of any medium is considered to undergo one of two effects, viz. either to pass into the medium on which it impinges in a determinate direction, in which direction it is supposed to persevere through the entire medium; or to be reflected back from the surface into the medium from whence it came, following also and persevering in a rectilinear course. We are now about to accompany this great scrutineer of nature through a more subtle analysis of the process to which a beam of light is submitted when it encounters the surface which separates two media of different densities.

If it were possible to divide the medium which a ray of light penetrates into a series of plates, the thickness of which should be minute to an extreme degree, and to examine the state of the ray during its transmission through each of them, we should attain the end which we desire. Although it would perhaps be difficult to effect this very minute subdivision by direct mechanical means, yet numerous expedients present themselves, and those too of a character sufficiently familiar, by which the phenomena in question may be brought under examination. Indeed, these phenomena were long the subjects of daily observation, and may almost be said to have been the sport and toy of children; but, like many other natural effects, not less wonderful, which are continually passing under our eyes, they had failed to excite the attention or stimulate the curiosity of those who, by faculties and acquirements, were qualified to behold in them manifestations of the laws and principles on which the works of nature are constructed.

(51.) If a small quantity of soap be mixed with water, the latter acquires a tenacious or glutinous quality, in virtue of which it may be blown into bubbles, or it may be thrown into that state by mere agitation. Every one is familiar with the various colours which these bubbles reflect. Similar appearances are exhibited by glass when blown into bubbles of sufficient tenuity. Since these effects are not produced when the bounding surfaces of the medium are more distant from one another, we are compelled to suppose that when the light first enters the transparent medium, it is put into some state in which it does not continue during its entire course through the medium. This inference is as singular and important as it is inevitable. Suppose that the two surfaces of water impregnated with soap were at a distance of one inch asunder—a ray of light entering the first surface perpendicularly, would penetrate the water, and passing through the second surface, would issue from the water at the other side, preserving its original direction. Now, suppose that the second surface of the water, instead of intercepting the course of the ray at the distance of an inch from the first surface, meets it at a distance from that surface, equal to the thickness of a certain part of the soap-bubble, to which we have alluded—the ray will no longer be allowed to pass out in its original state at the second surface. On the contrary, if it be white solar light, that part of it which has a certain colour, say red, will be reflected back in the direction from which it came, while the remainder only of the ray which, combined with red, would produce white, will be transmitted. It therefore follows that, in this instance, after the ray has penetrated the water through a space equal to the supposed thickness of the bubble, that portion of it which is red is put into such a state, that were it to encounter the second surface, it could not penetrate it, and would be reflected. This state, however, does not continue; for when the white ray is allowed to proceed further into the water before it is intercepted by the second surface, it will be brought into a state in which it will penetrate that surface, and be transmitted into the ambient medium. Such is an example of the class of facts which form the basis of the experiments and investigations which we are now about to explain.

(52.) The property which we have instanced in water and glass is common to all transparent media. The evanescent and fluctuating nature of a water-bubble renders it an inconvenient object of experimental inquiry. Glass is better, but still is difficult to procure, and to retain in the highly attenuated state which is necessary to manifest the desired effects. By the following contrivance, Newton rendered *air*, though at the first view an unpromising agent, available for the purposes of deliberate and close experimental observation.

He procured a double convex lens, the object-glass of a fifty foot telescope, and consequently having a degree of convexity so small as to be scarcely perceptible. On this he placed the plane surface of another lens, so that the two surfaces were in absolute contact at the centre, the distance between them increasing with the increased distance from that centre. A lens of air was thus inclosed between two glass lenses. This air lens was plane on one side and concave on the other, losing all thickness at the centre at which may be conceived an infinitely small space, filled by the point of contact of the glass lenses. Taking this point of contact as a centre, let us suppose a number of concentric circles to be traced on the lenses as represented in *fig.* 41. Let the smallest circle be called 1, the next 2, the next 3, and so on. It is plain that the thickness of the air under the circle 1, is less than under the circle 2. In like manner the thickness under the circle 2 is less than under the circle 3, and so on. A section of the lenses exhibiting the thickness of the air between the lenses under the several circles 1, 2, 3, &c. is represented in *fig.* 42.

Fig. 41.

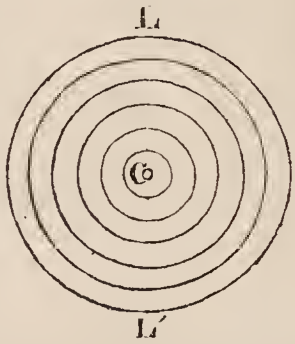
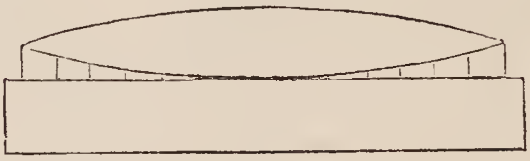


Fig. 42.



Upon exposing these lenses to a beam of light, a very minute black spot was observed at the centre. Immediately around this circular spot was a ring of blue colour, which gradually emerged from the black, so as to assume the perfect blue tint at some distance from the black spot. This blue ring was surrounded with a white one, into which it in like manner gradually melted, assuming tints more and more dilute, until it became absolutely white. This white ring was again bordered by a yellow one, which in its turn was tinted off in a red ring.

In this series of coloured circles the *blue* of the first circle was very faint; the *white* of the second was brilliant. The gradual tints assumed by the yellow of the third ring in passing into the red of the fourth, produced between them an *orange* ring.

A second series of rings succeeded, the first of which, surrounding the red ring of the last series, was *violet*, after which appeared in regular succession four other rings of *blue*, *green*, *yellow* and *red*. In this series the green was yellowish, the yellow brilliant, and the red partaking of a crimson hue.

After this came a third series. The first ring in this series surrounding the red of the last was purple, which was regularly succeeded by four rings of *blue*, *green*, *yellow* and *red*. Of these the green was brisk and copious, being a rich grass green, and the yellow was particularly splendid; but the red had a more faded appearance, and partook more of the pink and crimson, than the vermilion.

After these succeeded a fourth series, consisting of two colours, green and red. The green, in passing into the red, exhibited a yellowish *pink* hue. Then succeeded three other series, each consisting of two rings; the inner ones being various shades of green, and the outer ones various shades of red; each of the colours become more and more dilute, as the diameters of the rings increased.

(53.) Due consideration of these phenomena suggested some very important conclusions. Of the light which penetrated the glass within the central black circle none was reflected, for in that case the circle would take the colour of the reflected light. The incident light was, therefore, in this case, either stifled and absorbed by the glass, or was transmitted. To ascertain this, the eye was placed behind the lenses, so as to receive the transmitted light. The central spot now appeared white, proving that all the light incident on the glass was here transmitted. Again, of the light incident upon the first ring of the first series, the rays composing a bluish colour alone were reflected. The remaining rays were transmitted, as appeared by viewing the lenses on the other side; the colour of the first ring being that which was complemental to its reflected colour, or that which combined with the reflected colour would produce white. In the same way, each ring of each series was found to transmit the colour complemental to that which it reflected, which was proved by viewing the light through the lenses.

These phenomena were attended with many circumstances, which rendered it probable that some connexion subsisted between the colours of the reflected and transmitted light, and the thickness of the air-lens, at the place where these colours were produced. The same colour was observed to be arranged in a circle round the centre of the lens. It was evident, that in all parts of such a circle the thickness of the air-lens was the same. Again, in passing from one concentric circle to another, the tint was observed to undergo a change. In different concentric circles the thickness of the air-lens was different. Here, then, were two important steps towards the discovery of a connexion between the colour of the light, and the thickness of the air, which reflected or transmitted it.

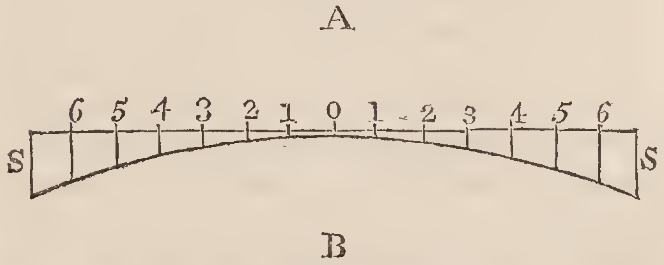
By pressing the glass lenses together, so as to force them into closer contact, the diameter of each circle, at which the air-lens had a given thickness, would obviously be increased. If it were true, that the colour of the reflected light depended on the thickness of the air at the points of reflexion, it would follow, that upon pressing the glass lenses together, each coloured circle would be enlarged. It was accordingly found, that upon applying such *pressure*, the central spot was increased, and each coloured circle expanded its dimensions, and retreated from the centre. These indications were further confirmed, by pressing the lenses more closely at one side of the centre than at the other, the colours still retreating from the points of closest contact.

(54.) Aware of the heterogeneous nature of solar light by previous investigations, Newton considered it probable that these coloured rings were not the effects of one simple cause, or of a single action of the transparent medium on the solar ray, but conjectured that it might rather be the result of the combined actions on all or several of the component parts of light. To simplify the phenomena, and thereby facilitate the analysis, he determined to expose the lenses successively to the different species of homogeneous light, and to observe and carefully note the separate effects of each. He concluded, that these effects being severally known, there could be no difficulty in combining them, so as to account for the phenomena produced by compound solar light.

With this view, he decomposed a sunbeam, by means of a prism, and casting successively on the lenses the several coloured lights in the spectrum, he observed and carefully noted the phenomena. In each case the rings appeared, and even in greater numbers, than in the case of the compound solar light. They, however, no longer exhibited any variety of colour, the central spot being now surrounded by rings of the same colour as the light cast upon the lenses, separated by dark rings, in which, like the central spot, all light seemed to be transmitted, and none reflected. Upon looking through the lenses towards the light, the intermediate rings just mentioned, as well as the central spot, appeared of the colour of the prismatic light, to which the lenses were exposed; and, on the other hand, those rings which by reflected light appeared coloured, were now dark, no light being transmitted.

Let S S, *fig*. 43, be a section of the air-lens, and suppose a beam of homogeneous red light projected on it from the direction A, and perpendicular to its surface. The centre of the lens being *o*, let 1 be the place of the first ring of red light, as viewed from A. At 2 will be a dark ring, at 3 a second ring of red light, at 4 a dark ring, and so on, the central spot *o* being dark. Now, let the lens be viewed from B, so as to receive the rays transmitted through it. The central spot *o* will appear red, the first ring 1, which before was red, will be dark, the ring 2 will be red, 3 dark, and so on; all the rings which were dark, when viewed from A, being red when viewed from B, and *vice versâ*.

Fig. 43.

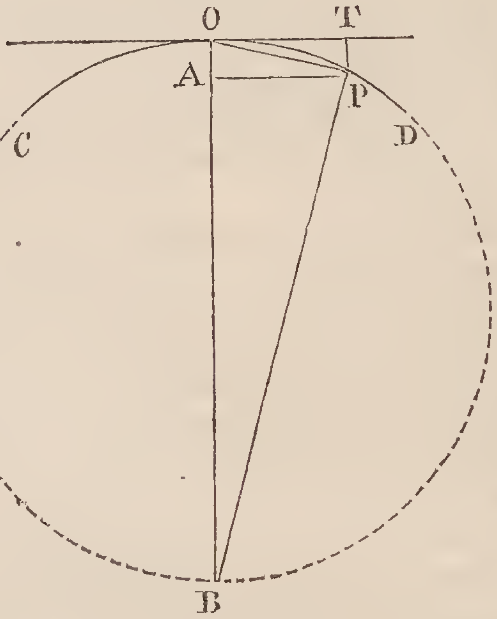


Upon exposing the lenses to orange, yellow, and the other species of homogeneous light, similar effects were observable; the bright rings always taking the colour of the light incident on the lenses, and being separated by dark rings, which being viewed from B appeared bright, the bright ones, as in the case of the red light, appearing dark. One difference, however, was remarkable, viz. that the rings formed by the less refrangible rays were larger than those formed by the more refrangible. Thus the first red ring was larger than the first orange one and this larger than the first yellow ring, and so on, the first violet ring being least.

(55.) The existence of a connexion between the colour of the reflected and transmitted light, and the thickness of the air-lens, being now manifest, Newton applied his attention to measure this thickness at the places of the several dark and bright rings. The extreme minuteness of the magnitude to be ascertained rendered the application of direct measures impracticable. The first principles of elementary geometry, however, furnished a method of effecting the measurement with the greatest accuracy.

* Let O T, *fig*. 44, be the plane side of the air-lens, and C D the concave side, and let the circle, of which C D is an arc, be completed. Let O B be its diameter. By the principles of optics, the length of O B may be deduced, from observing the focus of the convex glass lens used in the experiments, provided the refracting power of the glass be known. Let it be required to ascertain the thickness T P of the air-lens at T. Draw the line P O, and from P draw P A parallel to T O. By actually measuring the diameter of the ring at the distance T, the line T O, or P A, will be determined; and from the extreme minuteness of T P, the line P A may be considered as practically equal to P O. The right angled triangles O P B, and O A P, are similar; and,

Fig. 44.



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* This investigation may be omitted by those who are not familiar with the elements of geometry.

therefore, O P bears the same proportion to O B, as O A, or T P, bears to O P. Now, since the magnitudes of O P, and O B, are known, we know how many times O B is greater than O P. Then O P will be the same number of times greater than O A, or T P. Thus, if O B were 182 inches, and O P 8-79ths of an inch, the thickness T P is found by common arithmetic to be about $\frac{100}{1774784}$th part of an inch.

Calculating in this manner, Newton found a very singular analogy to subsist between the thicknesses at which the bright and dark rings of each colour were produced. Let the thickness at which the first bright ring of any homogeneous colour is produced, be called 1; the thickness at the next bright ring will be 3; the next 5; the next 7; and so on; the thicknesses of the successive bright rings being represented by the odd integers. Again, the thickness of the air at the dark ring, which immediately succeeded the first bright ring, was found to be twice its thickness at the first bright ring; and therefore the central spot being considered as the first dark ring, the second dark ring will be at the thickness 2. The third dark ring was found to be at the thickness 4; the fourth at 6, and so on: the thicknesses of the air at the several dark rings being represented by the even integers.

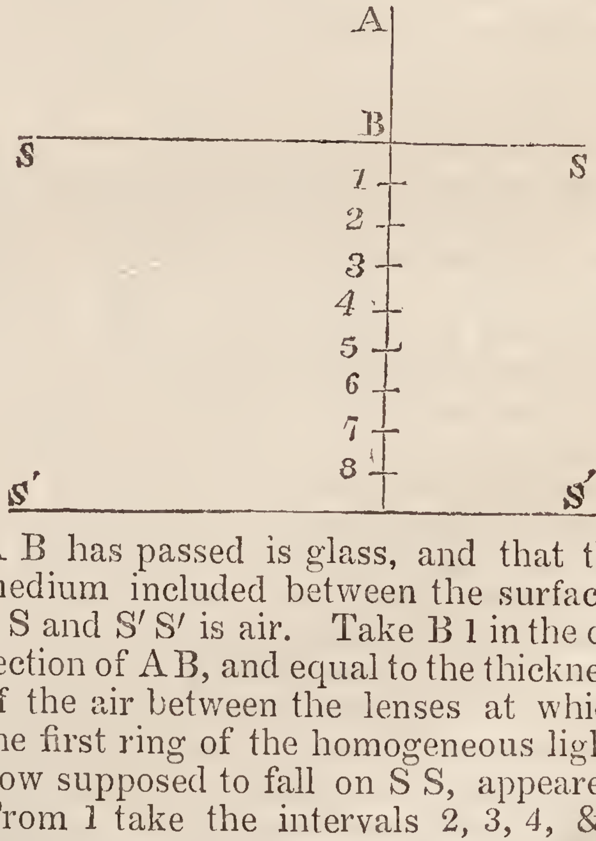
The proportion which we have now explained was found to prevail among the rings, whatever might be the colour of the light projected on the lens; but the absolute magnitudes of the rings was, as we have stated, different in each kind of light. This will, perhaps, be better understood by example. Suppose an inch divided into 180,000 equal parts, and let one of those parts be the thickness at which the first ring of a certain colour, say green, appears, homogeneous green light being projected on the lens. At a thickness equal to two of these parts will be a dark ring. At a thickness equal to three of these parts will appear the second bright green ring, and so on alternately.

The rings of the other colours will succeed each other in a similar manner, with this difference, that the thickness at which the first ring appears will be less for the more refrangible rays, *i. e.* those of a bluish tint, and greater for the less refrangible rays, which take the yellow or red hues, and that the intervals between the rings will also be less for the former rays than for the latter. Hence we may easily perceive how the succession of coloured rings is produced when compound solar light is projected on the lens. In this case each component part of the light forms its own set of rings, and the rings of one colour intermixing with those of another, form the several series of coloured rings already described.

(56.) All that has been observed respecting the rings produced by the light reflected from the lens will apply, with the requisite modifications, to the rings produced by the light transmitted through it. These latter, however, are much less vivid than the reflected rings.

Before we proceed further in our account of these phenomena, it may be useful, in impressing them on the memory of the reader, to give some account of the manner in which Newton explained them. He considers that every ray of light, in its passage through the surface which separates two media of different densities, is put into a certain transient constitution or state, which, in the progress of the ray, returns at equal intervals, and disposes the ray at every return to be easily transmitted through the next refracting surface, and between the returns, to be easily reflected by it. Let A B (*fig.* 45) be a ray of pure homogeneous light falling perpendicularly on a refracting surface, S S. Let us suppose that the medium through which A B has passed is glass, and that the medium included between the surfaces S S and S′ S′ is air. Take B 1 in the direction of A B, and equal to the thickness of the air between the lenses at which the first ring of the homogeneous light, now supposed to fall on S S, appeared. From 1 take the intervals 2, 3, 4, &c. equal to B 1. The action of the surface, S S, upon the ray, is *supposed* to

Fig. 45.



be such as to put it in a state in which it would be easily transmitted by another similar refracting surface, such as S′ S′, if that surface received the ray immediately after its passage through S S. But this state of easy transmission does not continue. When the light has arrived at 1 it is in a state of easy reflection, so that if it were intercepted at 1 by such a surface as S′ S′, it would be reflected back in the direction 1 B A. After passing the division 1, the state of the ray is again changed, and when it has arrived at the division 2, it is again in a state of easy transmission, as at B. If the surface, S′ S′, therefore, met the ray at 2, the ray would pass freely through it in the direction 3, 4, &c. In passing from 2 to 3 the ray again changes its state, and is found at 3 to be in the same disposition to be reflected as it was at 1; and such reflection would, in fact, take place if the surface, S′ S′, intercepted the ray at 3. In this manner the ray passes alternately into states of easy transmission and reflection, at the successive points, 4, 5, 6, &c.

(57.) These alternate states of the ray Newton calls *fits*, the light being in *fits of easy reflection* at the points 1, 3, 5, &c.; and in *fits of easy transmission* at the points 2, 4, 6, &c. The spaces B 2, 24, &c., or 13, 35, &c., he calls the *interval of the fits*. Although these phrases imply a theory or hypothesis, yet Newton intends them merely as names for *effects* which are known to exist, and distinctly disclaims the adoption of any hypothesis, endeavouring in every case to render his inferences independent of everything except the result of experiment or observation.

In describing the phenomena of physical science it is extremely difficult, if not impossible, to avoid expressions and terms which imply a theory or a supposed cause for effects. Every writer, but more especially he who promulgates new facts, should be cautious to remind the student that the *language of causation*, the use of which in physics is inevitable, is nothing more than a method of expressing the classification of effects; and that, when we are said to "discover the cause" of any appearance, nothing more is to be understood than that we have found a class of phenomena to which it belongs and must be referred. There is no philosopher who seems more conscious of the necessity of this than Newton; and, accordingly, in the introduction of the phraseology to which we have just alluded, he warns his reader that he does not pretend to affirm "what kind of action this (the fits) is; whether it consists in a circulating or a vibrating motion of the ray, or of the medium, or of something else. Those that are averse to assenting to any new discoveries but such as they can explain by an hypothesis, may for the present suppose that, as stones, by falling upon water, put the water into an undulating motion, and all bodies, by percussion, excite vibrations in the air; so the rays of light, by impinging on any refracting or reflecting surface, excite vibrations in the refracting or reflecting medium or substance, and by exciting them agitate the solid parts of the refracting or reflecting body, and by agitating them cause them to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting medium or substance, much after the manner that vibrations are propagated in the air for causing sound, and move faster than the rays, so as to overtake them; and that when any ray is in that part of the vibration which conspires with its motion, it easily breaks through a refracting surface; but when it is in the contrary part of the vibration, which impedes its motion, it is easily reflected; and by consequence, that every ray is successively disposed to be easily reflected or easily transmitted, by every vibration which overtakes it. *But, whether this hypothesis be true or false, I do not here consider.* I content myself with the bare discovery that rays of light *are*, by some cause or other, alternately disposed to be reflected or refracted for many vicissitudes."

(58.) By the observation of air between glass lenses, Newton ascertained the *fact* that, after passing through a certain thickness of air, a ray would be reflected or transmitted, according to the degree of thickness of the air and the species of the light. By actual admeasurement he ascertained the least thickness at which each species of homogeneous light would be reflected, a magnitude which will easily be perceived to be equal to half the *interval of the fits*. We have already observed that this interval is different in different kinds of light, being greater for the less refrangible rays than for the more refrangible. The following Table exhibits the interval for lights of the different degrees of refrangibility. If an inch be supposed to be divided into ten millions of equal parts, the number of these parts, in the interval for each ray, is expressed in the second column.

	Ten millionths of an inch.	Difference.
Extreme rays	133	0
Red rays	128	5
Intermediate	123	5
Orange rays	120	3
Intermediate	117½	2½
Yellow rays	113½	4
Intermediate	109½	4
Green rays	105½	4
Intermediate	101½	4
Blue rays	98	3½
Intermediate	94½	3½
Indigo rays	92½	2
Intermediate	90½	2
Violet	87	3½
Extreme rays	83½	3½

That the magnitudes, which appear in the third column of this Table, should be subjects of accurate computation, founded on measurements performed by the hand of man, must, we conceive, be matter of the greatest wonder and admiration. An inch being divided into five million of equal parts, a distance equal to one of these parts is ascertained by positive measurement, in the estimation of the intervals of the fits of indigo rays and those intermediate between them and the violet! Magnitudes of such minuteness far exceed even the powers of imagination. We have, perhaps, a distinct idea of the hundredth part of an inch, by imagining the tenth of an inch divided into ten equal parts. But when we are required to conceive one of these ten parts divided into fifty thousand equal parts, imagination altogether fails, and we cease to attach to the name of such a magnitude any positive conception. Nevertheless, this magnitude is, as we have seen, capable of measurement as accurately as any other, however gross and perceptible, so far does the power of reason exceed that of the imagination.

(59.) The principles which we have now explained will be found sufficient to account for all the phenomena of colours reflected, or transmitted by transparent media. Let us suppose that two glass surfaces are placed parallel at the distance of 120 ten-millionth parts of an inch, inclosing between them a plate of air. If pure orange light fall perpendicularly on the glass, it will pass through, being transmitted freely by the plate of air between the plates of glass. For on passing from the first glass plate into the air it is in a fit of easy transmission; and since the distance between the plates is equal to the interval between the fits (see Table), it will be again in a fit of easy transmission when it encounters the surface of the second plate and will consequently pass through. If, in this case, the eye be placed behind the second plate, the coloured light will be perceived to be transmitted.

Now suppose the two plates to be placed parallel as before, but only at half the former distance asunder. The orange light, on meeting the surface of the second plate, will be in a fit of easy reflection, and will consequently return through the plate of air in the direction from which it came. Having the same space to move through, it will be in a fit of easy transmission when it has again reached the surface of the first plate, and will consequently be transmitted. If the eye be placed before the first plate, it will be coloured over with the orange light.

Let us now suppose that the plates are placed at the distance of 240 ten-millionths of an inch asunder. This is equal to two intervals of the fits of orange light, and therefore that light being in a fit of easy transmission when it encounters it, will be transmitted at the surface of the second plate. An eye placed behind the second plate will receive the transmitted light, and the plate will take an orange hue.

In the middle of the interval between 120 and 240, that is, at the distance of 180 ten-millionths of an inch from the first plate, the ray will be in the middle of the second interval of its fits, and will therefore be in a fit of easy reflection. If it encounter the second surface at this distance, it will consequently be reflected. When it has returned to the distance 120, it will again be in a fit of easy transmission, which fit will return again after passing through the distance 120, at which point it will again meet the surface of the first plate, and will be transmitted through it. Thus an eye placed before the first plate will perceive the orange light reflected.

Thus the space followed by the ray in passing through the air being divided into parts equal to 60 ten-millionths of an inch, at each alternate point of division the orange ray will be in a fit of easy transmission, and at the intermediate points it will be in a fit of easy reflection. In the one case, the orange light transmitted will appear to an eye placed behind the second plate, while to an eye placed before the first no colour is apparent. In the other case the reverse will happen, the orange light being perceptible to an eye placed before the first plate, while no colour appears to an eye placed behind the second.

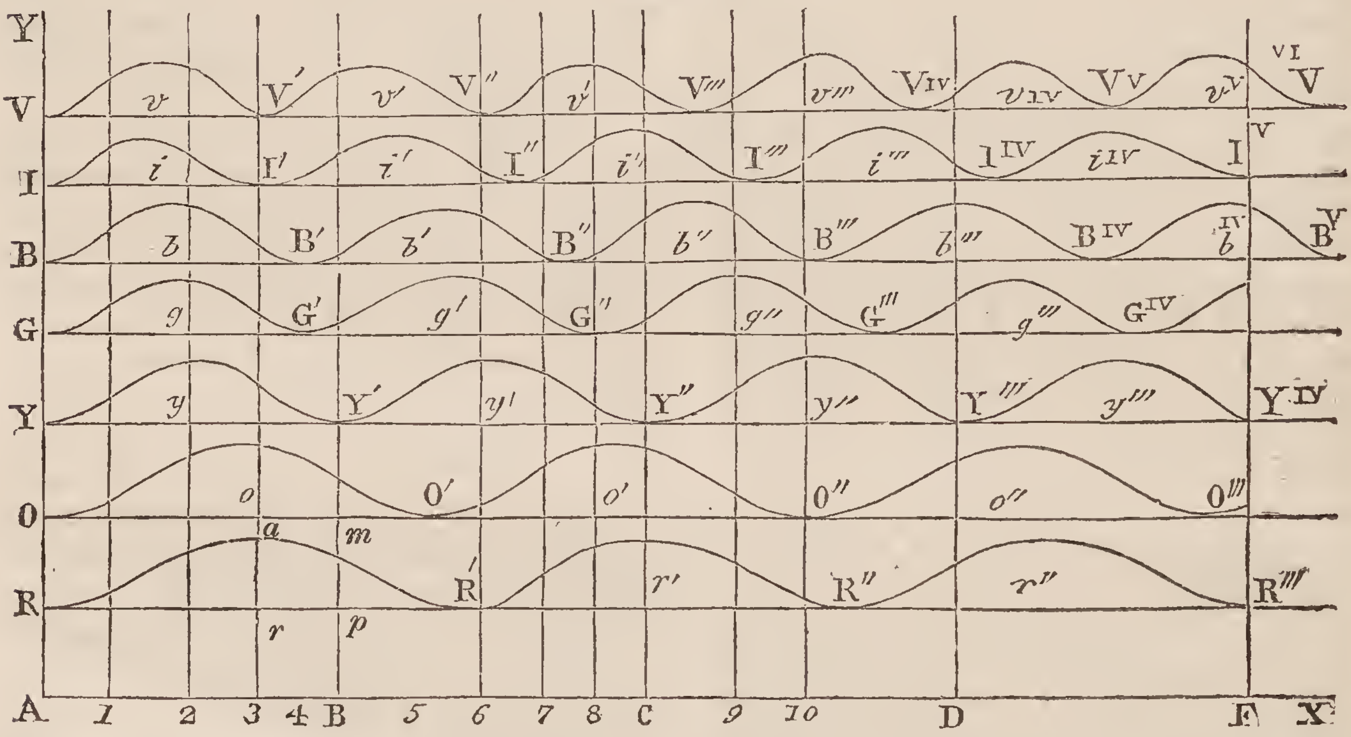
All that has been here observed of the orange light will be equally applicable to the red, yellow, and all the other colours, the interval of the fits only being different. It should, however, be observed, that the ray does not pass suddenly into its fits of reflection and transmission on arriving at the several points of division which we have mentioned, but passes gradually from its complete fit of easy reflection to its complete fit of easy transmission, and *vice versâ*, being, in the intermediate space, in a state to be partially reflected and transmitted.

(60.) When a beam of white solar light falls perpendicularly on the plates, each component part is put into fits separately, and in the same manner as would happen if that part alone had been incident on the plate. The thickness of the plate of air may be such, that several component rays may meet the second surface in fits of easy reflection, while the other parts meet it in fits of easy transmission. In this case, the tint exhibited to an eye placed before the first plate will be one which is compounded of the colours of those rays which meet the second plate, in fits of easy reflection, while the tint exhibited to an eye behind the second plate will be compounded of the colours of those rays which meet the second surface in fits of easy transmission. It will happen frequently that the second surface will encounter a ray in such a manner as to divide the interval of the fits unequally, so that the light will be partly reflected and partly transmitted. In this case, the tint seen on each side of the plates is determined as before, by the composition of the colours reflected and transmitted, due regard being had to their quantities.

(61.) Newton has given an ingenious scale for determining the colours reflected by the second surface, at any proposed distance from the first. We shall here, however, adopt another method of illustration, not differing in principle from that of Newton, but better adapted for popular illustration.

Draw two lines[*] AX (*fig.* 46) and AY at right angles, and taking any seven points R, O, Y, G, B, I, V on the line AY, draw through them lines parallel to AX. Let RR′ represent the interval of the fits of red light, and let this space be repeated, so that R′R″, R″R‴, &c. shall be equal to RR′. Draw a waving curve line touching the parallel through R in the points R, R′, R″, &c., and let the points, at which the distance of the curve from the parallel is greatest, be situated exactly in the middle between every two successive points of contact, that is, perpendicularly above the points *r*, *r*′, *r*″, &c. When the thickness of the air between the plates is equal to RR′, or RR″, or RR‴, &c. no red light is reflected, the ray being at those points in a fit of easy transmission. On the other hand, in the middle of each interval, or at the thick-

Fig. 46.


Y
V
I
B
G
Y
O
R
A
1
2
3
4
B
5
6
7
8
C
9
10
D
F
X
v V′ v′ V″ v′ V‴ v‴ V^IV v^IV V^V v^V V^VI V
i I′ i′ I″ i″ I‴ i‴ I^IV i^IV I^V
b B′ b′ B″ b″ B‴ b‴ B^IV b^IV B^V
g G′ g′ G″ g″ G‴ g‴ G^IV
y Y′ y′ Y″ y″ Y‴ y‴ Y^IV
o O′ o′ O″ o″ O‴
a m
R′ r′ R″ r″ R‴
r p

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[*] This scale is used by Mr. Herschel in his able "Treatise on Light."

nesses R r, R r', R r'', &c. the reflection of red light is most intense, the ray being at these points in a fit of easy reflection. From the first entrance of the red ray at the plate A Y until it reaches r, its disposition to be reflected is increasing, and consequently, the curve from R to r may be so shaped, that its increasing perpendicular distance from R r may be proportionate to the quantity of red light reflected at each increasing thickness. If this be done, the perpendicular distance of the curve from r will represent the quantity of red light reflected when the ray attains its first fit of easy reflection. As the ray passes from r to R′ it gradually changes its phase, and the reflected light constantly decreases like the distance of the curve from r R′, until at length, like that distance, it dwindles into nothing at R′, the entire light being here transmitted, the ray having attained its fit of easy transmission. The same process is repeated as the ray passes from R′ to R″, from R″ to R‴, and so on.

It thus appears, that the quantity of red light reflected by the plate of air intervening between the two plates of glass may be exhibited. Take R p equal to the distance between the plates, or the thickness of the plate of air. Draw $p\,m$ perpendicular to Rp and meeting the curve at m. Then $p\,m$ will bear the same proportion to $r\,a$ as the quantity of red light reflected at the thickness Rp bears to the quantity re-reflected at the thickness R r when the ray is in a fit of easy reflection. It is evident that the quantity of red light reflected at any other thickness may be similarly found.

Let O O′ be taken to represent the interval of the fits of orange light, and let this interval be repeated O′ O″, O″ O‴, &c., as in the former case. Let a curve be drawn as before, touching the parallel at the points which mark the fits of easy transmission, and such that its distance from the parallel will always be proportionate to the quantity of orange light reflected at each thickness of the plate of air. The other curves are to be drawn in the same manner, the distances YY′, GG′, BB′, I I′, V V′ representing the intervals of the fits of the yellow, green, blue, indigo, and violet lights respectively. It is evident that, by this scale, the quantity of light of each colour which is reflected at any given thickness may always be exhibited, and the colours which compose the tint, perceived by reflection, may thus be determined, both in quantity and quality.

It will be observed, that the intervals R R′, O O′, Y Y′, &c., continually diminish in passing from the red to the violet light. This is conformable to what has been explained; the interval of the fits being shorter for the more refrangible the lights. It is to this circumstance that the coloured rings between the lenses is owing; for if the fits of all the component parts of light were equal, the rings would be alternately white and black.

To explain more fully the manner of determining a tint corresponding to a given thickness, let the line A X be divided at 1, 2, 3, &c. Let A 1 be a thickness much less than R r, half the interval of the fits of red light. Through 1 draw a line parallel to A Y, and crossing all the curves. The parts of this line intercepted between each curve and the corresponding parallel to A X, express the quantities of the respective colours reflected by the air at this thickness. It thus appears, by inspection, that the quantities of red and orange are small; of yellow not much more, but that the proportion rapidly increases as we approach the blue and violet. The excess of light of a bluish tint, therefore, which enters the reflected light, will give that light a corresponding character. If the thickness of the plate of air be much less than A 1, the lines representing the reflected light will gradually disappear, and no light will be reflected. Accordingly, it was found in the experiment with the glass lenses described in (54), that at the centre, where the glasses were in contact, and for a small distance round it, a black spot was perceivable, arising from the absence of reflected light. In this case, the air immediately around the centre was too thin to reflect the light in any sensible quantity, as appears by the scale which we are now describing.

Let a parallel to A Y be drawn through 2. The lines which now represent the reflected lights are nearly equal. The red tints have not reached their maxima, and the blue tints have passed theirs. The intermixture of these produce a brilliant white.

Referring again to the experiments with the glass, we found that the blue ring which immediately succeeded the central black spot was followed by a ring of splendid white. Here, then, the thickness of the air between the lenses had so far increased as to be represented by A 2.

As the parallel to A Y is moved towards 3, the lines which are intercepted between the upper curves and their respective bases rapidly diminish and disappear, while those which correspond to the bases Y Y′, and O O′ attain their maxima. Hence it appears, that by this increase of thickness the reflection of the bluish lights is subdued, and the yellow and orange tints appear. When the parallel arrives at 3, the line *r a* representing the red light attains its maximum, the blues and violets altogether disappearing. Here the reflected tint will be red.

The first white ring which appeared between the lenses was succeeded by a yellow which passed through an orange into a red one. The increased thickness of the air between the lenses, in receding from the centre, accounts for this succession of colours as explained above.

As the parallel to A Y moves towards 4, 5, the lines representing the quantities of the red lights reflected gradually diminish and disappear, the indigo being on the increase, and the violet at its maximum. Hence the light reflected at this thickness will have a violet hue. Such is, in fact, the first ring of the second series between the lenses (54). As the parallel arrives at 5, the violet is on the decrease, the blue attains its maximum, and the reddish tints vanish. The colour reflected will, therefore, be blue, and corresponds to the blue ring in the second series between the lenses.

When the parallel to A Y arrives at 6, the lines representing the reds and violets disappear, the blue and green are only partially reflected, and the yellow is at its maximum. The partial reflexion of the blue and green mixed with the intense yellow, produces a yellowish green. This thickness of the air is that at which the green ring is reflected in the second series of rings between the lenses. After the parallel passes 6, the yellow predominates, the other lights being but faintly reflected, as appears by inspecting the curves. The yellowish green last mentioned, therefore, gradually changes into a bright yellow. This corresponds to the yellow ring of the second series.

As the parallel approaches 8, the red and orange increase; the greens, yellows, and blues nearly vanish; the violets are copiously reflected, but the stronger influence of the red and orange gives the reflected light a glowing crimson hue. This corresponds to the last ring of the second series between the lenses.

At C, the red and orange are copiously reflected; the yellow, green, and violet are not reflected in any perceptible quantity, but the indigo is abundant and the blue considerable. The result of the composition of these lights is a rich purple of a ruddy character, which corresponds to the first ring of the second series between the lenses. By following in this manner the parallel as it moves from A Y, we shall be able to trace distinctly the lights reflected by plates of air of every degree of thickness, and we shall perfectly account for the succession of coloured rings, as far as they are observable in the experiment with the lenses.

(62.) We have here constructed curves to represent the quantities of reflected light of seven distinct degrees of refrangibility. But it must be remembered that solar light consists of rays of every degree of refrangibility between certain extreme limits. To describe the phenomena perfectly, there should therefore be an infinite number of other curves between each pair of those already exhibited, and with bases of intermediate magnitude. Two such curves having bases nearly equal, correspond to lights which differ but little in refrangibility, and which have no perceptible difference in colour. When the thickness of the plate of air becomes considerable, the number of repetitions of the bases of two curves, whose bases have a very small difference, will be so great, that the points of contact will be separated by a considerable interval. Accordingly, it must at length happen, that when one ray is most copiously reflected, another of very nearly the same colour will not be reflected at all. As this is necessary to explain why a thick plate of air will only reflect white light, and that but faintly, we shall explain it more fully.

Let Y Y′, instead of representing the interval of the fits of the yellow light, represent that of the fits of green light differing a little in refrangibility and not perceptibly in colour from the light whose fits are at the interval G G′. Suppose that the difference between Y Y′ and G G′ is the fortieth part of G G′; after passing through a thickness equal to twenty repetitions of G G′, the highest point of the curve Y Y′ will correspond to the lowest of G G′, so that although no green light of the refrangibility of G be reflected, yet light of the same colour, but of the refrangibility of Y, will be reflected in abundance; so that at considerable thicknesses, green light, of some degrees of refrangibility, must always be reflected. Now, what we have to observe of green light is equally applicable to light of all other colours: so that it follows, that light of all colours will be both reflected and transmitted at considerable thicknesses.

(63.) Since a mixture of lights of all colours constitutes white light, it follows that the light reflected and transmitted by a thick plate of air will always be white; but since some is transmitted, and some reflected, neither the reflected nor transmitted light will be so intense as the lights which are reflected and transmitted by very thin plates.

The recurrence of the fits of lights of nearly equal refrangibility do not suddenly attain the state which we have described. They approach it gradually. Hence we may account for the dilute appearances of the colours reflected as the thickness of the air increases, as is perceived by observing the rings of colour at a considerable distance from the centre of the lenses. They become faint by degrees, and finally disappear.

(64.) Another circumstance attending the phenomena of the rings is accounted for on the grounds to which we have just adverted. When homogeneous light was projected on the lens, the rings appeared in much greater numbers, and more distinctly defined than when compound solar light was used. In fact, in this case their number seemed quite interminable. The colours were here not diluted and neutralized by the admixture of rays of all hues, as we have shown to be the case when white light was projected on the lenses.

(65.) Seeing that the interval of the fits changed with every change of refrangibility of the light, the problem to determine the relation between the interval of the fits and the degree of refrangibility obviously presented itself. This, however, Newton failed to solve. He fancied that the intervals of the fits of the seven coloured lights bore an analogy to the lengths of a string which sound the seven musical notes.

(66.) Hitherto we have conceived the light to fall perpendicularly on the plate of air, and therefore to be reflected perpendicularly by it. If the light enter the air obliquely to its surface, it will be reflected at the same obliquity. In this case the light is affected by the oblique incidence in a singular manner. The interval of its fits is lengthened, and bears a certain proportion to the length of the interval when perpendicular. Newton succeeded in detecting this proportion, and in showing how the length of the fit depended on the obliquity of the light.

Although the phraseology of mathematics may be considered in some degree necessary to express this curious relation, yet we hope to render it intelligible to the general reader, if a moderate portion of attention be given to the following explanation. Let the line P D (*fig.* 46.) represent the interval of the fits of any species of homogeneous light incident perpendicularly on a thin

Fig. 47.

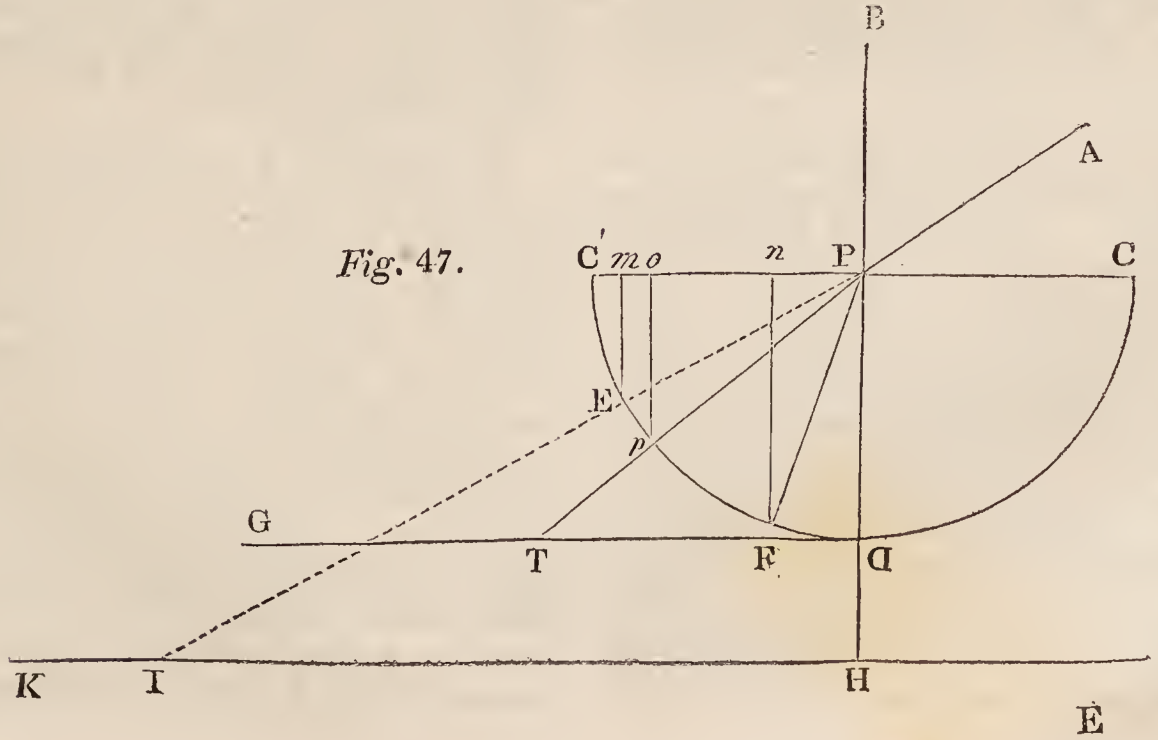


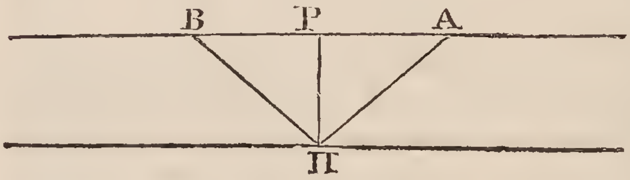
plate of air, having previously passed through a plate of glass. We are required to find the interval of the fits when the same ray enters the air obliquely to its surface, as in the direction A P.

With P as a centre, and P D as a radius, let a semicircle C D C′ be described, and let C C′ be considered as the surface of the plate of air. Let P F be the direction which the ray AP would take, if it were refracted by passing from air into glass. From F, draw F *n* perpendicular to C P. Continue the line A P to the circle at E, and from E draw E *m* perpendicular to C P. Divide the interval *m n* into 107 equal parts, and let *o* be the point of division nearest to *m*. From *o* draw *o p* perpendicular to C P, and meeting the circle in *p*. From D draw D G, touching the circle at D, and from P draw P *p*, and continue it to meet D G at T. Continue P D below D, and take P H equal to P T. Through H draw H K perpendicular to H P, and continue P E until it meet it at I. Then P I will be the interval of the fits of the oblique ray, and P H will be the thickness of a plate of air through which the ray passes between two successive fits.

It appears from hence that the interval of the fits of an oblique ray depends on three things; first, the angle of obliquity; second, the refrangibility of the light; and third, on the refracting power of the media which bound the thin transparent plate through which the light passes. The relation by which the interval depends on these elements, appears complex in the preceding explanation. This complexity is, however, only apparent, and arises from the necessity of throwing our account of it into a popular form. Expressed in the language of Trigonometry and Algebra, it is sufficiently simple.*

(67.) The manner in which Newton deduced this relation from observation of the rings may easily be conceived, by recurring to the method by which he discovered the interval of the fits, when the incident light was perpendicular. In that case, the thickness of the plate of air, which reflected the colour in each ring, was found by measuring the diameter of the ring. When he viewed the rings obliquely, he found them enlarged, and, consequently, the thickness of the air at which the light was reflected was increased. Its increased thickness was computed in the same manner as before, by measuring the diameter of the ring. By a careful comparison of the thickness thus deduced with the thickness which reflected the same colour perpendicularly, he discovered the method already explained of finding the magnitude P H by knowing P D and the obliquity. The thickness at which a given tint will be reflected at a given obliquity being found, the interval of the fits was easily discovered. In the case where the ray penetrated the medium perpendicularly, twice the thickness of the air was equal to the length of the course of the ray in passing through it and returning, and this, therefore, was the interval of the fits. But when the ray, as in the present case, penetrates the air obliquely, the course of the ray in passing through it and returning is more than twice the thickness of the air. Let P H (*fig.* 48) be the thickness of the air, and let A H be the ray entering, and H B emerging. The lines A H and H B taken together, form the course of the ray within the plate of air. These lines, therefore, taken together, or twice A H, is the interval of the fits.

Fig. 48.



In explaining the laws which govern these phenomena, we have referred constantly to a thin plate of air inclosed between two surfaces of glass. The presence of air, or any other material agent, is not necessary for the production of the effects. The lenses which we have described (54) would exhibit the same rings of colour if placed under an exhausted receiver. All that is necessary to produce the phenomena is, that two refracting surfaces should be placed close together, so as to include between a thin transparent space. In passing the first surface the rays will be put into fits of easy reflection and transmission, and those which meet the second surface in a fit of easy transmission will penetrate it, while those which meet it in a fit of easy reflection will be reflected from it.

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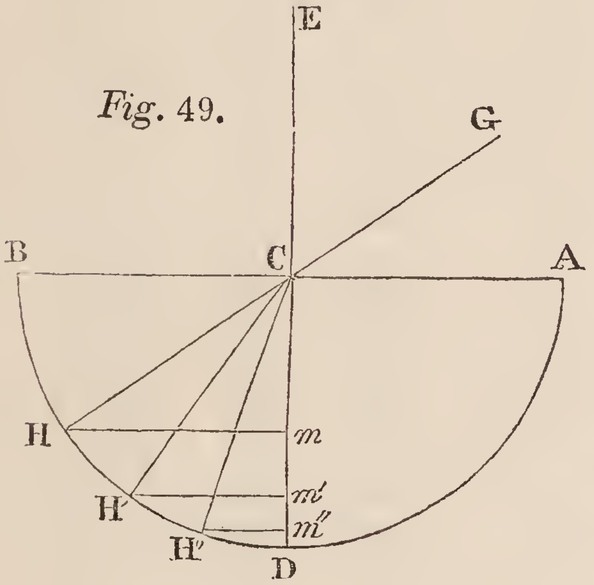
* It is very doubtful, as Newton does not give the details of his observations at great obliquities, and as the making the observations with accuracy at obliquities beyond 75° must be a matter of great difficulty, whether the construction is to be depended on; particularly as the observations, as far as from 0° to 75°, agree sensibly with the simple formula, secant < obliquity (or, if we use the figure in the text, P H, instead of being equal to P T, will be equal to the distance from P to the intersection of P E produced with D T). This formula is consistent with the theory of undulations. Biot does not say that he has repeated these observations at great obliquities, and found them to be correct. He is evidently at a loss to know how they were made.

(68.) The interval of the fits changes with the refracting power of the thin medium traversed by the light. The relation of this interval to the refracting power was detected by Newton, and may be exhibited as follows:

With a centre C (*fig.* 49) and a radius C A, describe a semicircle, and from any point G, through the centre C, draw the right line G C H. From H draw $H\,m$ parallel to B A, and meeting at m a line E D drawn through the centre perpendicular to B A. Let $H\,m$ represent the length of the interval of the fits when the ray traverses a vacuum. To find the length of a fit when the medium traversed by the ray is water, draw C H in that direction which a ray of light coming in the direction G C would take in passing from a vacuum, through the surface B A, into water. Let this direction be $C\,H'$, and from H' draw $H'\,m'$ parallel to B A. This line $H'\,m'$ will be the interval of the fits when the ray passes through water. Again, if the medium be glass, draw $C\,H''$ in the direction which the ray G C would take in passing from a vacuum into glass, and $H''\,m''$ is the interval of the fits.

Fig. 49.



Thus it appears that the greater the refracting power of the medium traversed by the light, the shorter will be the interval of its fits.

Newton discovered this law by introducing transparent liquids between the lenses, whose refracting powers were greater than that of air. "Upon wetting the object glasses at their edges the water crept in slowly between them, and the circles thereby became less and the colours more faint, insomuch that, as the water crept along, one-half of them, at which it first arrived, would appear broken off from the other half and contracted into a less room. By measuring them I found the proportion of their diameters to the diameters of the like circles made by the air, to be about seven to eight, and consequently, the intervals of the glasses at like circles, caused by the water and air, to be as three to four*. Perhaps it may be a general rule, that if any other medium more or less dense than water be compressed between the glasses, their intervals, at the rings caused thereby, will be to their intervals caused by interjacent air, as the sines are which measure the refraction out of that medium into air." These *sines* are the lines $H\,m$, $H'\,m'$, $H''\,m''$, &c. in our explanation.

(69.) It is remarkable, that whatever be the matter of which the then transparent medium traversed by the light is composed, the same colours will be reflected in the same order. This arises from the circumstance of the fits being regulated by the same law in all substances, one differing from another only in the length of the interval of the fits, and whenever the length of the fit for any one species of homogeneous light undergoes any change arising from a change in the refracting power of the medium, the intervals for all the other species of homogeneous light undergo a *proportional* change.

(70.) The scale for determining the tints corresponding to different thicknesses of the transparent medium would, accurately constructed, be a very exact method, not only of ascertaining those tints, but also the primary colours of which they are composed. Newton has given the following Table of the thickness of air, water, and glass, which reflect tints corresponding to the several series of rings described in (54). The numbers express millionths of an inch.

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* The intervals of the glasses, being in the proportion of the squares of the diameters of the rings, are more accurately as 49 to 64; very nearly the same ratio.

THE THICKNESS OF COLOURED PLATES AND PARTICLES OF AIR, WATER, AND GLASS.

		Air.	Water.	Glass.
Their Colours of the first Order	Very Black	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of Black	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{20}$
	White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	$5\frac{1}{6}$
	Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$.
Of the second Order	Violet	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indigo	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	14	$10\frac{1}{2}$	9
	Green	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright Red	$18\frac{1}{5}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Of the third Order	Purple	21	$15\frac{3}{4}$	$13\frac{11}{20}$
	Indigo	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue	$23\frac{2}{5}$	$17\frac{11}{20}$	$15\frac{1}{10}$
	Green	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Blueish red	32	24	$20\frac{2}{3}$
Of the fourth Order	Blueish Green	24	$25\frac{1}{2}$	22
	Green	$35\frac{2}{7}$	$26\frac{1}{6}$	$22\frac{3}{4}$
	Yellowish Green	36	27	$23\frac{2}{9}$
	Red	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Of the fifth Order	Greenish Blue	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Of the sixth Order	Greenish Blue	$58\frac{3}{4}$	44	38
	Red	65	$48\frac{3}{4}$	42
Of the seventh Order	Greenish Blue	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy White	77	$57\frac{3}{4}$	$49\frac{2}{3}$

A comparison of this Table with the scale in fig. 46 will give a tolerably accurate notion of the reflected tints and their composition. But it also answers the further purpose of measuring the thickness of a medium too minute to be estimated in any other way. Thus the size of the minute parts of natural bodies may be determined from their colours. "If two or more thin plates be laid upon one another, so as to form one plate equalling them all in thickness, the resulting colour may be determined. For instance: Hook observed that a faint yellow plate of Muscovy glass laid upon a blue one constituted a very deep purple. The yellow of the first order is a faint one, and the thickness of the plate exhibiting it is $4\frac{3}{5}$, to which add 9, the thickness exhibiting blue of the second order, and the sum will be $13\frac{3}{5}$, which is the thickness exhibiting the purple of the third order."

CHAPTER VII.

The Theory of Colours, continued.

(71.) IN explaining the theory of colours, it was shown that the colours of natural bodies arose from an aptitude in them to reflect the rays of some colours rather than those of others. The experiments and investigations noticed in the preceding Chapter, now enabled Newton to advance a step further in the inquiry into the causes of the colours of natural bodies. He accordingly proceeds to discuss the principles to which bodies owe this *aptitude* to reflect some lights rather than others, and to trace this aptitude to the internal constitution and essential nature of the bodies.

A very close analogy may be observed between the phenomena of reflection and refraction. It has been proved that those species of light which were most refrangible were also most reflexible. But it further appears that those surfaces at which light is most powerfully refracted have a proportionately intense power of reflection.

A *surface* which separates glass from air has a considerable refractive power. That its reflective power is also great, will be seen by looking into the side of a prism at its base, the eye being placed so as to view the base obliquely and the side perpendicularly. The reflection of light from the base will be so intense, that every object seen in it has all the vivid splendour of reality.

The surface which separates glass from water has a much less refracting power, and, accordingly, we find the reflective power proportionately diminished. If the prism just mentioned be laid with its base upon water, the splendour of the reflection will be considerably impaired, and the images seen in the base will no longer have the intense brilliancy which was observable when the medium below the base was air. To this cause the diamond owes its lustre. Its refracting power is greater than that of most other substances. In conformity with the principle here laid down, its reflecting power is proportionately great, and we find the reflection of light from its inner surfaces of a corresponding intensity. On the other hand, if a surface separates two media of equal refracting powers, no refraction takes place in passing from the one medium to the other, and so neither do we find any reflection. Thus, at the point at which the lenses touched in the experiment described in (54), no light was reflected at whatever obliquity it was viewed.

(72.) Newton, therefore, concludes that the reason why uniform pellucid substances, such as water, glass, or crystal, reflect no light except from their surfaces, is, because every part within those surfaces has an uniform refracting power, or has in every place the same density. He infers that reflection of light may always be taken as an indication of a change of density in the medium at that point where the reflection is made, and that it is a test by which a change of density may be as certainly ascertained as by refraction.

A ray of light falling on the surface of a body in a fit of easy transmission, enters it. If, after penetrating to a small depth, it meet with a point at which there is a change of density, and be at the same time in a fit of easy reflection, it will retrace its course. According to this view, every body, how opaque so ever it be, is transparent to a certain depth within its surface. Experiments sufficiently evince the truth of this proposition. If a very thin lamina of the most opaque substance be suspended before a hole through which a beam of light is admitted into a dark room, it will be manifestly transparent, and light will be perceptible through it, provided it be sufficiently attenuated.

From all this it seems not impossible that the intimate particles or molecules of opaque or coloured bodies, are separated by minute pores or spaces, which are either entirely void or filled with some subtle material of a different density from the molecules of the body, in the same manner as a liquid pervades the particles of a solid substance which it holds in solution. If we admit that light penetrates the external surface of opaque bodies, a fact which experience proves, we must also admit the existence of spaces within that body, *filled by some medium* differing in density from the molecules of the body itself; for, without this difference of density, there could be no reflection, and, consequently, no opacity.

(73.) There are many experiments by which phenomena are elicited, which seem to support the hypothesis that the discontinuity of parts is the cause of opacity. If the pores of an opaque body be filled with any substance of nearly the same density or refracting power, it loses its opacity, and becomes diaphanous. Paper and oil have nearly equal refracting powers. Dry paper is opaque; but, when steeped in oil until its pores are in a great degree filled with that fluid, it acquires a proportionate degree of transparency. In the same manner, linen cloth, and many other substances of very imperfect transparency, will have that quality perceptibly increased by being steeped in liquids whose refracting power is nearly equal to their own.

On the other hand, the most translucent bodies may be rendered opaque by extricating from their pores the matter which pervades them, which may be done by separating their parts by mechanical division; or the subtle pervading agent may be banished by heat or chemical action. Thus, pulverised glass and the shavings of horn are no longer transparent. Wet paper becomes opaque upon evaporating the moisture by heat; and salts and other substances, held in solution, lose their transparency by precipitation.

On such grounds Newton assumes that the parts of opaque bodies are separated by interstices; but he even goes further. He affirms that these interstices must have certain determinate magnitudes, and there are limits within which they cannot come. In the experiment with the lenses (54), the black central spot, within which the transmission of the light was total, was much larger than the point of contact of the glass, and no light was reflected until the air attained a certain thickness. In like manner, if a soap-bubble be examined, it will be found that as the liquid which forms it subsides, the top becomes so thin as to reflect no light, and, consequently, appears black. In order, therefore, to reflect light, the distances between the particles of a body must be such as to allow the pervading medium, whatever it be, to exist in sufficiently thick parts; otherwise the distances between the particles would be less than half the interval of the fits, and the ray, throughout the whole thickness of the medium, would be in a fit of easy transmission.

This Newton considers to be the case with water, glass, translucid stones, and other transparent substances. That there are spaces between their particles is beyond doubt. But those spaces, relatively to the pervading medium, are circumstanced in the same way as the air immediately around the point of contact of the lenses; they are too small to reflect light, and the ray never has space enough in any of them to pass from a fit of easy transmission into one of easy reflection, and consequently it is wholly transmitted.

(74.) This theory derives some further support from the effects of the solution of solid substances in liquid. The particles, by a subdivision, inconceivably minute, are brought into closer contact; and, although the solid after solution, fills a greater space, and the *aggregate* of the interstices of its parts must therefore be increased, yet the subdivision is so minute that every *single interstitial space* is very much diminished. This diminution is carried so far, that it is less than the interval between a fit of easy transmission and one of easy reflection, and consequently the light is transmitted, and the solution is diaphanous.

(75.) There are many analogies which support the opinion, that the colours of natural bodies are produced upon the same principle, governed by the same laws, and attended with effects in all respects the same as those produced by their transparent plates, as explained in the last chapter. One of these thin transparent plates, provided it be of uniform thickness, will, in every part, reflect the same colour. Let it be slit into threads, or broken into fragments; every piece will separately reflect the same colour as the entire plate. A heap of these fragments will still appear of the same colour; and if a natural body be considered as a mass of such fragments, it must, on the same grounds, exhibit the same colours.

It will be recollected, that thin plates, viewed at different obliquities, shifted their tints, the length of the fits, and, therefore, the species of the light reflected, changing with every change in the obliquity of the light to the plate. In conformity with this, many natural bodies vary their hues with the point from which they are viewed. The plumage of birds, and more especially of the peacock, presents a splendid instance of this. Changeable silks, and almost all dyed clothes, are attended with a similar effect. A spider's web, when finely spun, exhibits colours. All these phenomena bear a close and obvious analogy to those which occupied our attention in the last chapter.

(76.) But, further, some substances reflect one colour and transmit another, like the air between the lenses in (54). Examples of this will be found in leaf gold, some species of stained glass, and the infusion of lignum nephriticum. The coloured powders used by painters change their tints by mere grinding. The parts being thus reduced in size, reflect various colours, in the same manner as a transparent plate would by reducing its thickness.

No example more strongly supports Newton's reasoning than those which are so common in chemical experiments, in which various changes of colour are produced by the admixture of different species of liquids. Two liquids, each of which is perfectly transparent, will frequently, when mixed, be strongly coloured. This may be conceived to arise from the mutual action of the corpuscles of the two liquids: they either create or destroy the connexion of particles, and cause the molecules to swell or shrink, whereby not only their bulk but their density may be changed.

This change of the magnitudes of the particles and their interstices is an adequate cause for the change of colour, according to Newton's theory.

It is observable in experiments such as those described in the last chapter, that when the thin transparent medium which reflects the colours is more dense than the surrounding medium, the colours are much more vivid and brisk, and less liable to shift their tints with a change in the direction in which they are viewed. This will be evident, by comparing the effect of thin films of glass or mica, the ambient medium being air, with that of a thin plate of air, inclosed between two pieces of plate glass. From this circumstance, Newton supposes that the parts of bodies on which their colours depend must be denser than those which fill the remaining spaces within the surface of the body. For the colour of a body being generally produced by light reflected at all angles, if the particles reflecting colour were rarer than the surrounding medium, all tints would be reflected at different obliquities, and so the body would appear white or grey. But if the parts reflecting colour be more dense than the others, the lights reflected nearly perpendicular will, by predominating over the oblique ones, give their own colour to the body.

(77.) All the preceding reasoning led Newton directly to the inference, that *the colour of a body furnishes a means of determining the magnitude of the ultimate transparent corpuscles of which it is composed.* Many circumstances render it probable, as Newton conceived, that the parts of bodies have, for the most part, the same refractive density as those of water or glass. This being assumed, it follows that the diameter of the corpuscle of a body, which has any proposed colour, is equal to the thickness of a plate of water or glass, which would reflect the same colour, and which may always be determined by the Table, p. 52, and by the Scale, fig. 46. Thus, if it be desired to know the diameter of a corpuscle, which, being of equal density with glass, will reflect green of the third order; $16\frac{1}{4}$ expresses the number of millionth parts of an inch in it.

(78.) The received opinion respecting the cause of reflection of light was, that its particles impinged upon the hard surface of the solid parts of the reflecting body, and were reflected by the reaction of that surface, in the same manner as an elastic sphere is reflected when it strikes a hard plane. This opinion seemed to be countenanced by the law of reflection of light, which is the same as that which regulates the reflection of all elastic spheres impinging upon hard surfaces. Newton, however, held that light does not impinge on the solid and impervious parts of bodies, but is reflected without having encountered these surfaces. He shows many difficulties which attend the hypothesis, that light impinges on the solid parts of bodies in reflection, among which the following may be noticed.

If light, after passing through glass, be reflected by the surface, the ambient medium being air, the reflection will be stronger than it would be if the light had passed through the air and been reflected by the surface of the glass, and much stronger than if the ambient medium had been water. This effect would lead to the conclusion, that the particles of air repelled the light with greater force than those of either water or glass. But it is still more unaccountable on this theory, that upon withdrawing the air from about the glass by an air-pump, the reflection of the light from the surface which separates the glass from the vacuum is still stronger than in either of the former cases.

If the light, after passing through the glass, be incident on the surface more obliquely than at an angle of 41°, it will be wholly reflected; but, at all obliquities less than this, it will be transmitted. Hence, if we admit that the impact on solid parts is the cause of reflection, and therefore the penetration of interstices the cause of transmission, we are compelled to suppose, that at obliquities greater than 41°, all the parts of the light encounter solid parts of air, but at less obliquities they all pass into pores or interstices; and, on the other hand, that in passing through air, and impinging upon glass, it never fails to meet pores enough to transmit nearly the whole of it, even at the most oblique incidences. Some may suppose that the light, after passing through the glass, is reflected—not by the parts of air contiguous to the glass, but by the last particles of the glass itself. This supposition, however, is scarcely intelligible, and, besides, is contradicted by wetting the surface of the glass, which immediately affects the reflection, which could not be the case, if the reflection were made by the parts of the glass.

It has been shown that two kinds of homogeneous light — say blue and red — might be incident at the base of a prism, with the same obliquity, and so that the red should be wholly transmitted, and the blue wholly reflected. In this case it seems inconceivable, that the red rays should everywhere fall on pores, and the blue rays on solid parts. A similar difficulty attends the explication of all the phenomena of thin plates explained in the last chapter, for we must there suppose, that the transmitted rays, being all of certain species, meet only with pores while the rays of the complemental colours encounter nothing but solid parts.

Still greater difficulties attend the explanation of the phenomena of reflection of light at polished surfaces, according to this theory. The substances used in polishing glass and other surfaces, only bring the roughness of the surface to a very fine grain, so that the inequalities become too small to be perceptible either to sight or touch. Hence, since innumerable inequalities exist, irregularly distributed over the most polished surface, it follows, that if the light were reflected by impinging on the surfaces of the solid parts, it would be as irregularly scattered about by such a surface as by the roughest.

(79.) Newton, therefore, considers that light is reflected—" not by a single point of the reflecting body, but by some power of the body which is evenly diffused over its surface, and by which it acts upon the ray without immediate contact."

By a comparison of the refracting powers of substances with their densities, Newton traced an evident connexion between them. In all bodies, whether solid, liquid, or aëriform, except such as be of an unctuous or sulphureous nature, the refracting powers are very nearly in the proportion of their densities. A body of an unctuous or sulphureous nature, compared with one which is not so, has a greater relative refracting power than the proportion of their densities indicates. But even in this case, bodies of this nature, when compared with one another, have refracting powers in a ratio not very remote from that of their densities.

The sulphureous principle, therefore, seems to increase the action of bodies on light; and Newton conjectures that this principle, existing more or less in all bodies, may be the cause of all the phenomena of reflection and refraction. " And as light congregated by a burning glass acts most upon sulphureous bodies to turn them into fire and flame, so, since all action is mutual, sulphurs ought to act most upon light. For that the action between light and bodies is mutual, may appear from this consideration; that the densest bodies which refract and reflect light most strongly, grow hottest in the summer sun, by the action of the refracted and reflected light."

This last observation of Newton has been since proved incorrect. The most dense bodies, if transparent and colourless, do not grow hot. However, the theory which has formed the subject of this chapter, and which has been given substantially as Newton himself has left it, must be received as matter of hypothesis. To discuss its merits, and compare it with subsequent experiments and theories, would greatly exceed the limits of the present treatise. Our object here is to give a succinct popular account of " Newton's Optics," and not to discuss the relative merits of the different theories which have been advanced to represent the phenomena.

Chapter VIII.

Thick transparent Plates.

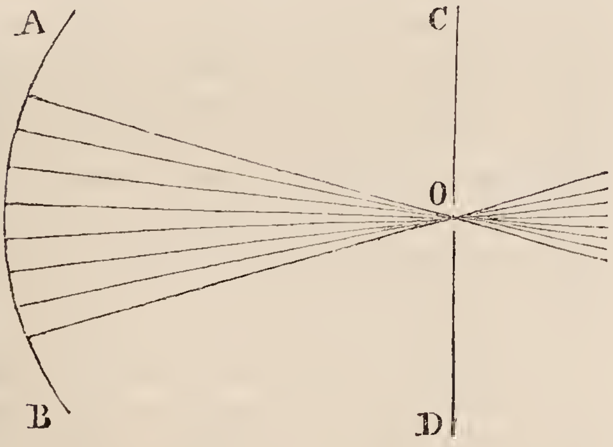
(80.) The colours produced by the reflection and transmission of light by thin transparent plates are not peculiar to these, but may also be exhibited, under certain circumstances, by plates of considerable thickness. By a series of experiments with concave glass reflectors, silvered on the convex side, Newton made these phenomena manifest, and applied his hypothesis of the fits of easy reflection and transmission successfully, in accounting for them, and in reducing them to the laws which were explained in the sixth chapter.

Before we proceed to describe these experiments, it is necessary to remind the reader of the several effects which a beam of light undergoes when it encounters the surface of a transparent medium. A part is reflected, according to the regular law of reflection, and a part entering the medium is regularly refracted. But besides these, which may be considered as the principal parts, there is still another portion which is scattered about in all directions around each point of incidence, both within and without the refracting medium. It is this portion of the incident light which serves to render the surface visible to an eye placed in any situation with respect to it. For if no light proceeded in any other directions than those of the regularly reflected and refracted rays, the surface would be invisible to every eye, except to one placed in these directions. It is to the portion of light scattered thus irregularly that the phenomena, which we are now about to describe, are to be traced.

Newton procured a concave glass speculum or mirror, silvered on the convex side, and in every part exactly a quarter of an inch thick. This he placed in a darkened chamber opposite the window, through which a beam of light was admitted. The radius of the sphere on which the mirror was ground was about six feet, so that the centre of the mirror was at that distance from its surface. At the centre of the mirror he placed a white opaque paper screen, having a small hole in it, and so adjusted the mirror and the screen, that the beam of light, passing through the hole in the screen, should fall upon the mirror, and be reflected back from the mirror to the same hole.

Let A B (*fig.* 50) be the mirror, C D the screen placed before it, so that the hole O shall be at the centre of the mirror. The rays of light, passing from O to the mirror, will fall perpendicularly on it, and consequently all the regularly reflected rays will return to O; and as far as these rays are concerned, the side of the screen presented to A B will be as dark as if no light passed through O to the mirror. Nevertheless, upon observing the screen presented towards A B, Newton found this not to be the case. He observed upon the paper "four or five concentric irises or rings of colours like rainbows, encompassing the hole. These rings, as they grew larger, became fainter and diluter, so that the fifth was scarce visible. Yet, sometimes, when the sun shone very clear, there appeared to be faint lineaments of a sixth and a seventh."

Fig. 50.



There was a very obvious analogy between these rings, and those exhibited between the lenses in 54. The colours did not succeed each other in the order of the reflected rings, but in that of those which were in that case transmitted. In the centre, at and around O, was a white round spot. This was skirted or fringed with a dark grey, which insensibly brightened into a violet ring. This was followed by a circle of indigo, one of pale blue, a greenish yellow, a vivid pure yellow, and, finally, a red, which deepened into a purple on the outer edge. Such was the first iris which surrounded the white central spot.

This was encompassed by a second series of coloured rings, of which the first was a dark purple, the outer boundary of the red in the former series. This was followed in succession by circles of blue, green, yellow, and red.

The colours of the third and fourth series were green and red. Those of the fifth were so faint as to be scarcely distinguishable.

In order to trace their connexion with rings exhibited by air inclosed between the lenses, Newton now measured their diameters, and found exactly the same proportion subsist among them as prevailed among the rings seen by transmission in that case. In order to make the relation of these phenomena still more manifest, he now transmitted through the hole O, not a beam of compound solar light, but of pure homogeneous light, obtained in the usual way by a prism. Transmitting through O in succession each of the colours, he observed the effects on the screen. Rings now only appeared of that colour which fell upon the speculum. If the speculum were illuminated with red, the rings were totally red with dark intervals; if with blue, they were totally blue, and so of the other colours. With whatever colour they were illuminated, the same proportion subsisted among their diameters as was observed with the lenses.

But if the colour was varied, they varied their magnitude. In the red they were largest; in the indigo and violet least; and in the intermediate colours, yellow, green, and blue, they were of several intermediate sizes, answering to that colour.

Hence it was plain, as in the case of the lenses, that when the speculum was illuminated with white light, the several colours of the rings were produced by the superposition of the rings which were separately formed by projecting successively on the speculum the several component elements of white light. The several tints of the rings produced by the white light admitted, in this case, of being found by such a scale as was explained in 61.

The diameters of the corresponding rings of different colours varied with the refrangibility of the light, and varied exactly according to the same proportion, as in the case of the rings seen between the lenses.

In order to ascertain whether the colours seen upon the screen were the mere effects of light and shade, and not to be attributed to the lights proceeding from the mirror, Newton placed his eye where the rings appeared plainest, and directing his view towards the mirror, he beheld the speculum all tinged over with waves of colours, like those seen between the lenses; and, like these, the rings swelled and contracted as they were viewed more or less obliquely by moving the eye from or towards the centre of the speculum. A bystander, during this experiment, observed upon the eye of the observer the same coloured light as he perceived in the speculum.

On comparing the manner in which these rings were produced with that in which the like phenomena were caused, as described in Chapter VI., the only differences which are observable are the thickness of the glass, and the posterior surface of it being silvered. The thin transparent medium, forming a soap-bubble, has the same figure as the speculum used in the present case, with this difference, that the colours are seen on the convex side, whereas, in the present, they are seen in the concave side. This circumstance, however, so far from impairing the analogy, renders it more perfect when the order of colours is considered, for the rings are not those seen by reflexion in thin transparent mediums, but by transmission. Hence, if the soap-bubble were viewed upon the concave side by an eye placed within it, the colours would be identical with those exhibited by the concave speculum in the present case.

To ascertain whether the silvering upon the back of the speculum had any part in producing the phenomena of the rings, Newton tried the effect of a similar concave glass, without silvering. The result was the production of the same rings, but with more faint colours, owing to the reflecting power of the second surface being diminished by the want of silvering. It therefore appeared that the circumstance of the back of the speculum being silvered, had no other effect than that of increasing the intensity of the colours.

Notwithstanding the identity of the two phenomena, as well as the complete similitude which existed between the manner in which they were produced, Newton did not feel himself warranted in applying the theory of the fits of easy reflection and transmission in accounting for them, until he established the fact that the two surfaces of the glass were indispensable for their production. Were it possible for a single surface, *i. e.* the concave surface alone, to produce the phenomena, this theory would have been quite inapplicable, and altogether inadequate to explain them. To reduce this question to the test of experiment, Newton procured a concave speculum of polished metal, which reflected only from one surface. On presenting this to the light, in the same manner as the glass speculum, no rings were produced. This result was decisive of the point, and proved that the two surfaces were necessary to the production of the rings.

It was plain that the light, *regularly reflected* from the speculum, had no part in these phenomena; for, as all the incident light radiated from the centre of the sphere, it fell perpendicular on the speculum, and was therefore reflected back perpendicularly to the same centre. The rings, therefore, must owe their existence to the light irregularly scattered by the surfaces of the glass. By the experiment already mentioned to have been made with the metallic speculum, it appeared that they could not be produced by the light scattered by the irregular reflection of the first surface alone; for, if that were possible, the metallic speculum, by the surface of which the light was thus scattered, would have exhibited them. The cause of the phenomena was, therefore, to be looked for in the light irregularly refracted by the glass, and reflected by the posterior surface. The rays thus irregularly refracted by the first surface fell upon the second surface at various obliquities, and were reflected by it regularly and irregularly, so as to return through the first surface at various obliquities.

We must here call the recollection of the reader to a property of thin plates already mentioned. A plate of a certain thickness reflects or transmits a certain colour, the rays of light being perpendicular to it. The rays being still perpendicular, every change of thickness will produce a corresponding change of hue in the reflected and transmitted lights. The very same effects, the same shifting of tints, which is thus produced by a change of thickness, the rays remaining perpendicular, may also be produced by changing the direction of the rays, the thickness of the plate remaining unvaried. Thus a plate of a given thickness, at a certain obliquity, will transmit red light, at another yellow, at another green, and so on. Also, if a ray of a particular colour gradually increases its obliquity from the perpendicular direction, it will be alternately reflected and transmitted for many successions.

The glass mirror used in the experiment to which we now refer was, in every part, of exactly the same thickness; therefore the rings of colour could not, as in the case of the lenses, be ascribed to a regular variation of thickness. The light, however, reflected irregularly by the silvered surface of this speculum, encountered the first surface after reflection at various obliquities. Those rays which met the first surface at obliquities proper for transmission, were transmitted to the screen, where they depicted their proper colour, and those which fell upon it at other angles were intercepted. Thus, suppose a ray of homogeneous red light radiated from the centre to the speculum, all the regularly reflected rays returned to the centre. Those which being irregularly reflected by the second surface met the first surface at obliquities proper for transmission, were propagated in corresponding directions to the screen, and they produced a red tint. These rays being regularly disposed in circles round the axis of the speculum, depicted circles of red light on the screen, and the rays returning to the first surface at intermediate obliquities not being in fits of transmission, were intercepted, and thus caused the dark circles between the luminous red ones on the paper. What we have here said of red light may be equally applied to homogeneous lights of other colours; and, hence, will easily be collected the cause of the various coloured rings produced when the light which emanates from the centre of the speculum is compound solar light, this effect being nothing more than the result of the simultaneous exhibition of the rings of each colour.

(81.) Newton points out one difference between the effect of the speculum on the light and that of the thin plates, described in Chapter VII. In the latter case the colours are produced by alternate reflections and transmissions of the light at the second surface of the plate, after one passage through it; but here they pass from the first surface to the second, and then return from the second to the first, there being either transmitted to the screen or reflected to the silvered surface, according as they are in fits of easy transmission or reflection.

By measuring the diameters of the rings of the different lights, and comparing this with the distance of the screen from the speculum, Newton ascertained the obliquities at which the lights of different colours emerged from the speculum, and thence derived the angles at which they were incident on the first surface after reflection at the second. On comparing these with the obliquities corresponding to the lights of different colours deduced from the theory established in Chapter VII., he found a perfect accordance.

(82.) In order, however, to put the matter to a more decisive test, he calculated the effect which the thickness of the glass which formed the speculum would produce upon the diameters of the several rings, and found that, according to the theory of fits of easy reflection and transmission, the squares of the diameters of corresponding rings produced by different mirrors should be in the inverse proportion of the thickness of the mirrors. He, therefore, procured another speculum, ground on both sides, to the same sphere with the former. Its thickness was $\frac{5}{62}$ parts of an inch, and the diameters of the first three bright rings, measured between the brightest parts of their orbits, at the distance of six feet from the glass, were 3, $4\frac{1}{6}$, $5\frac{1}{8}$ inches. Now, the thickness of the other glass being $\frac{1}{4}$ of an inch, was to the thickness of this as $\frac{1}{4}$ to $\frac{5}{62}$, or as 62 to

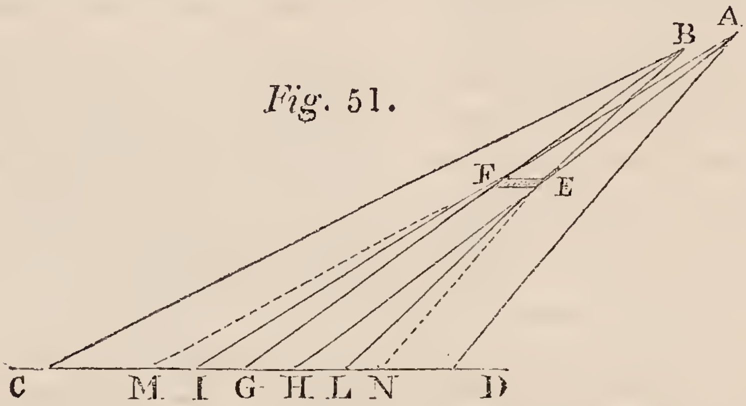
20, or 31 to 10, or as 310,000,000, 100,000,000, which numbers are very nearly as the squares of 17,607 and 10,000, and in the proportion of the first of these to the second, are the diameters of the bright rings made by the thinner glass, 3, $4\frac{1}{6}$, $5\frac{1}{8}$, to the diameters of the corresponding rings made by the thicker glass, $1\frac{1}{14}$, $2\frac{3}{8}$, $2\frac{1}{12}$. This perfect accordance of the phenomena with the results of theory was considered by Newton to be conclusive as to the validity of his hypothesis.

Chapter IX.

Experiments on the Inflexion of Light.

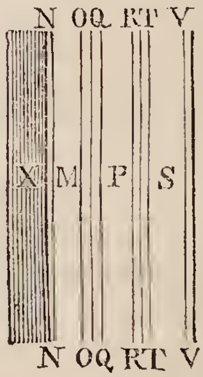
(83.) In the account of the state of optical science before the time of Newton, which was given in the first chapter, the discovery of the diffraction or inflexion of light by Grimaldi was noticed. This phenomenon was too striking to escape the attention of Newton. He accordingly instituted some experiments with a view to the investigation of this property; but he seems not to have had leisure or inclination to pursue the subject to as great an extent, or to obtain results so satisfactory as those which have been discussed in the preceding chapters. In the commencement of his third book, however, be describes some experiments which he made, and hazards some conjectures on the probable cause of these phenomena, which, as well as some other matters connected with physical science in general, and with the phenomena of heat and light in particular, he proposes in the form of queries. He states that he was interrupted in his investigations; that he could not afterwards think of taking these things into further consideration; and as he had not finished his design, he leaves these queries, in order to a further search to be made by others.

Fig. 51.



The experiments which led Grimaldi to the discovery of inflexion were the following. Through a small aperture A B (*fig.* 51) he admitted a beam of light into a dark room. He observed that the light thus admitted, spread itself in the form of a cone, and illuminated a large portion of a screen C D, held perpendicular to its direction, and at some distance from the hole, the illuminated part of this screen increasing with the increase of distance. When an opaque body E F was held in the light so as to cast a shadow on the screen C D, this shadow was found to be much larger than it would have been if the light had passed the edges of E F in right lines. From B draw the straight line B F G and from A the line A E H. Also from A draw the right line A F I, and from B the right line B E L. Now, if the light passes the edges of E F in right lines, it is plain that the space G H will be altogether deprived of light, and that from the limits G H to I and L respectively there will be a partial or penumbral illumination; but that beyond the limits I L, the illumination of the screen will be as complete as if the opaque object E F were not present. This was, however, found not to be the fact, for the shadow extended beyond I and L to wider limits, as M and N.

Fig. 52.

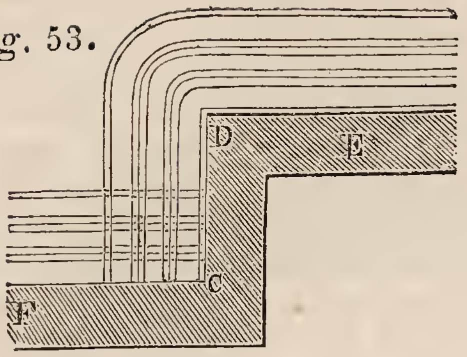


Beyond this shadow, and skirting it, Grimaldi observed three coloured fringes, the broadest and brightest of which was next the shadow. If X (*fig.* 52) represent the edge of the shadow, the space N N was *blue*, M was colourless, and O O red. In the second fringe, Q Q was faint blue, P colourless, and R R red. This fringe was narrower than the first and more faint. The third fringe T T, S and V V, was similar to the other two, but still narrower and fainter.

When the opaque body has a salient angle as D (*fig.* 53), the fringes were arched round it, and at a re-entrant angle they intersected each other as at C.

Newton repeated and varied these experiments of Grimaldi. He pierced a plate of lead with a pin, making a hole

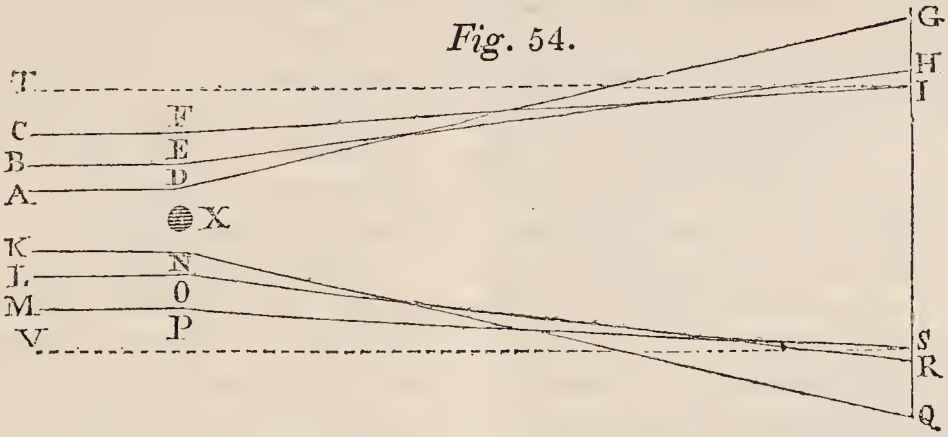
Fig. 53.



in it, the diameter of which was the 42nd part of an inch. Through this hole he admitted a beam of the sun's light, and found that the shadows of hairs, pins, straws, and such slender substances, appeared, when received upon a screen, to be much larger than they would be if the light passed their edges in unbroken straight lines. A human hair, the breadth of which was the 280th of an inch, held at 12 feet from the hole, cast a shadow upon a screen held at 4 inches from it, which was the 60th part of an inch broad, that is, above four times broader than the hair. Like effects were observable with other opaque bodies, and at all distances.

These effects were found to be entirely independent of the refractive power of the air, or of the nature of the body whose shadow was formed. When the hair was surrounded with moisture, or inclosed with a liquid between glass planes, the effect was the same. The scratches made on glass, and the veins in plates of that material, produced shadows of the same kind. Thus whatever was the origin of this effect, it was evident, that it did not proceed from the refractive powers of bodies, nor on any quality connected with the nature or properties of bodies of particular species.

Fig. 54.



The way in which Newton conceived the rays to be affected in passing the body may be thus explained. Let X (*fig*. 54) be a section of the hair at right angles to its length. Let A D, B E, C F, K N, L O, M P, be rays of light approaching the hair in parallel directions. The ray A D is deflected at D in the direction D G, and falls upon the screen at G. In like manner the ray K N, at the same distance below the hair, is deflected in the direction N Q, meeting the screen at Q. Newton supposes that the force which deflects the light diminishes as the distance from the hair increases. Consequently, the rays B E and L O will be less deflected than A D and K N, and their deflected directions E H and O R will cross those of the nearer rays, if the screen be sufficiently distant from the hair.

In like manner, the more distant rays C F and M P will be still less deflected, and will also cross both the last rays. This continues until the distance of the ray from the hair becomes so great, that the deflecting power is lost, and the rays proceed in straight lines to the paper as T I and V S. These rays at I and S bound the shadow, and at the hair include within them the deflected rays, all of which they must cross between the hair and the screen.

The continued intersections of the deflected rays with each other between the hair and the screen form curves, which are concave towards the shadow, and the distance between which increases as the distance from the hair increases. It is evident also that the illuminated space on the screen, immediately beyond the boundaries L S of the shadow is exposed to the light of the deflected rays between T I or V S and the hair, as well as to the direct rays which proceed without T I and V S and parallel to them.

The shadows of bodies, such as metals, stones, glass, wood, horn, ice, &c., exposed to light in this way, were skirted with three coloured fringes. The colours observed by Newton in these fringes were as follows: the first or innermost fringe was violet, deep blue, light blue, green, yellow, and red. The second and third fringes which immediately succeeded this were blue, yellow, red; but their colours were faint, Upon measuring the breadths of these fringes at their brightest parts, and also the breadths of their intervals, Newton found them nearly in the same proportion, and so that the fringes and intervals taken in regular succession were as the following quantities:

$$1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{5}}$$

His next experiments were on the shadows produced by the edges of sharp knives placed across the aperture through which the light was transmitted. The light admitted between the edges being received upon a screen at some distance, the edges were moved towards one another, until their distance did not exceed the four hundredth part of an inch. The light upon the screen now parted in the middle, and formed two parallel lines. The intermediate shadow was so black and dark that all the light which passed between the knives seemed to be bent, and turned to the one side or the other. As the edges approached, the shadow grew broader, and the lines of light narrower, until at length, when the edges were in actual contact, the light wholly vanished. This experiment Newton considered conclusive of the greater deflection of the rays which were nearer the body.

From these experiments, he concluded that the light which formed the first fringe passed the edge of the knife at a distance not less than the eight hundredth part of an inch; that the light of the second fringe passed the edge of the knife at a greater distance than that of the first, and the light of the third fringe at a still greater distance.

From these and other circumstances, Newton concluded that the distances at which the light forming the fringes passed the knives are not altered by the approach of the knives, but that the angles at which the light is inflected are increased by their approach; the knife which is nearer each ray determining the direction in which it is bent, and the other knife increasing the deflection.

When the rays fell very obliquely on the ruler, at the distance of the third of an inch from the knives, the dark lines between the first and second fringes bounding each shadow, intersected each other at the distance of the fifth of an inch from the termination of the light. Hence Newton computed the distance between the edges, at the concourse of these lines, to be the hundred and sixtieth part of an inch. Since this concourse is in the middle of the light which passes between the edges, it follows that one-half of the light passes each edge at a distance not greater than the three hundred and twentieth part of an inch.

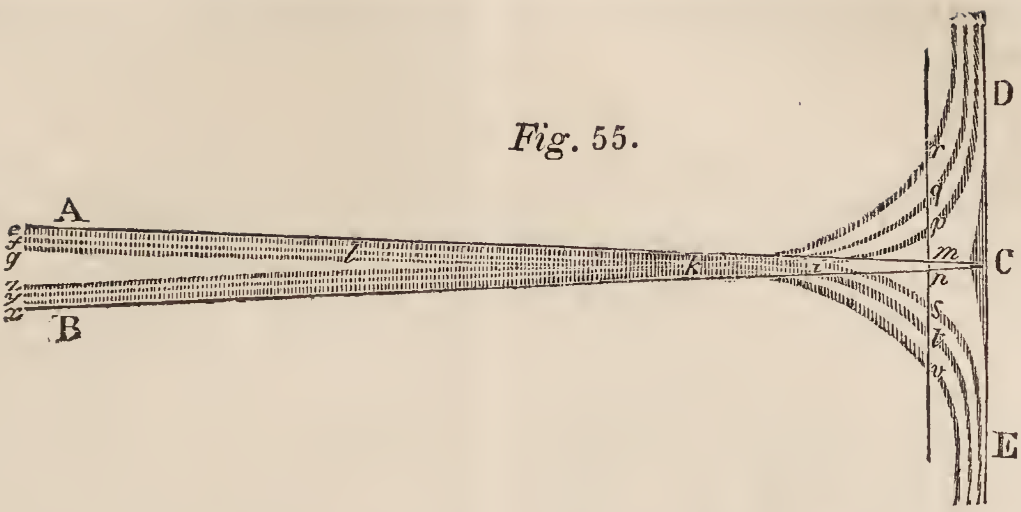
Upon increasing the distance of the ruler from the knives, he found that the distance of the concourse of the dark lines before mentioned, from the termination of the light, was more than the fifth part of an inch. Hence it appeared that the light, which in this case passed between the knives at the concourse of the dark lines, passed at a greater distance from the edges than the hundred and sixtieth part of an inch.

From these experiments Newton concluded that "the light which makes the fringes, is not the same light at all distances from the knives; but when the ruler is held near the knives, the fringes are made by light which passes the edges at a less distance, and is more bent than that which forms the same fringes at a greater distance." It will be perceived that this is consistent with the explanation of the phenomena of inflection already given.

When the shadows were received upon paper at a great distance from the knives, the fringes assumed the form of hyperbolic curves, being nearly straight where the distance between the edges was considerable, but bending into arches after intersecting. Let C A, C B, *fig.* 55, represent the projections of the edges of the knives upon the paper. The entire light would fall within the angle A C B, were there no inflection. Through C draw D E equally inclined to C A and C B. The curve *e i s* represents the boundary of the shadow of the blade A C; *f k t* represents the dark line which separates the first and second fringe, and *g l o* the dark line which separates the second and third fringe. The lines *x i p*, *y k g*, *z l r*, are similarly related to the shadow of the other edge.

The two systems of curves, which are perfectly similar, intersect at the points *i*, *k*, *l*; so that the shadows of the edges are marked by the lines *e i s* and *x i p*, until the intersection of the fringes, and then each of those lines crosses the fringe corresponding to the *other* edge. Then those lines cross the fringes, distinguishing them from another light which begins to appear at *i*, and illuminates the triangular space *i p* D E *s*.

Fig. 55.



Newton now repeated the first experiment described in page 219; but, instead of using compound solar light, he used homogeneous light, obtained by a prism placed behind the hole through which the light was transmitted. He found the shadows of all bodies held in this light bordered with fringes, not as before of different colours, but of the colour of the transmitted light. Those made by the red light were largest, and those made by the violet, least; the intermediate colours having intermediate breadths.

From these experiments he inferred that the rays which formed the fringes in the red light passed by the body at a greater distance than those which formed the violet or any of the intermediate colours. So that the action of the body on the less refrangible rays at a given distance was equal to its action on the more refrangible rays at a less distance, and thus occasioned the tints of the fringes without changing the colour or properties of any component part of the solar light.

Such was the state to which Newton's experimental investigations had arrived, when they were unfortunately interrupted. In the queries, however, annexed to this book, and which have been already alluded to, he throws out some suggestions for the consideration of future inquirers. The following queries relate to the subject with which we have just been engaged.

1. Do not bodies act on light at a distance; and by their action bend its rays? and is not this action strongest at the least distance?

2. Do not the rays which differ in refrangibility differ also in flexibility? and are they not, by their different inflexions, separated from one another, so as, after separation, to make the colours in the three fringes above described? and after what manner are they inflected to make those fringes?

3. Are not the rays of light in passing by the edges and sides of bodies bent several times backwards and forwards with a motion like an eel? and do not the three fringes of coloured light above-mentioned arise from three such bendings?

4. Do not the rays of light which fall upon bodies, and are reflected or refracted, begin to bend before they arrive at the bodies; and are they not reflected, refracted, and inflected by one and the same principle, acting variously in various circumstances?

The theory hinted at in these queries is sufficient to represent the principal phenomena of inflexion; but it must be confessed that the undulatory theory affords rather a more satisfactory generalization of those complicated effects, and has been more generally adopted in the investigations of modern philosophers. Neither the material nor undulatory theory can be said to be satisfactorily established; but it would seem that every phenomenon which can be brought under the former, can also, with equal facility, be explained by the latter; while there are some known effects in strict accordance with the latter, which cannot, without great difficulty and the introduction of gratuitous hypothesis, be accounted for by the former. For the most part, however, the language of either may be translated into the other.

To discuss the question respecting the experiments which have been just described, and the inferences drawn from them, it would be necessary to refer to the original experiments and reasoning of Grimaldi, and the later investigations of Dr. Young and Fresnel. Such details are, however, foreign to the objects of the present treatise.

The remaining queries which terminate this book relate to the probable connexion and causes of heat and light, the sense of vision, the cause of gravitation, and other subjects in physics, which cannot properly find a place here. To this, however, we must make exception of that (the 26th) in which he points out the polarity of light, to the merit of having first suggested which he has an undoubted right. "If the planes of perpendicular refraction of one piece of Iceland spar be at right angles with those of another, the rays which are refracted after the usual manner by the first crystal will be all refracted after the unusual manner in passing through the second crystal; and the rays which are refracted after the unusual manner in passing through the first crystal will be all of them refracted after the usual manner in passing through the second." Hence Newton concluded, that the rays have *sides*, possessing different properties, in virtue of which they will be differently refracted, according to the direction in which the crystal is presented to them. If the ray be successively transmitted through two crystals, and is placed in the same manner with respect to the planes of both, it will be refracted in the same manner by both; but if that side of the ray which looks towards the coast of unusual refraction in the first crystal be at right angles with that side of the same ray which looks towards the coast of unusual refraction in the second crystal, the ray shall be refracted after several manners in several crystals. There is nothing more required to determine whether the rays of light, which fall upon the second crystal, shall be refracted after the usual or after the unusual manner, but to turn about this crystal, so that the coast of its unusual refraction may be on this or on that side of the ray. *And therefore every ray may be considered as having four sides or quarters, two of which opposite to one another incline the ray to be refracted after the unusual manner, as often as either of them are turned towards the coast of unusual refraction, and the other two, whenever either of them are turned towards the coast of unusual refraction, do not incline it to be otherwise refracted than after the usual manner. The first two may therefore be called the sides of unusual refraction.*

We cannot better conclude the rapid sketch which we have attempted to give of this extraordinary production of human genius, than by quoting some precepts which have been most admirably illustrated in the works of this illustrious philosopher. "The main business of natural philosophy is to argue from phenomena without feigning hypotheses, and to deduce causes from effects until we come to the very First Cause, which certainly is not mechanical; and not only to unfold the mechanism of the world, but chiefly to resolve these and such like questions: What is there in places almost empty of matter, and whence is it that the sun and planets gravitate towards one another without dense matter between them? Whence is it that nature doth nothing in vain? and whence arise all that order and beauty which we see in the world? To what end are comets, and whence is it that planets move all in one and the same way in orbits concentric, while comets move in all manner of ways, in orbits excentric; and what hinders the fixed stars from falling on one another? How came the bodies of animals to be contrived with so much art, and for what ends were their several parts? Was the eye contrived without skill in optics, and the ear without knowledge of sounds? How do the motions of the body follow from the will, and whence is the instinct of animals? Is not the sensory of animals that place to which the sensitive substance is present, and into which the sensitive species of things are carried through the nerves and the brain, that there they may be perceived by their immediate presence to that substance? And these things being rightly dispatched, does it not appear from phenomena, that there is a Being incorporeal, living, intelligent, omnipresent, who, in infinite space, as it were in his sensory, sees the things themselves, intimately and thoroughly perceives them, and comprehends them wholly by their immediate presence to himself; of which things the images only, carried through the organs of sense into our little sensoriums, are there seen and beheld by that which in us perceives and thinks. And though every step in this philosophy brings us not immediately to the knowledge of the First Cause, yet it brings us nearer to it, and on that account is highly to be valued. * * * * And if natural philosophy in all its parts shall at length be perfected, the bounds of moral philosophy shall also be enlarged. For so far as we can know by natural philosophy what is the First Cause, what power he has over us and what benefits we receive from him, so far our duty towards him as well as that towards one another, will appear to us by the light of nature."

(4)

TRAILL, Thomas Stewart, the Elder

THE THERMOMETER AND PYROMETER.

THE ancients were unacquainted with any more certain mode of marking the variations of temperature, than the indications of the senses, and the limited knowledge derived from observing the melting or combustion of different substances. In modern times, instruments have been invented for noting variable degrees of heat and cold, which, under the designation of *thermometers*, or *thermoscopes*, *pyrometers*, or *pyroscopes*, are now in general use in every part of the civilized world. Their names are derived from the Greek terms θεϱμος, πυϱ, signifying *heat*, *fire*, and μετϱον, σϰοπος, a *measure*, an *investigator*.

The principle on which all such instruments are constructed, is *the change of bulk which every body undergoes by alteration of its temperature.*

All homogeneous bodies, except water, within a few degrees of its freezing point, expand by heat and contract by cold.* Their expansion, then, may afford a relative measure of the increase of temperature; and their contraction, of its diminution. This law holds good in gases, liquids, and solids; and, accordingly, matter in those three states of existence has been employed in the construction of instruments for measuring the intensity of heat and cold.

The changes of volume which gases or aeriform bodies undergo, were first employed for this purpose; liquids, such as spirit of wine, oils, or mercury were next used; and lastly, the changes in the bulk of solids were applied to measure the variations of higher temperatures, which would have too much expanded gaseous and liquid bodies.

The designation of *thermoscope* or *pyroscope* might be, with most propriety, applied to such instruments; but, in conformity to common usage, it is proposed in this treatise to apply the general term *thermometer* to the instruments depending on the expansions of aeriform and liquid bodies, and *pyrometer* to those in which the expansion of solids is the measure of the elevation of temperature; and the subject will be treated under the following heads.

I. Of the common Thermometer.
  1. Its history and construction.
  2. The precautions necessary in its construction and graduation.
II. Of the Pyrometer.
III. Of Register Thermometers.
IV. Of the Differential Thermometer, and its modifications.
V. Of some peculiar applications of the Thermometer.
VI. Of the imperfections common to all instruments for the indication of heat.

CHAPTER I.

Of the Common Thermometer.

§ 1. *History and Construction of the Thermometer.*

THE invention of the thermometer, like almost every other discovery of great utility, has been claimed for different philosophers; and national vanity has occasionally been enlisted in support of the pretensions of rival claimants. There seem, however, but two whose titles are worthy of notice.

The Italian writers generally give the honour to their countryman *Santorio Santorio*, long a physician at Venice, and afterwards a professor at Padua, who flourished about the beginning of the seventeenth century; and who had obtained just celebrity by his discovery of the insensible perspiration of the animal frame: the Dutch philosophers as unhesitatingly ascribe it to *Cornelius Drebbel*, a physician of Alkmaar, who appears to have enjoyed a high reputation as a chemist, a mathematician, and an inventive mechanical genius.

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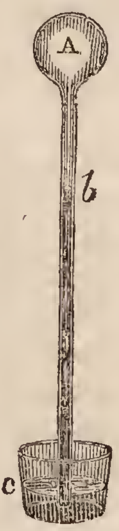
* Clay, a seeming exception, is not a homogeneous substance, of which afterwards.

Santorio expressly claims the invention as his own,* and he is supported by Borelli † and Malpighi; ‡ the title of Drebbel is considered as undoubted by Boerhaave § and Musschenbroek.|| It would now be difficult, perhaps, to decide the controversy; but it is worthy of remark, that Santorio, who was born in 1561, and died in 1636,¶ did not publish his claim to the invention till 1626;** and, although thermometers are alluded to by Robert Flud, within the first quarter of that century, yet as he travelled both in Germany and Italy for six years, we can draw no inference from that circumstance. Certain it is, that thermometers were constructed about the same time, both in Italy, and in Holland, on the same principle; and though the instruments of Drebbel were well known in Holland and England, before the fame of Santorio appears to have reached the North-West of Europe, the most recent writers have generally considered the latter as the real inventor of the thermometer. It is, however, by no means improbable that each may be justly entitled to the merit of a discoverer.

Be this as it may, the instrument was, from its imperfect construction, of little use in the hands of either, and required the successive labours of different philosophers to render it a tolerably accurate indicator of the variations of temperature.

The thermometer ascribed to Santorio and to Drebbel, is precisely the same in form and principle. It consists of a glass tube, with a ball blown on one of its extremities A, (*fig.* 1,) and having the other end open. A portion of the air in the ball is expelled by heat, and then the open end of the tube is immersed in any liquid contained in the cup *c*. As the ball cools, the included air diminishes in volume, and the liquid is forced into the stem, as at *b*, by the pressure of the atmosphere, until it replaces the volume of air which was expelled by the heat. When a heated body is applied to the ball A, the air will again be expanded, and depress the liquid in the stem; and, if this stem be a cylinder, a scale of equal parts applied to it will enable the observer to form some idea of the difference between the relative temperature of bodies applied to the ball. On the removal of the heated body, the volume of the included air again diminishes, and the liquid again rises in the stem by atmospheric pressure, until the elasticity of the air within the instrument is in *equilibrio* with that of the surrounding atmosphere. Instruments constructed on this principle are termed *air thermometers;* because their action depends on the elasticity of air; and from their having been originally employed to mark the changes of atmospheric temperature, they are described by the older writers under the name of *weather-glasses;* a denomination also given to barometers.

Fig. 1.



Drebbel appears to have devised a variety of the instrument more delicate in its indications. The globular form of the common bulb, and its small size, rendered it less susceptible of slight changes than a flattened bulb of larger diameter; and Boerhaave describes the bulb of Drebbel's thermometer, as composed of two shallow segments of large spheres, as in *fig.* 2. A, united at their edges, and in *fig.* 2. B, where it is seen in profile.

Fig. 2. A.

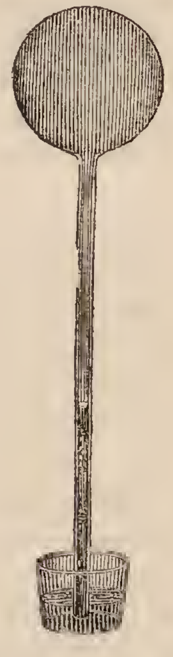
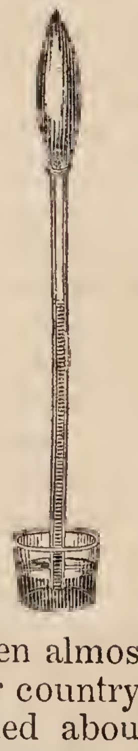


Fig. 2. B.



In the obscure, and often almost unintelligible, writings of our countryman, Dr. Robert Flud, published about the beginning of the seventeenth century, frequent mention is made of the thermometer, or, as he calls it, *speculum*

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* Comment. in Galen. et in Avicen.

† De Motu Animalium. Prop. clxxv.

‡ Opuscula Posth. p. 30.

§ Elementa Chemiæ, tom. i. p. 152.

|| Elem. Phil. Nat. § 780.—Tentam Exp. Acad. Cim.

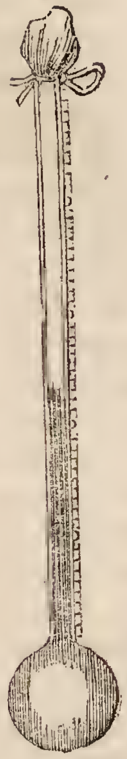
¶ Tiraboschi Storia, tom. viii. P. 1, 323.

** Commentaria in Avicennam.

Calendarium; and the common air thermometer is repeatedly figured in his singular work, *De Philosophia Moysiaca,** with its stem equally divided into an ascending and descending series, each of 7 degrees, respectively appropriated to winter and to summer. It is obvious, that the size of an air thermometer, on such principles, is only limited by convenience, and the length of the column of liquid which the pressure of the atmosphere can sustain in the tube. As originally made, they were unwieldy, they could not be applied to high temperatures, and were, besides, liable to two very important objections, as indicators of the atmospheric changes of temperature,—they were liable to be affected not only by heat and cold, but by the varying pressure of the atmosphere; and the scales adapted to them were arbitrary, and without fixed points for the comparison of observations made with different instruments.

The first objection was foreseen and obviated by the scientific members of the Florentine academy *del Cimento,* assembled under the auspices and patronage of Fernando II., Grand Duke of Tuscany. In the first article in the published transactions of that learned body,† we find a full description and delineation of a thermometer from which the influence of atmospheric pressure is excluded. The expansion of spirit of wine is employed to ascertain the temperature, instead of the dilatation of air; and the instrument is sealed *hermetically,* as it is termed, or has its orifice closed by melting the glass, after the introduction of as much spirit as fills the bulb and a portion of the stem. The method employed by the Florentine academicians is nearly that still used by the makers of the instrument; namely, by heating the bulb in the flame of a lamp, to expel the air, and then immersing the open end of the tube in the liquid destined to fill the thermometer. As the ball cools, the atmospheric pressure forces the liquid into the stem and ball, to supply the *vacuum;* and the orifice is closed by melting with the blowpipe the end of the tube, from which any excess of the liquid may be previously expelled by again heating the ball. (*Fig.* 3.)

Fig. 3.



The Florentine academicians appear also to have been aware of the necessity of adapting some fixed scale to the tube; but their attempts were not very successful. They described the thermometer as consisting of a ball and tube of such relative size, "that on filling it to a certain mark of its neck with spirit, the cold of snow and ice will not cause it to fall below 20 degrees measured on the stem; nor, on the other hand, the greatest heat of summer expand it more than 80 degrees."* This method is undoubtedly erroneous, inasmuch as the last point could be of no determinate temperature; and their method of graduation is in itself rather rude. The tube is directed to be divided by compasses into ten equal parts, these divisions are to be marked "by a little button of *white* enamel; and these may be further subdivided by the eye, and the intermediate degrees marked by buttons of glass, or of *black* enamel."

This instrument was variously modified by them to suit different purposes. The ball was occasionally enlarged, and the tube reduced in thickness to render the instrument more sensible; and in the work already quoted, we find a figure of a thermometer of this sort, with the stem spirally twisted to render it more portable, and less liable to accident.

Another invention of those philosophers to indicate changes of temperature may be here noticed. It consisted of hermetically sealed spherules of glass, of different specific gravities, introduced into a wide tube filled with pure spirit. The degree of the Florentine thermometer at which each sank was noted, and by hanging this instrument in an apartment, it somewhat slowly showed the variations of the temperature of the surrounding air.† Imperfect as these attempts were, they paved the way to very important improvements in thermometers.

The indefatigable Boyle appears early to have turned his attention to the improvement of the thermometer, and his first attempts were on the air thermometer, or the weather-glass, as it was then styled. He rendered the instrument more convenient, by making one reservoir for the liquid and for the air at the bottom of the tube; and thus the thermometer might be conveniently dipt in a fluid, or applied to any body for ascertaining its temperature. "The thermometer," he says, "being made by the insertion of a cylindrical pipe of glass (open at both ends) into a phial or bottle, and by exactly stopping with sealing wax, or very close cement, the mouth of the phial, that the included air may have no communication with the external, but by the newly mentioned pipe."[*] If a portion of any liquid sufficient to cover the lower extremity of the pipe, be contained in the bottle, it is obvious, that the expansion of the enclosed air will elevate the included liquid in the cylindrical pipe; and this liquid will again descend on the contraction of the enclosed air: *fig*. 4, 5. Mr. Boyle likewise showed that no dependence could be placed on the indications of *open* air thermometers, under different degrees of atmospheric pressure; and he states, that on plunging the bulbs of different thermometers in liquids of very different specific gravities, as mercury and water, the liquor in the stem stood at unequal heights, though both had been long exposed to the same temperature.

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* Folio, Goydæ, 1638.

† Saggi di Naturali Esperienze.

* Saggi di Naturali Esperienze, p. 4.

† Saggi, p. 10.

Fig. 4.

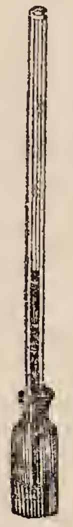
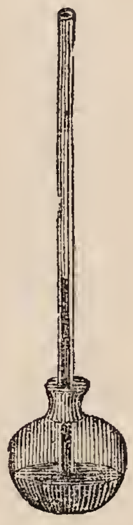


Fig. 5.



The Florentine thermometer was about that time introduced into England, and duly appreciated by both Boyle and Hooke. The specimen seen by these philosophers was filled with *colourless spirit*, but they made use of spirit of wine, tinged by cochineal, "of a lovely red;" and, says Boyle, "'tis pleasant to see how many inches a mild degree of heat will make the tincture ascend in the cylindrical stem of one of these useful instruments."[*] Boyle was fully aware of the imperfection of the scales hitherto applied to the thermometer, and sought to discover a remedy. He proposed to obtain a fixed point in the scale, by marking the height of the liquid in the stem of the instrument, when the ball was placed in thawing oil of aniseeds; a point which he preferred to that of thawing ice, because the former could be readily obtained at any time of the year. His method of making two or more comparable thermometers, however, would be found extremely difficult, if not impossible, in practice; it is best explained in his own words. "For if you put such rectified spirit of wine into a glass, the cavity of whose spherical, and that of its cylindrical part, are as near, as may be, equal to corresponding cavities in the former glass, you may by some heedful trials, made with thawed and recongealed oil of aniseeds, bring the second weather-glass to be somewhat like the first; and if you know the quantity of your spirit of wine, you may easily enough make an estimate, by the place it reaches to in the neck of the instrument, whose capacity you also know, whether it expands or contracts itself to the 40th, the 30th, or the 20th part, &c. of the bulk it was of, when the weather-glass was made."[†]

Boyle mentions that an "*ingenious man*"[‡] had proposed the freezing of distilled water, as a fixed point in the scale of thermometers; but he himself evidently gives the preference to the congealing point of aniseed oil. Dr. Halley proposed to regulate the scale by the uniform temperature of such a cavern as that under the Observatory of Paris, or the point at which *spirit boils;* and he also suggests the fixing of the scale from the *boiling of water*. This point he considered as an invariably fixed one, not liable to alteration from external circumstances; and the same idea was entertained by Amontons. With a single point so fixed, the method attempted by Boyle, Halley, and Hooke was to calculate the proportion of the stem to the ball, and thus to determine the increase in bulk of the whole liquid, by a certain temperature. Dr. Hooke describes a method of obtaining this by comparing the expansions of the thermometer to be graduated, with those of the liquid in an accurately formed cylinder of metal, two inches in diameter and depth, and having cemented to its top a glass pipe, just $\frac{1}{10}$ of the diameter of the cylinder:[*] measure off two inches of the stem, above the cylinder of metal, and divide the space between them into 10 equal parts, so that each division of the stem will = $\frac{1}{1000}$ of the capacity of the cylinder. The thermometer to be graduated has the commencement of its scale, or 0°, fixed by marking the point at which the included liquid stands in the stem, when the bulb is plunged in distilled water just beginning to freeze; and the rest of the process he details in these words. "Fill this cylindrical vessel with the same liquid wherewith the thermometers are filled, then place both it and the thermometer you are to graduate in water that is ready to be frozen, and bring the surface of the liquor in the thermometer to the first mark, or 0°; then so proportion the liquor in the cylindrical vessel, that the surface of it may just be at the lower end of the small glass cylinder; then very gently and gradually warm the water, in which both the thermometer and the cylindrical vessel stand, and as you perceive the tinged liquor to rise in both stems, with the point of a diamond give several marks on the stem of the thermometer, at those places which, by comparing the expansion in both stems, are found to correspond to the divisions of the cylindrical vessel; and having by this means marked some few of the divisions on the stem, it will be very easy by these to mark all the rest of the stem, and accordingly to assign to every division a proper character."[†] This ingenious method is, however, more difficult in execution than any one, unacquainted with such operations, will readily suppose; and it presupposes, what is not easy to accomplish, a very perfect adjustment of the metallic cylinder and the glass stem in the standard instrument.

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[*] Works of Hon. Robert Boyle, folio, vol. ii. p. 247.

[*] Works, vol. ii. p. 249.

[†] Works, vol. ii. p. 247.

[‡] He undoubtedly alluded to Hooke.

Dr. Hooke appears invariably to have used in his thermometers spirit of wine "highly tinged with the lovely colour of cochineal, which he deepened by pouring in it some drops of common spirit of urine."

The sagacity of our illustrious Newton saw the importance of improving thermometers. He appears to have been early aware of the inconvenience of spirit as a thermometric fluid, and employed linseed oil to fill his thermometer. It has the advantage of being able to endure a very considerable temperature, without endangering the bursting of the tube, and therefore can be applied to a higher range of temperature than a spirit thermometer. It has the disadvantage, however, to be more sluggish in its movements, and to adhere much to the inside of the tube, while it differs greatly in its fluidity at different temperatures. Newton perceived the convenience of having two fixed points in the construction of the scale; and he used the freezing and boiling points of water as the most suitable for this purpose.[*] His method of graduating his oil thermometer is given in the *Principia.* The oil, at the temperature of melting snow, was supposed to consist of 10,000 equal parts, which, when heated to the temperature of the human body, expanded to 10,256; at the temperature of water strongly boiling to 10,725; and at that of tin beginning to congeal, to 11,516 parts. In the first instance the ratio of expansion is as 40 to 39; in the second as 15 to 14; and in the third as 15 to 13 nearly. Hence, by taking the temperature of the oil in the ratio of the rarefaction and assuming 12 as the heat of the human body, the temperature of water briskly boiling will be 34 degrees, and of congealing tin 72 degrees.[†]

Newton continued his scale of temperature farther by observing the rate of cooling of heated bodies, until he could apply his thermometer to them, on the principle that equal decrements of temperature take place in equal times. It was thus he estimated the temperature of iron heated to the utmost intensity of a small kitchen fire equal to 194 degrees, and in a fire of wood about 200 or 210 degrees of the same scale.

It is perhaps unfortunate for the philosophy of heat that more sublime and dazzling objects drew Newton to other pursuits. Though he led the way to just views of the subject, neither he, nor any of his predecessors, appear to have been aware of the influence of the varying atmospheric pressure on the boiling points of liquids; nor do any of them seem to have considered that the varying expansions of the thermometric liquids at different temperatures, and the expansions of the glass of the instrument, must have materially affected every attempt to subdivide the stem of the thermometer into fractional parts of the whole bulk of the contained liquid.

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[*] Micrographia.

[†] Micrographia, p. 39.

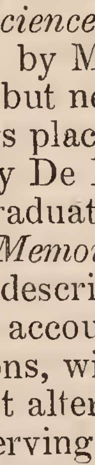
[*] Phil. Trans.

[†] "Ponendo caloris olei ipsius, rarefactione proportionalis, et pro calore corporis humani scribendo 12, prodest calor aquæ ubi vehementer ebullit partium 34, et calor stanni ubi liquescit prodest postea 72." Princip.

One of these questions, however, seems to have about that time engaged the attention of philosophers, *viz.* whether equal increments of temperature caused equal expansions of the thermometric fluid. Dr. Brooke Taylor tried the experiment with an oil thermometer, by mixing definite portions of hot and cold water, and measuring the temperature of the mixture. His conclusion was in the affirmative, but the delicacy of his instruments was unequal to the solution of this nice problem, although he has the merit of pointing out how the problem is to be solved.

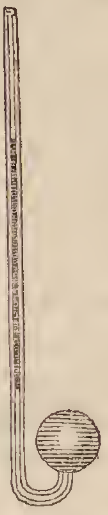
The construction and uses of thermometers early engaged the attention of the French *Académie des Sciences;* and several were constructed by Mr. Hubin for that learned body; but neither these, nor the thermometers placed in the observatory of Paris by De La Hire, appear to have been graduated on any fixed principle. The *Memoirs of the Academy* contain several descriptions of thermometers, and an account of many interesting observations, with these instruments; but the first alteration in their construction deserving of notice is the air thermometer of Geoffroy, which from the short description appears to be an improvement on that of Boyle, inasmuch as it is not affected by atmospheric pressure. He describes the tube as without any opening, except one, which descends almost to the bottom of the ball, and there dips into a small portion of coloured liquid.[*] There is no figure given in the original, and but a very rude one in our *Philosophical Transactions*,[†] seemingly from the description. It is not stated how the ball was joined to the tube, but it was most probably by cement, as represented in *fig.* 6.

Fig. 6.



M. Amontons clearly saw the importance of fixed points in the thermometric scale, and proposed to obtain them from the boiling point of water.[*] His thermometer consisted of a tube four feet in length, ending below in a ball bent upwards, as in *fig.* 7, and open at the other extremity. The measure of the temperature was the elasticity of a given portion of air included in the ball, and subjected to a pressure equal to *two atmospheres*, by adding to the usual atmospheric pressure that of a column of mercury of 28 French inches. Each half-inch of his tube is therefore equal to one inch under the usual pressure; and hence at a mean pressure of 28 French inches, the volume of the compressed air is really equal to 56 inches under the usual pressure.

Fig. 7.



In passing from the mean temperature of a Parisian spring to the heat of boiling water, Mr. Amontons found that these 56 inches were increased by one-third, or 18 inches 8 lines, and therefore he fixed the boiling point of his scale at $56+18,8=74$ inches 8 lines. To measure this on Amontons's principle a tube of 47 inches is quite sufficient; for 74 inches 8 lines minus 28 inches, the atmospheric pressure which need not be considered in the length of the tube, is equal to 46 inches 8 lines; and, indeed, as in Amontons's process, the compression at high temperatures is rather more than in the duplicate ratio of the air we breathe, the mercury in boiling water will not rise above 45 of his scale.[†] There is a slight discrepancy between the original account of Amontons's thermometer and that given by Martine, who states its boiling point at 73 inches, and its freezing point at $51\frac{1}{2}$ inches; but, according to the Academicians, the latter will be at 52 inches and about 8 lines. The ingenious contrivance of the double pressure enabled him to apply the instrument to measure the temperature of boiling water, by a tube less than four feet in length.[‡]

Although the idea of Amontons was a fine approximation to an universal standard for a thermometric scale, the instrument is liable to such objections that its principle seems scarcely ever to have been put in practice, except by its inventor and the Marchese Poleni.* It is difficult to construct two instruments which shall correspond, from the varying expansibility of air according to its moisture or dryness; the indications are liable to be affected by the fluctuations of atmospheric pressure; it is liable to be deranged by the escape of a portion of the included air, when the instrument is moved about; it is, moreover, too unwieldy, and very liable to be broken.

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[*] Mém. Acad. tom. xiii. p. 120. It was read in May, 1700.

[†] Phil. Trans. vol. xxiii. p. 962.

[*] Mémoires de l'Acad. for 1702.

[†] Mémoires de l'Acad. des Sciences, tom. xv.

[‡] Mém. Acad. des Sciences, tom. xv. for 1702

Much about the period when those attempts to perfect the thermometer were made in France, important improvements on it were effected in the north of Germany and in Holland, by the introduction of quicksilver as the thermometric fluid.

The objections we have stated to the use of the spirit thermometers, and to the oil thermometer of Newton, led the way to the employment of quicksilver in the construction of the instrument. Dr. Halley alludes to several advantages of quicksilver as a thermometric fluid, but seems to have rejected it on the ground of its slight expansion by heat,† although this objection might have so easily been obviated by increasing the disproportion between the bulb and the diameter of the tube. On this account the claim set up for his title to priority of invention may justly be denied. It is most probable that science is indebted for this great improvement to Roëmer, the celebrated astronomer of Dantzic, to whom the invention is ascribed by Boerhaave, as well as the first idea of the scale now known as that of Fahrenheit. Boerhaave further adds, that as early as 1709, Roëmer observed with that instrument a natural cold so intense as to sink the mercury to the beginning of the scale.‡ Thermometers of this construction began to be made by Daniel Gabriel Fahrenheit, a native of Dantzic, who afterwards lived at Amsterdam, in so admirable a manner, that he has generally been considered the original inventor; they were speedily spread over the north of Europe under his name, and still maintain their ground in several countries, especially in Britain.

It has commonly been alleged, that at the time when Roëmer's or Fahrenheit's scale was proposed, its zero was derived from the artificial cold produced by a mixture of salt and snow, then supposed to be the lowest possible reduction of temperature. This, however, seems to be inaccurate: Boerhaave* gives a different account of the matter, which is repeated in the *Philosophical Transactions.*† The zero was fixed from "the lowest cold observed in *Ysländ,*" (Iceland); which was supposed to be as low a temperature as was likely to become the object of philosophic investigation: but when artificial methods of reducing the temperature of bodies much lower, and occasional natural colds brought the mercury below that point, a scale of equal parts was extended below the 0°; the ascending series of degrees being distinguished by sign + or *plus*, and the descending series by the sign — or *minus*.

The principle which dictated the *peculiar division* of the scale is as follows. When the instrument stood at the greatest cold of Iceland, or 0 degree, it was computed to contain 11,124 equal parts of quicksilver; which, when plunged in melting snow, expanded to 11,156 parts; hence the intermediate space was divided into 32 equal portions, and 32° was taken as the freezing point of water: when the thermometer was plunged in boiling water, the quicksilver was expanded to 11,336 parts; and therefore 212° was marked as the boiling point of that fluid.‡ *In practice*, Fahrenheit determined the divisions of his scale from two fixed points, the freezing and boiling of water: *the theory* of the division, if we may so speak, was derived from the lowest cold observed in Iceland, and the expansions of a given portion of mercury.

The mercurial thermometer was used by the Italian philosopher Renaldini before the end of the seventeenth century: and he proposed, in 1694, an ingenious method of graduating it between the freezing and boiling points of water, by successive mixtures of determinate weights of boiling and ice cold water.

The great advantages of Fahrenheit's thermometer over every other previous invention, consisted in its applicability to a greater range of tempera-

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[*] Phil. Trans. No. 421.

[†] Phil. Trans. vol. xvii. p. 652.

[‡] Boerhaavii Chemiæ, tom. i. p. 720.

[*] Chemiæ, tom. i. p. 720.

[†] Vol. xliv. p. 680.

[‡] For 11156 — 11124 = 32, and 11336 — 1124 = 212.

ture, from the freezing to the boiling point of quicksilver, in its not soiling the containing tube, and in its receiving the impressions of heat and cold more readily, while its density rendered capillary tubes filled with it perfectly visible; and thus the instrument became more portable and delicate. We may also remark, that at the period of its invention, there was no other scale in use that could pretend to vie with it in accuracy; and it still possesses the peculiar advantages, that from the lowness of its 0°, the observer is seldom troubled with *negative* degrees, and from the number of its divisions has rarely, in ordinary operations, to use fractions of a degree.

We are indebted also to Fahrenheit for the knowledge of the fluctuation of the boiling point of water, according to the difference of atmospheric pressure.* Le Monnier, in 1739, confirmed this fact, by noting the temperature of boiling water on the top of Mount Canigou, one of the Pyrennees; and in 1744 it was fully established by Martin Folkes, who found that water boiled on the summit of Pic du Midi 15° of Fahrenheit's scale lower than at Bagneres; and at the latter place $3\frac{1}{2}$° lower than at Bordeaux; while he proved that elevation in the atmosphere had no sensible influence on the stability of the freezing point.† These facts led to an important correction in fixing the boiling point of water or other liquids.

It would now be a waste of time to describe minutely the various thermometers which were in use in France and England before the time of Fahrenheit. They were all without fixed points in the scale; and though they were vaunted as constructed after the models in the Royal Observatory at Paris, or in the apartments of the Royal Society of London, they gave most discordant results. An analysis of the most noted of them has been elaborately and ingeniously attempted by Dr. Martine in his valuable Essays, and the results presented in the very convenient form of a tabular view. We shall therefore pass at once to notice some of the other more accurate thermometers that have been employed in different parts of Europe, although the principle in them all is similar to what has been already described.

The thermometer with which the Dutch philosopher Cruquius made the observations published in the *Philosophical Transactions*, (vol. xxxiii. No. 381,) was an air thermometer, on which he states the freezing point of water to be indicated by 1070°, and boiling water by 1510°: the lowest known cold, which seems to have been the beginning of his scale, he gives = 1000°.

The objections to the thermometer of Amontons are clearly stated by Reaumur,* who proposed to adopt the freezing and boiling points of water as fixed points in the scale, but employed spirit as the thermometric fluid. He unquestionably fell into error when he stated that 1000 parts of strong spirit dilated to 1087.5 parts in passing from the freezing to the boiling point of water; for how could strong spirit sustain so high a temperature without being partially converted into vapour? His proposal was to use spirit of just such strength, that between these two temperatures it should expand from 1000 to 1080; and, commencing his scale or 0° at the freezing point of water, he made the boiling point 80°. The principle of this construction was good; but Dr. Martine has shown that from the large size of the bulbs of his thermometers, which were from 3 to 4 inches in diameter, and the short time they were immersed in the freezing mixture, they could not have acquired an uniform temperature; and accordingly Martine found their freezing point too high,† and the error in the boiling point from the cause already alluded to, must have been still greater.

These errors might have been obviated by the use of quicksilver instead of spirit. This was accordingly soon done; by whom first is uncertain, although there is strong reason to believe by De Luc; and the mercurial thermometer, with the 0° at the freezing point of water, and 80° as its boiling point, soon became general in France, and well known over Europe under the name of *Reaumur's Thermometer*. The only material objections to such a scale, when the instrument is accurately made, arise from the largeness of the divisions rendering fractional parts of a degree of frequent occurrence, and the elevation of 0° often introducing + and — degrees in a series of observations, even at common natural temperatures.

The mercurial thermometer of Mons.

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* Phil. Trans. xxxiii. No. 381.

† Phil. Trans. vol. xliii. p. 32.

* Mémoires de l'Acad. des Sciences, for 1730.

† Martine's Essays, Edin. 1792, p. 23.

J. De Lisle of St. Petersburg, differs little in principle from the instruments just mentioned; but its graduation is inverted. His 0° is at the boiling point of water, and he continues the graduation *downwards:* and conceiving the mercury, at that temperature, to be divided into 100,000 parts, he determined the degrees by the contractions of the whole mercury as it cooled, expressed in such parts.* The distance between the freezing and boiling points of water on this scale is 150°, as ascertained by Dr. Martine, who examined one of Dr. Lisle's original thermometers: but this thermometer seems to possess no advantage over those just described, and never came into general use except in Russia, where it is still employed.

Our countryman, Dr. Stephen Hales, employed another thermometer in his experiments on vegetable physiology. The 0° was at freezing water, and the highest point was ascertained by placing the instrument in hot water, on which wax was just beginning to congeal; the intervening space was divided into 100°.† This near approach to a true centesimal scale was defeated by the uncertainty of the upper point, arising from his using spirit instead of mercury in the tube, and the difficulty of ascertaining the exact moment of the congelation of the wax.

In the year 1742, the Swedish philosopher Celsius, professor at Upsal, divided *centesimally* the thermometer known in the north by his name, and which has, since its tacit adoption by the French chemists, obtained additional celebrity as the *Thermomètre Centigrade.* Celsius commences his scale at the freezing point of water, and divides the space between that point and the height of the mercurial column in boiling water into 100°. This appears a more natural and simple division than any that had been previously proposed, and it possesses several advantages; but it has two inconveniences of some importance in many practical operations. Thus, from the high position of the 0°, natural colds are frequently to be noted by a descending series of figures, and one column of observations may be hence embarrassed by + and − degrees; while from the large space intercepted between the degrees, the observer is frequently obliged to compute fractional parts of a degree.

M. de la Lande, in 1804, proposed a new thermometric scale, the 0° or mean point of which he would fix at the mean temperature of the earth; which he gives as = to 9°.5 of Reaumur's scale; and his degrees were to be the ten millionth part of the volume of the mercury in the instrument. Among the advantages of such a division, he considers the simplification of expression in meteorological observations—thus, 30° would express the heat of summer and cold of winter; 40° a hot summer and severe winter; while the smallness of the degrees would obviate the use of fractions of a degree. The boiling point of water would be at + 133, and the congelation of mercury at − 74°; ice would melt at − 18°, and the zero of Fahrenheit would be at − 44.*

This proposition has never been adopted; and its advantages seem overrated by the inventor. It only obviates one of the objections urged against the scale of Celsius, and is inferior in simplicity either to a millesimal division of the interval between the freezing and boiling point of water, or to the thermometric scale proposed by the late Dr. Murray of Edinburgh. That acute philosopher proposed to employ the freezing and boiling points of mercury itself as the extremes of his scale, and to divide the intervening space into 1000°. It is a more natural division than any hitherto proposed, inasmuch as it is taken from relations of the best thermometric fluid itself to heat: and if we suppose these two points to have been accurately fixed at − 40° and + 655° of Fahrenheit, the freezing point of water would be 99°, and its boiling point 347° on Murray's scale.

The advantages of this scale, are that it will very seldom, in natural temperatures, render the introduction of − degrees necessary, and the smallness of the divisions supersede the employment of fractional parts of a degree, in ordinary cases; two circumstances of considerable importance in a long series of thermometric observations.

Magellan informs us,† that M. Achard of Berlin invented a thermometer for ascertaining high temperature, which is a true *Pyrometer*, and might have been introduced in the next section. It consists of a ball and tube of semitranslucent porcelain, highly baked, containing a fusible alloy of two parts of bismuth, one of lead, and one of tin. In the temperature of the air, it remains solid in the tube; it becomes fluid about the boiling point of water; then, as a fluid, expands by increase of temperature; and its expansion being seen through the semitranslucent tube, which is divided into equal parts or degrees, becomes an indication of the temperature applied to the ball.

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* Phil. Trans. vol. xxxix. p. 221, for 1736.

† Vegetable Statics, vol. i. p. 58.

* Journal de Physique, 1804. Nicholson's Journal, 8vo. vol. ii. p. 61.

† Sur la Théorie du Feu Elémentaire, 1780.

This invention promises to be of considerable utility, and is capable of extension by the employment of less fusible metals. From the simplicity of its construction, it is rather surprising that it has not been more generally known, and employed in potteries, where the instrument could be easily made. An instrument on this construction would be a better method of uniting the scales of the common thermometer and pyrometer than any heretofore employed.

Of these various thermometric scales there are but three in very general use, *viz.* that of Fahrenheit, Celsius, and Reaumur. Fahrenheit's is chiefly used in Britain, North America, and Holland: the scale of Celsius was adopted by the French, and is now employed in most parts of the north and middle of Europe: Reaumur's was the only one used in France before the Revolution, and is still that best known in Spain and in some other continental states; but it is further important, as affording the terms in which numerous very valuable observations are recorded.

For these reasons it is useful to have formulæ for readily converting one scale into the equivalent degrees of the other two. The freezing point of *water* on Fahrenheit's scale is at 32°, and on those of Celsius and Reaumur at 0°, while it boils on each respectively at 180°, 100°, and 80°, above that point. Hence the degrees of Fahrenheit are to those of Celsius as $180:100 = 18:10 = 9:5$, and to those of Reaumur as $180:80 = 18:8 = 9:4$ —, or 9° of Fahrenheit are equal to 5° of Celsius and to 4° of Reaumur. Therefore, when we wish to convert the degrees of Celsius into those of Fahrenheit, we have to multiply the number of the former by 9, divide by 5, and add 32; to reduce the degrees of Fahrenheit into those of Celsius, the *converse* of the proposition will give the required result; that is, from the degree of Fahrenheit subtract 32, then multiply by 5, and divide by 9. When we wish to convert the degrees of Reaumur into those of Fahrenheit, we have to multiply by 9, divide by 4, and add 32; and subtracting 32 from the given degree of Fahrenheit, multiplying the remainder by 4, and dividing by 9, will give the equivalent degree of Reaumur's scale.

The following short formulæ will apply to each case:

1. $$F = \frac{9\,C}{5} + 32.$$

2. $$C = \frac{(F - 32) \times 5}{9}.$$

3. $$F = \frac{9\,R}{4} + 32.$$

4. $$R = \frac{(F - 32) \times 4}{9}.$$

These formulæ apply to all degrees above the freezing point of water; but when negative degrees of Celsius are to be converted into the equivalents on Fahrenheit's scale, multiply the degree of Celsius by 9, divide by 5, and the difference between the quotient and 32 is the required degree of Fahrenheit: or when negative degrees of Fahrenheit are to be reduced to their equivalents on the scale of Celsius, add 32 to the given degree of Fahrenheit, then multiply by 5, and divide by 9. By substituting 4 for 5, the same formulæ will apply to Fahrenheit and Reaumur, all which may be thus expressed:

1. $$-F = \frac{9\,C}{5} \backsim 32.$$

2. $$-C = \frac{(F + 32) \times 5}{9}.$$

3. $$-F = \frac{9\,R}{4} \backsim 32.$$

4. $$-R = \frac{(F + 32) \times 4}{9}.$$

The formulæ are convenient for reducing a few examples from one scale to another; but when they perpetually occur in reading or writing it is very useful to have comparative tables, from which, by one glance, the desired information may be obtained.

§ 2. *Precautions necessary to be observed in constructing accurate Thermometers.*

A general idea has been already given of the mode of constructing a thermometer, but where much accuracy is required there are many niceties that demand attention.

1. The tube should be of equal diameter throughout the whole stem. As obtained from the glass-house, the tubes are in reality frusta of very elongated hollow cones, which, by extension, become more or less nearly cylindrical; and as the divisions of the scale are usually equal, it is very important that the tube should not perceptibly differ from a true cylinder.

For these purposes, after a tube has been chosen by the eye as equal in calibre as possible, the best makers blow a bulb on it, and introduce a short column of mercury into the stem, perhaps an inch in length, which is accurately measured on a fine scale of equal parts, in different portions of the tube, as the column is, by the heat of the hand, moved from the bulb to the open extremity of the tube. Should the mercurial column subtend the same number of divisions on the scale in every part of the tube, it may be considered as a perfect tube for a thermometer.

The late Mr. Wilson, of Glasgow, introduced thermometric tubes of an *elliptical* bore. The advantage of this form is, that a very small column of mercury is much more visible when it is expanded at right angles to the line of vision. If due precaution be taken to ensure the equality of the tube this form answers well, especially for ordinary purposes; but where great nicety is required, we would recommend the cylindrical tube.

2. The form and proportion of the bulb may vary according to the purpose for which the instrument is to be applied. The larger the bulb in proportion to the stem, so much more delicately susceptible of changes of temperature will be the thermometer. The spherical bulb is to be preferred, for this shape is least likely to be affected by the varying pressure of the air; but when the bulb is very large this form renders the thermometer less susceptible of minute changes of temperature, and pyriform or cylindrical bulbs are usually adopted. All large bulbs are more or less sensibly affected even by slight pressure. An examination of more than fifty common thermometers, with large spherical bulbs, in the work-shop of an excellent artist, afforded the writer of this article an opportunity of observing that by slightly compressing their bulbs between the finger and thumb, the mercury in the stem rose and fell alternately several degrees, as the pressure was increased or diminished. The bulb and stem are usually in the same straight line, but for various purposes the bulb is occasionally placed at various angles to the stem.

In forming the bulb the mouth must not be employed to blow it, otherwise moisture will condense in the tube, which is expelled with much difficulty, and if suffered to remain, will greatly impair the value of the thermometer. Good instrument-makers use a small bottle of caoutchouc, or elastic gum, fastened by a thread on one end of the tube, while the other extremity is softened by the flame of a tallow lamp, urged by a blowpipe. By compressing the bottle, after the orifice of the softened end of the tube is closed by the aid of another rod of glass, a bulb is formed of any required size; but a neat workman will rarely consider the first blown bulb sufficiently well formed for his purpose. It is generally dilated till it bursts; the glass, while still soft, is compressed into a rounded mass, and a fresh bulb formed of a regular shape, and size proportioned to the calibre of the tube. Should the artist not intend to fill the tube immediately, he usually hermetically seals the other end of the tube to prevent the entrance of damp air or dust.

3. The precautions necessary in filling thermometers with mercury are exceedingly well given in Nicholson's *Chemistry*.*

The mercury should be clean, dry, and recently boiled, to expel air as much as possible. Mercury is often cleaned by thermometer-makers by agitating it in a phial, for some time, with sand, and then straining it through leather; for nice instruments it should be distilled from iron filings, or reduced from its sulphurets, in clean iron vessels, at a moderate heat.

The bulb to be filled is heated in the flame of a lamp, and the open extremity of the tube is immersed in the mercury; as the bulb cools, the pressure of the atmosphere forces the fluid into the tube and ball. Mr. Nicholson recommends, that the bulb should be but moderately heated at first; so as, on cooling, to become only half filled. He advises the open end of the tube to be kept under the surface of the mercury, and the instrument to be retained as nearly in the horizontal position as possible, while the flame of a newly snuffed candle is applied to the bulb, so as to boil the included mercury. Thus the remaining air will be expelled; and on removing the candle, the mercury will suddenly fill the ball and part of the tube.

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* Edition 3rd, p. 24.

4. To ensure a delicate thermometer, the mercury is next to be boiled in the thermometer. For this purpose a slip of clean writing paper is to be rolled tightly around the upper part of the tube, so as to form, beyond the orifice, a cup or cylinder capable of containing as much mercury as the bulb: secure this round the tube with a thread, put a drop of mercury into the paper cavity, and again apply heat to the bulb, holding the tube by the part covered by the paper. The mercury will soon boil, and about one-half of the contents of the ball will rush up into the paper cup. On removing the bulb from the candle, the mercury will suddenly return. Repeat this operation again and again, until the speedy boiling of the mercury, and the diminished noise and agitation, show that the whole has been well heated, and air and moisture expelled from it.

Should there be the least moisture in the tube before this part of the operation, it is very likely to burst the bulb; and the same accident is likely to happen, if the mercury be too strongly boiled the first or second time.

An experienced eye will readily judge what range of scale the thermometer will have; but this point can easily be ascertained, before the tube is closed, by heating the bulb in the mouth, and then immersing it in cold water or melting ice. When the latter is used, the operator can at pleasure fix how far from the bulb he will have the freezing point; for, by keeping the tube more or less filled, he can adjust that point to any desired height.

5. The tube is now to be *hermetically sealed*, that is, closed by the fusion of the glass at the upper extremity, which for this purpose is previously drawn to a capillary orifice. When it is intended to free the tube entirely from air, which is the best method with mercurial thermometers, heat is again to be gently applied to the bulb, which at the same moment is to be softened by another flame, and closed in the usual way, as soon as the mercury reaches the extremity of the tube. When the ball has cooled a little the sealing is rendered more secure by fusing the glass more fully around the top, so as completely to obliterate the orifice. If the vacuum be perfect, the mercury will fall to the extremity of the tube on inverting the thermometer, unless the calibre be absolutely capillary; in which case capillary attraction will overcome the force of gravity, and the mercury will retain its position in the tube, in every situation of the instrument.

Where there is a complete vacuum in the tube, the mercury must be well boiled before the sealing, as above directed; and when we choose a thermometer, the ready falling of the mercury, on inversion of the tube, is the best test we can have that the mercury has been well freed from air and moisture. This vacuum is not, however, so essential to the true action of the thermometer as was once supposed. A thermometer with a small dilatation of the tube when sealed, containing some common air, has lately been recommended as preferable to the instrument with a vacuum on the surface of the mercury.

M. Flaugergues* first called attention to the fact, that when old thermometers are placed in melting ice, they seldom fall quite so low as the mark of freezing on their stems, especially when the whole air has been expelled from them. This difference he found to amount sometimes to 0.9 of a degree. The same fact has been confirmed by MM. De la Rive and F. Marcet,† and also by Bellani‡ and Arago.§ The writer of this article possesses three thermometers; one very delicate, made by Ramsden, and two well made instruments by Lovi of Edinburgh, all which have been in his possession upwards of a quarter of a century. On lately placing them in a vessel filled with pounded ice, in a warm apartment, they all showed a slight elevation of the freezing point. That made by Ramsden has a capillary tube and small spherical ball; the other two have small pyriform bulbs, and the mercury readily falls to the extremity of the tube on inverting them: yet Ramsden's stood about 0.6 of a degree above the freezing point, and the other were just perceptibly above it.

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* Bibliothèque Universelle, tom. xx. 1823.

† Ib. tom. xxii.

‡ Giornale di Fisica, tom. v.

§ Annales de Chimie, tom. xxxii.

M. Flaugergues attributes this change to the effect of long continued atmospheric pressure on the bulbs of thermometers, in which there is no air to counteract it. De la Rive and Marcet give the same explanation, and remark how this circumstance must affect the result of all experiments on the cold produced in vacuo.

Arago is not inclined to attribute this elevation of the zero to atmospheric pressure on the bulb; since he found it equally affecting thermometers with very thick and very thin bulbs. He inclines to ascribe it to the disengagement of air, which either adhered to the glass or the mercury, and its accumulation in the upper part of the bulb, so as to affect the column in the stem.

The most complete observations on this point are those of Bellani,* who acknowledges two sources of variation in the zero of thermometers. That elevation of the zero, first noticed by Flaugergues, according to him, goes on gradually increasing for a limited period, but ceases after a year or two. He ascribes it to the extreme slowness with which glass once softened has the equilibrium among its particles restored. He found, that some months after graduation, a thermometer did not sink quite to the freezing point when immersed in melting ice; if laid by for some months, and again tried, its zero will be still higher; but after some time this irregularity ceases. He found that this effect was not diminished by leaving the thermometer open at the top, and it was sensible even in spirit thermometers.

The other irregularity noticed by Bellani is detected in the following manner.—Let a thermometer, having such a range that $\frac{1}{10}$ of a degree is appreciable, after lying by for some months, be plunged into melting ice, and its height accurately noted, then into boiling water, and again into ice, it will now stand *lower* by about $\frac{1}{10}$ of a degree than at its first immersion in the liquefying ice. This effect he ascribes to the extreme slowness with which the expanded glass can regain its former state of contraction, compared to the mercury.

These deductions appear to be perfectly just; and we are further indebted to Bellani for an ingenious method of showing that the air, if not wholly, is chiefly retained in thermometers and barometers by the glass, not by the mercury. He introduced a portion of unboiled mercury into a bulb, containing mercury which had ceased to give out any air, and found that this introduction did not renew the agitations which the first application of heat to the bulb had occasioned.

The difficulty of freeing thermometers from air is admitted by Arago, while he recommends boiling the mercury in the bulb as the best method of effecting the expulsion of the air; and he quotes some unpublished experiments of Dulong, to show the tedious manipulations which are necessary for this purpose.

We would recommend the boiling to be performed in the manner stated, until the agitation of the fluid caused by the air ceases; and after the tube is closed, the observations of Bellani would incline us to recommend, for delicate instruments, that the attempt to fix the freezing point should be deferred, until the glass might be supposed to have contracted to its state of equilibrium; after which, there would probably be little change in the dimensions of the bulb.

6. We come now to the last and most delicate step of the process, the adaptation of the scale to the instrument.

In the *manufacture* of thermometers this is conveniently done by plunging the new instrument, along with a standard thermometer, into two liquids at different temperatures: but the graduation of *this standard instrument* is a work of such nicety and importance, that a committee of seven members of the Royal Society was formed to investigate the subject, and their elaborate report is given in vol. lxvii. part ii., where all the requisite circumstances are distinctly noticed, and the best manipulations minutely described.

Two fixed points are sought; and the freezing and boiling points of water are most convenient for that purpose. To find the first, nothing more is necessary than to place the thermometer to be graduated, after it is filled, in melting snow or ice, in such quantity around the ball and tube, as to bring it to the desired temperature. When the mercury has become stationary in the tube, a mark is to be made on the tube with a file, just opposite to the top of the mercurial column; and that mark fixes the freezing point of the scale of the instrument. The determination of the boiling point is much more difficult, because it is affected by atmospherical pressure, and even by the form of the vessel in which the water is heated.

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* Bellani, Giornale di Fisica, tom. v.

The Committee of the Royal Society* recommend that the boiling point ought to be fixed under a barometrical pressure of 29.80 inches. For the graduation of the thermometer they recommend that the bulb should not be immersed in the water; because they found, that according to the depth of this immersion the mercury rose to a greater height in the tube. They recommend a vessel of tin plate, provided with a cover which fits easily on, and rendered steam-tight by a ring of woollen cloth between it and the vessel. This cover has two apertures—a chimney, with an area not less than half a square inch, and two or three inches high, to carry off the steam of the boiling water; and a hole for a cork, through which the thermometer tube is inserted in such a manner, that the ball does not touch the surface of the water, but may be surrounded with an atmosphere of steam; while no more of the tube should be above the cork than is sufficient to show the height to which the mercury rises when the water is briskly boiling. When all things are thus adjusted, a thin plate of metal is to be laid over the chimney, to prevent the escape of the steam as it is formed; heat is to be applied to the bottom of the vessel; and when the mercury has remained a few minutes stationary in the atmosphere of steam, its height is carefully to be marked with a file on the tube.

The water may be distilled, or any soft water, such as clear rain water, be used; for, if there be much saline ingredient in the water, this will affect the boiling point, and may lead to error.

Various mechanical contrivances have been proposed for more conveniently fixing the tube in the cover, but they are of little comparative importance. Some prefer plunging the ball into the water to the depth of two or three inches: in this case there is no necessity for a plate of metal on the chimney, nor for the tightness of the cover; but the adjustment of the boiling point is to be made for the barometer at 29.50 inches. To those unprovided with such a vessel the following method is recommended. Wrap several folds of linen, or flannel, round the tube, nearly as high as the supposed boiling point, which may be guessed at by previous immersion of the bulb in boiling water: hold the thermometer in an ascending current of boiling rain water about two or three inches below the surface; pour boiling water three or four times on the covering of the tube, at intervals of some seconds; and waiting a few seconds, after the last affusion, to allow the water to be in brisk ebullition, mark the height of the mercury in the tube, which will be the boiling point of the instrument.

Having thus obtained two fixed points, the freezing and boiling points of water, it is easy to mark off corresponding divisions on the scale which is to be graduated. If the tube be truly cylindrical, nothing more is necessary than to divide the intervening space into as many equal parts as it is intended to have degrees between those points. Should the tube not be of uniform bore, the size of the divisions ought to be accommodated to the inequalities of the tube. This may be done by taking intermediate points in mixtures of water at different temperatures; and after marking them on the tube, proportioning the size of the degrees, at short intervals, to the varying diameter of the tube. This method of graduating from intermediate points ought, in nice instruments, to be adopted, however true the tube may appear; but a tube with sensible inequalities is in general to be avoided.

Although it would be advisable to fix the boiling point when the barometer is at the height above recommended, this may be attended with serious inconvenience to artists; and philosophers have therefore investigated the correction to be made for every ordinary variation of atmospheric pressure.

The first considerable series of experiments on this subject are those of De Luc, in 1762, published in his interesting *Recherches sur les Modifications de l'Atmosphere*,* which were extended and verified by Sir George Shuckburg† in 1775 and 1778. Employing Reaumur's scale, De Luc ascertained, that if y represent the height of the barometer, T the height of the thermometer above the freezing point, expressed in hundredths of a degree of this scale, when immersed in boiling water; and a the constant number

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* Phil. Trans. vol. lxvii. part ii.

* Vol. i. 382; vol. ii. 338.

† Phil. Trans. vol. lxix. part ii.

10387, the following formula will express the height of such thermometer when plunged in boiling water under every variation of barometric pressure.

$$\frac{99}{200000} \log y - a = \mathrm{T};$$

or, as expressed in the more usual way of considering all the figures after the index as decimals, De Luc's formula would stand thus :

$$\frac{99 \times 100}{2} \log y - a = \mathrm{T}.$$

De Luc's researches and his formula are reduced to English measures, and adapted to Fahrenheit's thermometer by Horsley, in a valuable paper in the *Philosophical Transactions*;* where a table is computed for the direction of artists in adjusting the boiling point. It is unnecessary to give his equation of the boiling point, because the later experiments of Shuckburg, and of the Committee of the Royal Society, enable us to present a more complete table for the direction of British artists in correcting the height of the boiling point in every ordinary fluctuation of the barometer.

Barometer when the boiling point is found by immersion in		Correction in 1000ths of the interval between freezing and boiling of water.	
Steam.	Water.		
	30·60	10	Lower.
	·50	9	
30·71	·41	8	
·50	·29	7	
·48	·18	6	
·37	·07	5	
·25	·95	4	
·14	·84	3	
·03	·73	2	
29·91	·61	1	
·80	·50	0	
·69	29·39	1	Higher.
·58	·28	2	
·47	·17	3	
·36	·06	4	
·25	28·95	5	
·14	·84	6	
·03	·73	7	
28·92	·62	8	
·81	·51	9	
·70		10	
·59			

The use of this table requires no further explanation: but it is necessary to remark, that it presupposes the thermometric tube to be cylindrical, or of equal dimensions throughout, before the indications of the table can be received as *quite* correct; yet, unless the irregularity of the tube be considerable, a *small* correction will scarcely produce any sensible error in the instrument.

In proportioning the bulb to the tube, the eye and experience of the artist are usually judged sufficient for the purpose; or they are copied as nearly as possible from standard instruments. M. Durand has, however, thought it necessary to propose an algebraic formula for determining the proportions they ought to bear to each other; but there are practical difficulties in the way of its application, which render his formula an exercise rather of his own ingenuity than of utility to the artist.

During the various improvements of the common thermometer, the air thermometer was almost wholly neglected until of late years; but the attention of philosophers was directed to the changes of bulk which *solids* undergo by alterations of temperature, as a measure of the relative degrees of heat.

Chapter II.

History and Construction of Pyrometers.

1. The impracticability of applying the known modifications of the thermometer to bodies much heated, induced the celebrated Musschenbroek, before the middle of the last century, to employ the expansions of solid rods of metal to indicate the temperature of such bodies; and he gave the name of *pyrometer* to his invention.

As the expansions of solids are extremely minute, it was necessary to devise some method of rendering them perceptible; and the mechanism represented in *fig.* 8 was the Dutch philosopher's arrangement for this purpose. *a* is a metallic prism 5.8 inches in length and 0.3 in thickness, resting in a notch in the upright *i*, where it is secured by a screw, and heated by the lamp *b* with five wicks. The prism is pinned to the end of a bar *c*, which has twenty-five teeth in one inch of its length, and forms a rack sliding smoothly on the table of the instrument through the two holdfasts seen in the figure, and playing in the six-leaved pinion *d* on the same axis as the wheel *f*, which is furnished with sixty teeth. This wheel plays in another pinion *e*, of six leaves also, which is on the axis carrying the index *g*, which

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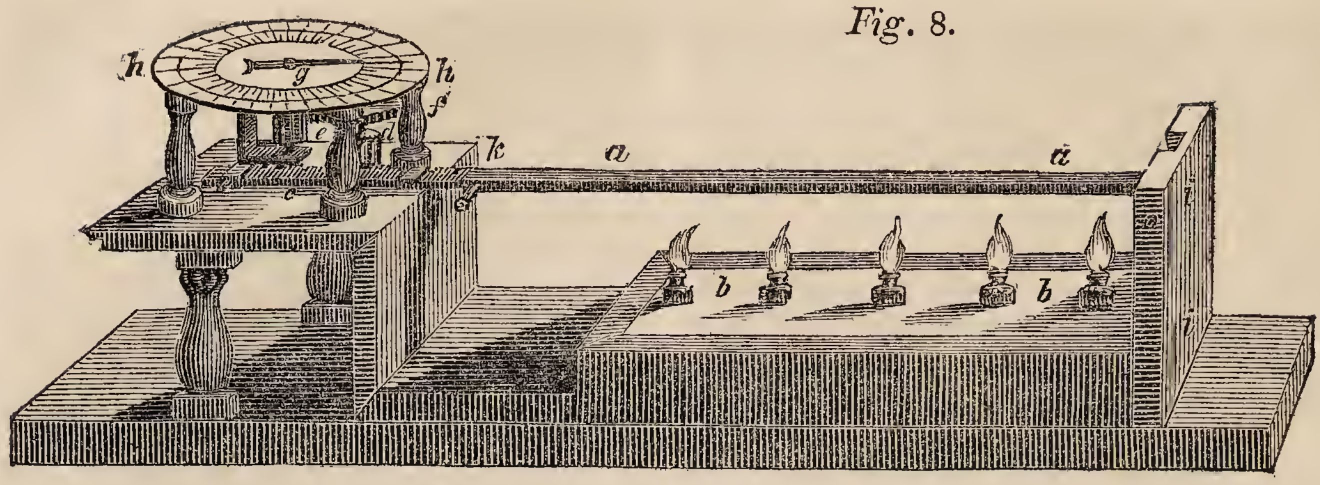
* Vol. lxiv. part i.

Fig. 8.



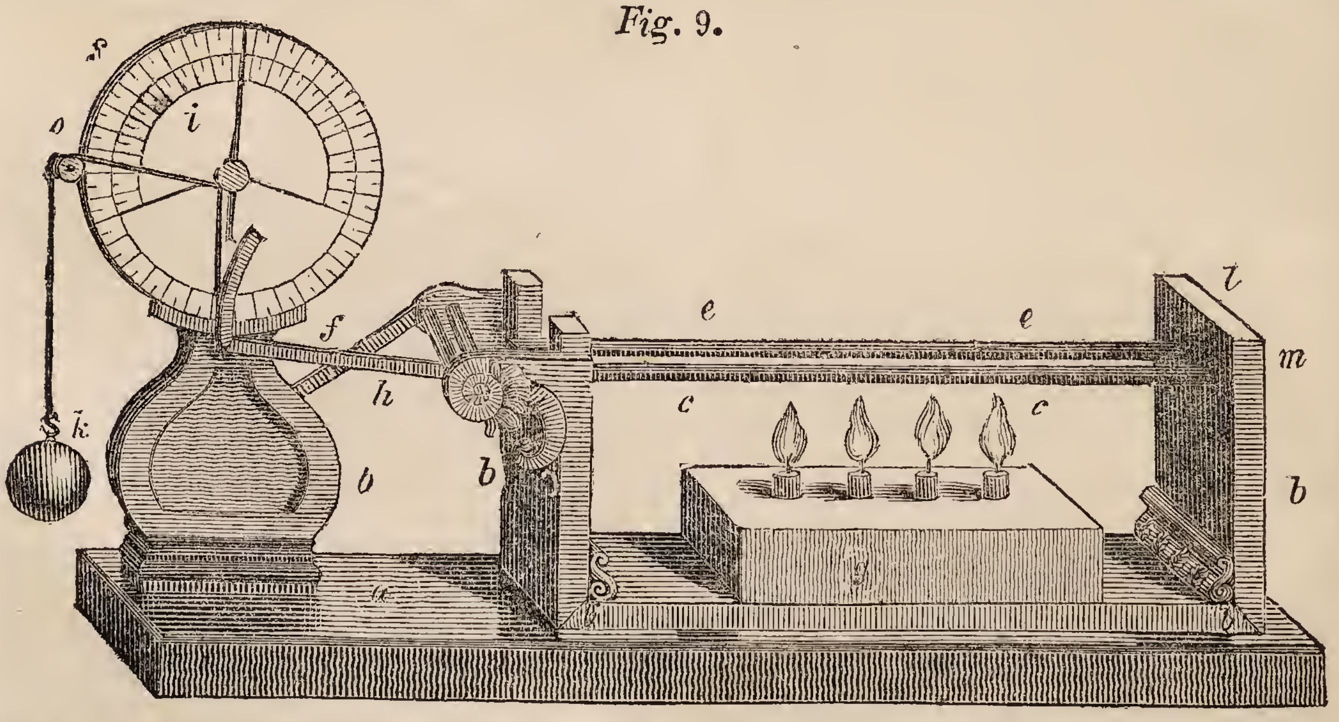
moves round the circle *h*, divided into 300°. The consequence of this arrangement is, that if the expansion of the metal were to push the rack *e* one inch forward, it would turn the pinion *d* $4\frac{1}{6}$ times round; and the wheel *f*, moving at the same rate, will carry the pinion *e*, and consequently the index $(10 \times 4\frac{1}{6}) = 41\frac{2}{3}$ round. Hence the index would have moved over $(41\frac{2}{3} \times 300)$, or 12,500 divisions of the scale; or each degree of the instrument is equivalent to $\frac{1}{12500}$ of an inch of the expansion of the prism *a*. Similar prisms of different metals applied in like manner to the instrument, enabled Musschenbroek to measure the different expansibility of steel, iron, copper, brass, and lead, with considerable accuracy:* but there is always some uncertainty in the movements of so many loosely connected teeth and pinions; and this pyrometer was improved by

2. Desaguliers,† who instead of prisms substituted cylinders, as wires are more easily procured than prisms of equal dimensions. For the first pinion he employed steel slightly roughened by the file in the same direction as the teeth. Thus a more equable motion was given to the instrument. The toothed wheel and second pinion were supplied by a wheel and roller, having grooves in their circumference for receiving a watch-chain, by which motion was communicated to the index. The dial plate was square and movable, in order to stretch the watch-chain as there might be occasion. A thin plate of rough steel $\frac{15}{100}$ inch wide, slightly convex towards the first roller, was substituted for the rack; and this last, which in Musschenbroek's pyrometer was made to travel lightly over a small bit of fine watch-spring, moved in Desaguliers over a well constructed friction wheel, or roller.

These changes improved the delicacy of the instrument very considerably; but it soon underwent other modifications.

3. The pyrometer of Mr. John Ellicot, of London, is seen in *fig.* 9, *a a* is a flat plate of brass screwed to a thick mahogany sole, to which the three brass uprights *b b b* are firmly attached.

Fig. 9.



The pyrometric pieces consist of two metallic bars: the flat one *c c* is of steel, and is that by which the expansions of all the other metals are to be compared together. Its extremity to the right passes through a hole in the upright, and is fixed to a spring which may be tightened by the screw *m*. Its other extremity is free, and presses against a *snail* on the axis of the lever *f*. The other bar *e e* is a prism of any metal, the right end of which rests on the end of the screw *l*, while its other bears on a snail on the axis of the lever *h*. When the bars are expanded by the heat of the spirit lamp *g*, they move the levers, to each of which is attached a slender watch-chain: the chain from the lever *f* passes round a pulley $\frac{1}{4}$ inch in diameter, fixed on the axis round which the inner graduated circle *i* of the dial moves; the chain from the lever *h* passes round a similar pulley on the axis of the index, as seen in the figure; and the expansions of this bar are marked by the index on the fixed outer circle. Both pulleys have a thread wrapped round them in a contrary direction on each, and then passing over the pulleys at *o* to the weight *k*, which acts as a counterbalance to bring back the index and movable circle as the bars cool. The index and circle are both adjusted to the beginning of their scales by means of the screws *l*, *m*, at the commencement of each experiment; and when the temperature applied expands the standard bar to a given degree, as indicated on the inner circle, the index will show on the outer circle the relative expansibility of whatever metal is applied to the instrument at *e e*.*

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*Tentam Acad. del Cimento.

† Desagulier's Experimental Philosophy, i. 421.

This instrument was chiefly intended by its ingenious inventor, a chronometer-maker by profession, for ascertaining the relative expansion of the metals usually employed in the construction of pendulums; an important object, for which many of the best pyrometers have been devised.

In this instrument the dial is about three inches in diameter; the levers two inches and a half in length, and the proportions of the several parts such that the expansion of $\frac{1}{20}$ inch in the bar will move the index wholly round the circle; or each degree will mark the $\frac{1}{7200}$ of an inch in the lengthening of the bar. From the mean of numerous experiments, Ellicot ascertained the following to be the relative expansions of seven metals:—

Steel.	Iron.	Gold.	Copper.	Brass.	Silver.	Lead.
56	60	73	89	95	103	149,*

which is more nearly in the ratio of the *conducting power* of the different metals, than of any other of their physical properties.

4. In the 44th volume of the *Philosophical Transactions* is a description of another pyrometer by Dr. Cromwell Mortimer, which, though less accurate and convenient than Ellicot's, is worthy of notice, especially as it may be employed to show the alterations of atmospheric temperature.

Fig. 10. A.

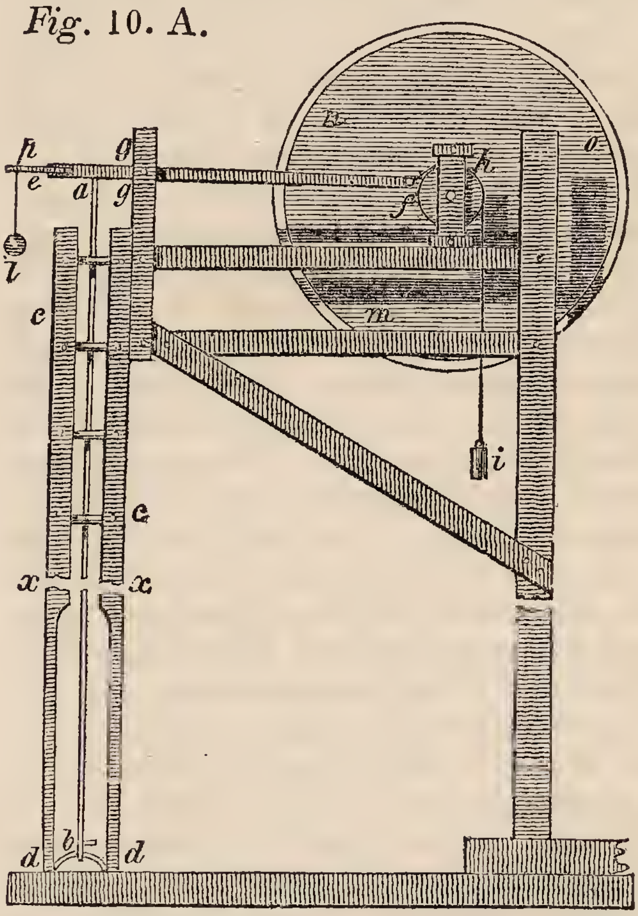
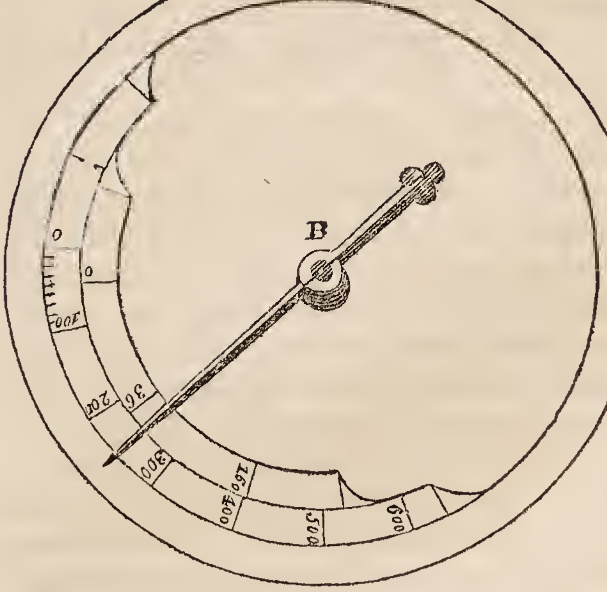


Fig. 10. B.



a, *b*, *fig*. 10, is a round rod of brass or steel, $\frac{1}{4}$ inch in diameter and three feet long, its upper extremity terminating in a hardened steel point one inch more in length, and entering a hole in a steel plate on the under side of the lever *e*, while its lower end rests on a point attached to the metallic plate at *d*. *c*, *d* are plates of iron joined at *d*, and at different other points, as in the figure: at *x*, *x* they are turned half round, to allow the application of heated bodies, as sand or water, to the bar, which is immersed in the heated bodies to a certain mark as at *b*. In the original instrument this mark was at $1\frac{1}{2}$ inch from the bottom: *e*, *f* is a lever moving round an axis in *g*. A string from the end of its longest arm passes twice round the pulley *h*, and is kept tight by a weight *i* of $\frac{1}{2}$lb., while there is another weight *l*, at the short arm of the lever, sufficient to counterbalance the weight of the longer arm, and to keep the point *a* in close contact with the lever. *m*, *n*, *o*, a dial, of which the face is seen at B, graduated to correspond to Fahrenheit's and Reaumur's degrees, which are indicated by an index fixed on the axis of the pulley *h*. The frame of the instrument is of oak. The lever from p to $a = 4$ inches; from a to $g = 1.5$ inch; from g to $f = 12$ inches; the pulley = 0.5 inch; the dial = 11 inches in diameter. In the original the melting point of different substances is indicated by their chemical signs in the outer circle of the dial.

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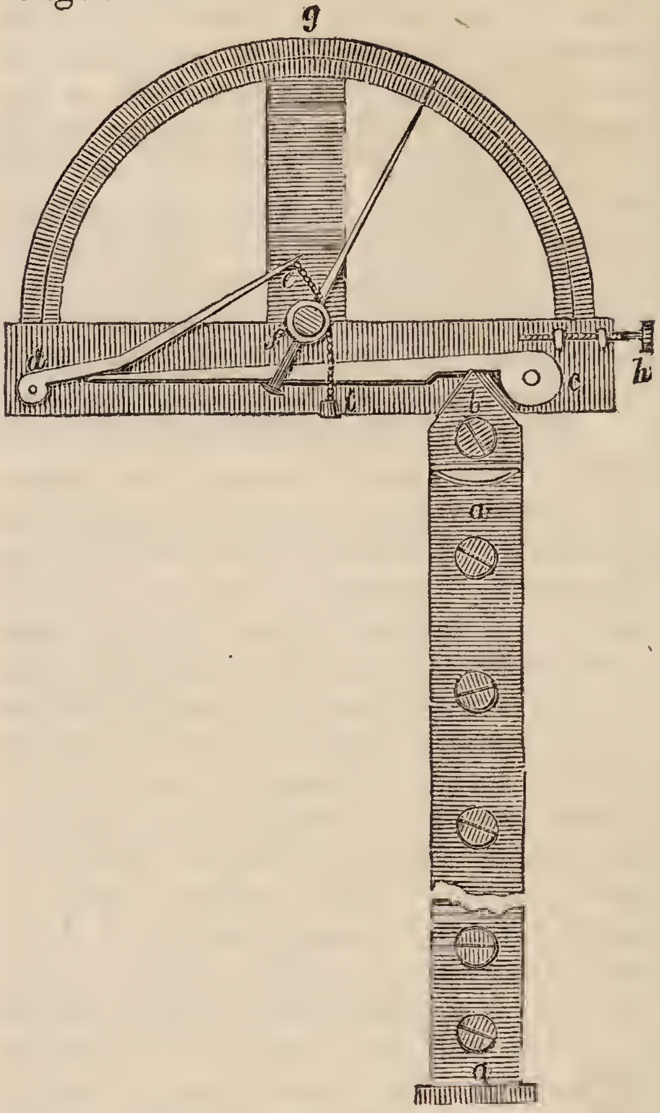
* This description is taken from an original instrument now before the author.

* Phil. Trans. vol. xxxix. p. 297.

This instrument appears to have been of considerable delicacy, and to have marked minute changes of atmospheric temperature very readily: but the size is inconvenient; and it must now be regarded rather as an instrument of curiosity than utility.

5. The pyrometer, figured 11, the invention of Mr. Froteringham, a Lincoln grazier, combines simplicity with considerable delicacy. It was also intended to indicate the changes of atmospheric temperature. *a*, *a* is a bar of iron four feet long and $1\frac{1}{4}$ inch wide, having a polished brass surface screwed to it with steel screws, which are fitted to short slips in the brass that allow the expansion of the iron bar, without that of the brass ornamental surface, to affect the hardened steel apex *b*. This apex moves the lever *c*, which raises the lever *d*; both turning on well made central disks. A chain from the extremity of the lever *d* is lapped twice round the pulley *f* on the axis of the index, which moves round a graduated circle *g*. The counterpoise *i* brings back the index as the levers fall. The screw *h* is for adjusting the index to the beginning of the scale. It is very obvious that such an instrument would be capable of showing the expansions of the bar in proportion to the difference between the arms of the levers; and, it is said, that the original instrument, in the library of a philosophical society at Spalding, indicated the changes of the heat of the weather with great precision.*

Fig. 11.



6. All these instruments, however, yield in accuracy to the invention of the celebrated Smeaton, which is described in the *Philosophical Transactions*.†

In this instrument the expansions of the metallic bars, heated by water, are measured by means of a micrometer screw; a principle which had been before employed by the great chronometer-maker Graham, for the adjustment of the rods of a pendulum.

From the principle of its construction, this instrument is called the Micrometer-Pyrometer, *fig*. 12.

The basis of this instrument *a*, *b*, *c*, *d* is of solid brass, which was chosen as

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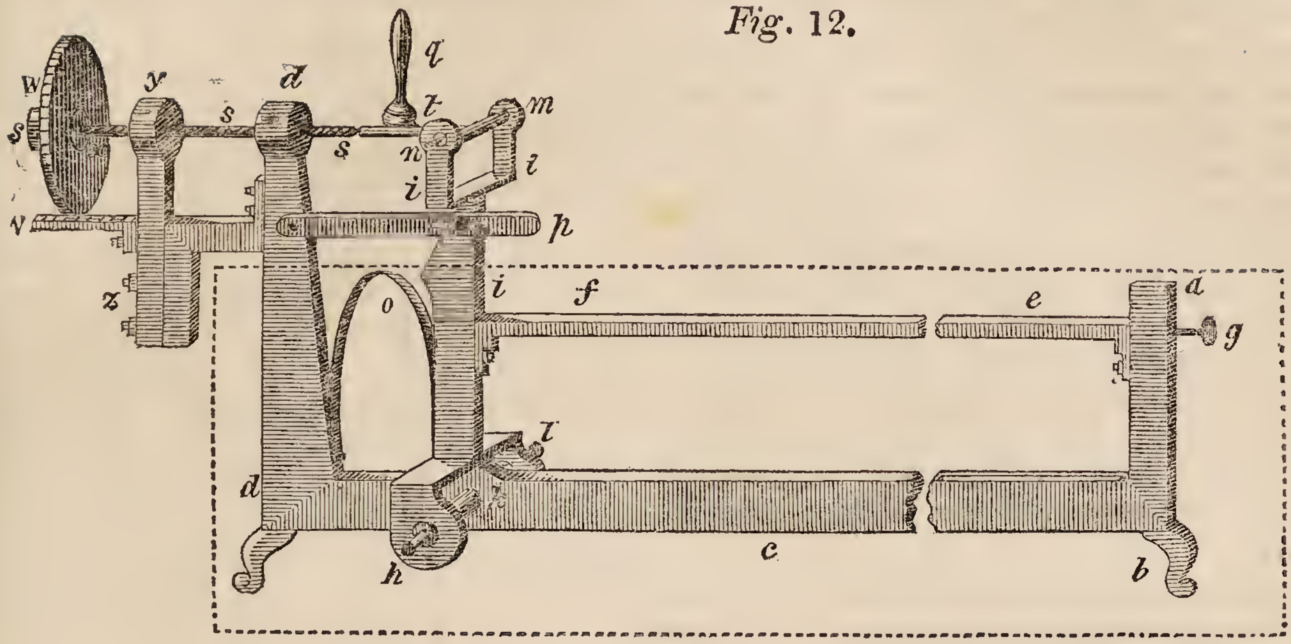
* Phil. Trans. vol. xlv. p. 125.

† Phil. Trans. vol. xlviii. p. 487.

Fig. 12.



of a mean expansibility among the metals. *e f* is the bar to be measured, resting on two notches, one attached to the fixed upright *a b*, and the other to the principal lever *h i*. *k* is a strong arbor fixed to the basis, and intended to receive the ends of two screws *h*, *l*, upon which the principal lever *h*, *i* turns; *o* is a slender steel spring intended to press the lever against the extremity of the bar; and *p* is a check-rod to support the lever, when the bar is removed. *t* is called the *feeler;* it is in the form of the letter T, and is suspended freely, but without shake, between the points of the screws *m*, *n*. *q* is the handle of the feeler, which is movable on a loose joint, so that the feeler may be moved by the handle without being irregularly affected by the pressure of the hand. The principal part of the instrument is *s*, the micrometer screw, and *w* the graduated circle or index-plate fixed on the screw, which indicates the revolutions of the screw on the index *v*. The micrometer screw passes through two solid heads perforated by a corresponding screw; the piece *y z* is made somewhat springy, and tends to draw the micrometer screw backward from *d;* by which its threads press uniformly against the corresponding threads in the holes, and keep the motion equable and easy.

When the instrument is used, its basis and the bar are immersed in a tin vessel containing water, as marked by the dotted line, which is heated by seven lamps applied below. The vessel is provided with a cover; and a delicate mercurial thermometer is suspended in the water, for regulating and ascertaining the temperature employed, which is not intended to exceed that of boiling water.

The expansion of the bar presses the lever and feeler towards the end of the micrometer screw, which, as well as the extremity of the feeler, is tipt with hardened steel. The handle *q* is laid hold of, and by it the feeler is moved up and down, while the screw is turned, until its steel point comes in contact with the end of the screw. Mr. Smeaton found that he could judge of that contact more accurately by the *ear*, than by the *eye* or the *touch*.

The turns of the index-plate counted by its edge and the divisions of the index, show the expansion of the bar; and its length when cool may be found in the same manner, either before or after the experiment above described. In this instrument the bar acts against the centre of a lever of the *second order*, the fulcrum of which is in the basis; and when both are expanded, the free extremity of the lever moves through a space double of the difference between the expansion of the bar and of the basis: hence, when we know the length of the lever from its axis to the point of suspension of the feeler, the distance from that axis to the point of contact of the bar, the number of threads of the micrometer screw in an inch, and the number of degrees on the circumference of the index-plate, we can compute the value of these degrees in fractions of an inch. In the original pyrometer the following were the proportions:

	Inches.
From axis of lever to point of suspension	5.875
—— fulcrum to point of contact	2.895
Length of 70 threads of the screw	2.455

Division of index-plate 100°.

Hence the value of each division of the index-plate will = $\frac{10}{5786\frac{1}{3}}$ds of an inch; and as, when the instrument was well adjusted, the difference of contact was very perceptible when the screw was moved through $\frac{1}{4}$ of a division, the $\frac{1}{23145}$th of an inch of expansion was determinable by this pyrometer, with which Mr. Smeaton ascertained the expansibility of many solids.

The following table is the result of his experiments, showing in 10,000dths of an inch the expansion of rods of different kinds of matter, in passing from the freezing to the boiling point of water.

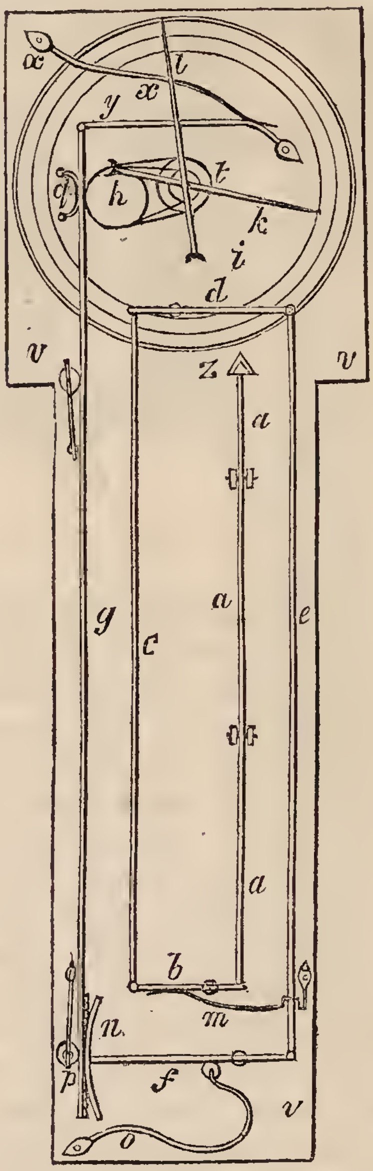
White glass barometer tube . .	100
Martial regulus of antimony . .	130
Bistered steel	138
Hard steel	147
Iron	151
Bismuth	167
Copper hammered	204
Alloy, 8 copper, and 1 tin . . .	218
Cast brass	225
Alloy, brass 16, tin 1	229
Brass wire	232
Telescope speculum metal . . .	232
Alloy, 2 brass, 1 zinc	247
Fine pewter	274
Grain tin	298
Soft solder, 2 lead, 1 tin	301
Alloy, 8 zinc, 1 tin, slightly hammered	323
Lead	344
Zinc	353
Zinc hammered out 1 inch per foot	373

These experiments correspond as nearly with the results obtained by Ellicot, as the difference of the instruments admit. They introduced a precision hitherto unknown in the law of expansion of solid bodies; and are still quoted with approbation in those nice disquisitions which have paved the way to the perfection of *horology*, and the modern refinements in *geodesical* operations, while they have extended our knowledge of the effects of heat.

7. The *metalline thermometer* of Mr. Keane Fitzgerald comes next in order of time; but it is chiefly applicable to mark the alterations of atmospheric temperature. Its general construction will be readily learnt from *fig*. 13.*

The basis of the instrument is a piece of well seasoned deal, on which a system of levers is fixed; *a a* is the pyrometric bar, 2 feet long, the upper extremity of which bears against the fulcrum *z*, while its other end rests on a small hemisphere of metal on the short arm of the lever *b*. The long arm of this lever is $2\frac{1}{2}$ times as long as the other; *b* is joined by a pivot to the rod *c*, 2 feet 2 inches in length, which bears against the short arm of *d*, and the long arm of *d* is $2\frac{3}{4}$ times as long as the former. The rod *e* is 2 feet 4 inches long, and is jointed to *f*, as in the figure. The long arm of *f* is 4 times the length of its short arm, and terminates in a slender arch-head, which is attached to the lower end of the rod *g* by a watch-chain, as in the figure. This last rod is 3 feet long, and is kept perpendicular by sliding between two friction rollers *p*, *v*, its connection with the arch-head, its suspension from the lever *y*, and its friction on the pulley *h*. The weight of the levers, &c. is counterbalanced by the springs *m* and *o*, and the spring of *y* is nearly neutralized by the pressure of *x*. The pulley *h* is fixed at 2 feet 6 inches from the lower end of *g*, and is 3 inches in diameter. Two cords fixed to the spring *q*, pass twice round

Fig. 13.



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* Phil. Trans. vol. li. p. 523.

the pulley *h* in different ways, and thence go over the pullies at *t*, respectively, 1 inch and $\frac{1}{4}$ inch in diameter. These last are put on the common axes of the indices *k*, *l*, in the same manner as the hands of a clock. The face of the dial is 12 inches in diameter, and from the construction, the index *l* ranges 48 times, and the index *k* 12 times as much as the bar *g*. The dial has on it three circular scales; the inner is divided into 240°, corresponding to those of Fahrenheit's thermometer; the middle is divided into 360°; and the outer into 1080 parts, marking 18 for each degree of the thermometer, and 12 for each degree of the circle.

This instrument may be used as a pyrometer in low temperatures; for the bar *a* is removable; and from the construction, each division of the outer circle is equivalent to an expansion of $\frac{1}{73840}$th of the bar.

Used in this way, Mr. Fitzgerald informs us that the dilatations of metallic bars 2 feet long, at the same temperature, were as follows:

		Divisions.
Spelter or zinc	=	1570
Zinc 18, copper 2 parts . .	=	1550
Brass	=	1120
Iron	=	785
Steel	=	695

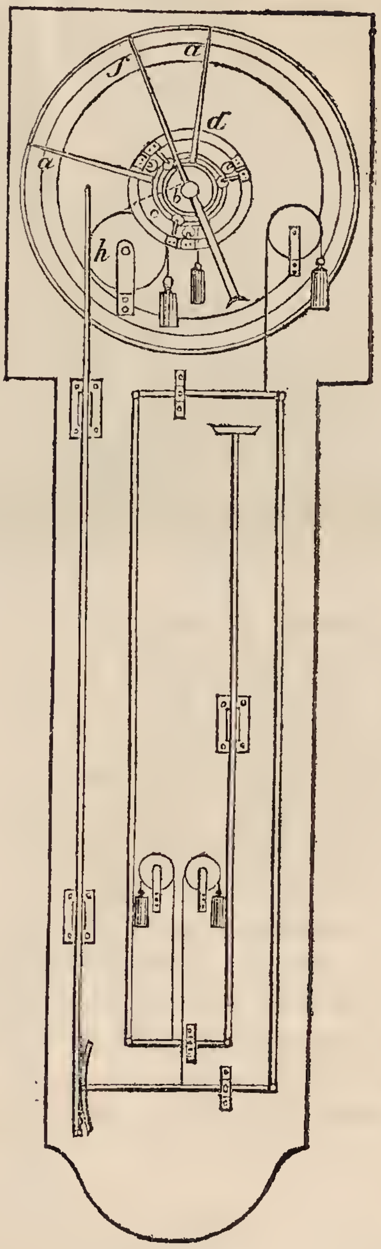
which agrees pretty well with the experiments of Smeaton and Ellicot.

When used as a thermometer the index *k* marks 74 divisions in passing from the usual extremes of temperature in our climate, and 212 divisions from freezing to boiling water.

Mr. Fitzgerald experienced some difficulty in proportioning the strength of the springs to the weight sustained by the levers, and he improved the instrument by the adoption of pulleys and counterpoise weights, as in *fig*. 14, which he ingeniously converted into a register thermometer, by adapting two light index hands *a*, *a*, fixed to two brass circles moving between friction wheels, attached to a fixed circle *d*. They were so nicely fitted as to move readily by a weight of 8 grains hung on them. These hands are moved in opposite directions, by a small stud in the under surface of the index *f*, which receives its motion from a cord passing from the pulley *h* round a small wheel on its axis.

This alteration of the instrument was intended only to note the changes of the atmosphere, which it seems to have done with much delicacy; for it had a range of 72 inches from the common changes of the heat of the weather in London; and it would show an alteration amounting to 50 or 60 degrees of its scale, when the pyrometric bar of the instrument was five or six times breathed upon.*

Fig. 14.



8. In Ferguson's *Lectures* two pyrometers, the invention of that great self-taught mechanician, are described.

Fig. 15 was merely intended to exhibit to his audience the expansions of bodies by heat, yet is worthy of notice.

a a, a mahogany board, on which are fixed four brass studs; of these *b* supports a screw for adjusting the pyrometric bar *f*, which rests in notches in the studs *c d*. The extremity of the bar presses against the crooked lever *g*, which acts on the index *i i*; the stud *e* holds the spring *h*, which brings back the index when the bar cools. The lever *g* (of the second order) has the portion between the point of contact of the

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* Phil. Trans. vol. lii. p. 146.

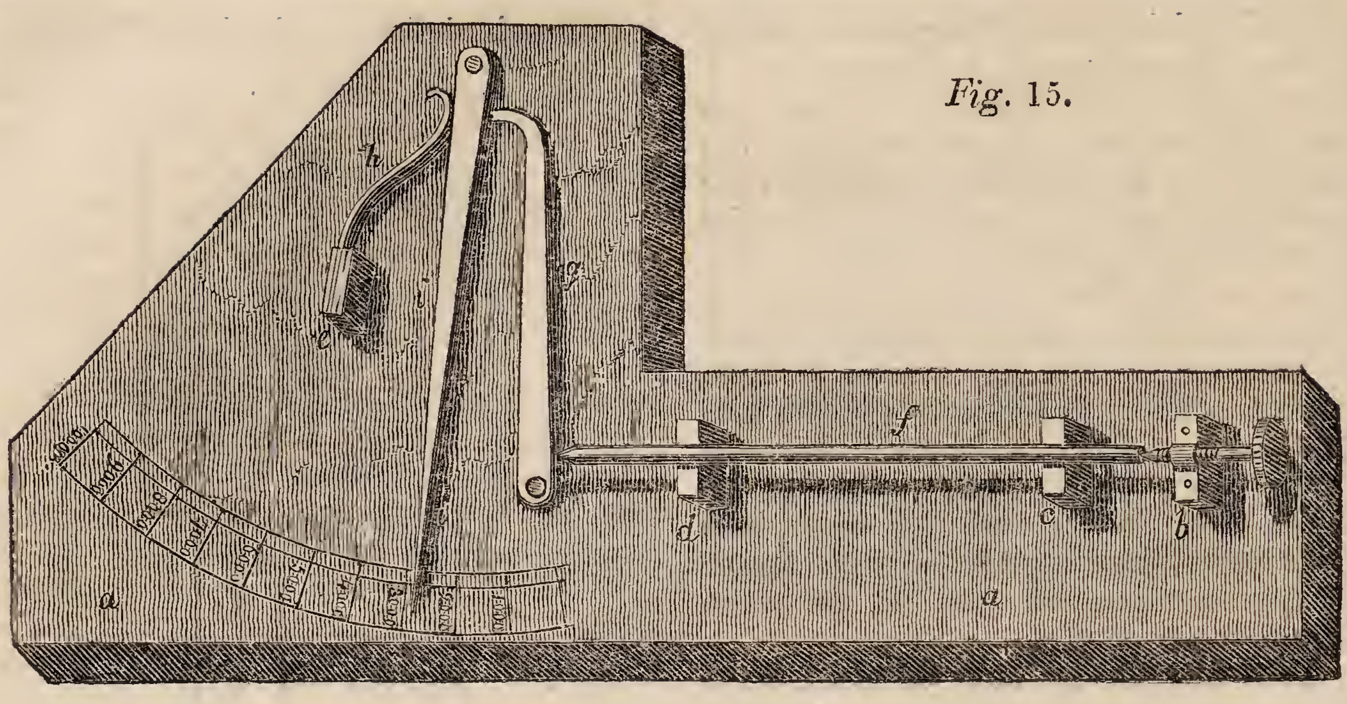


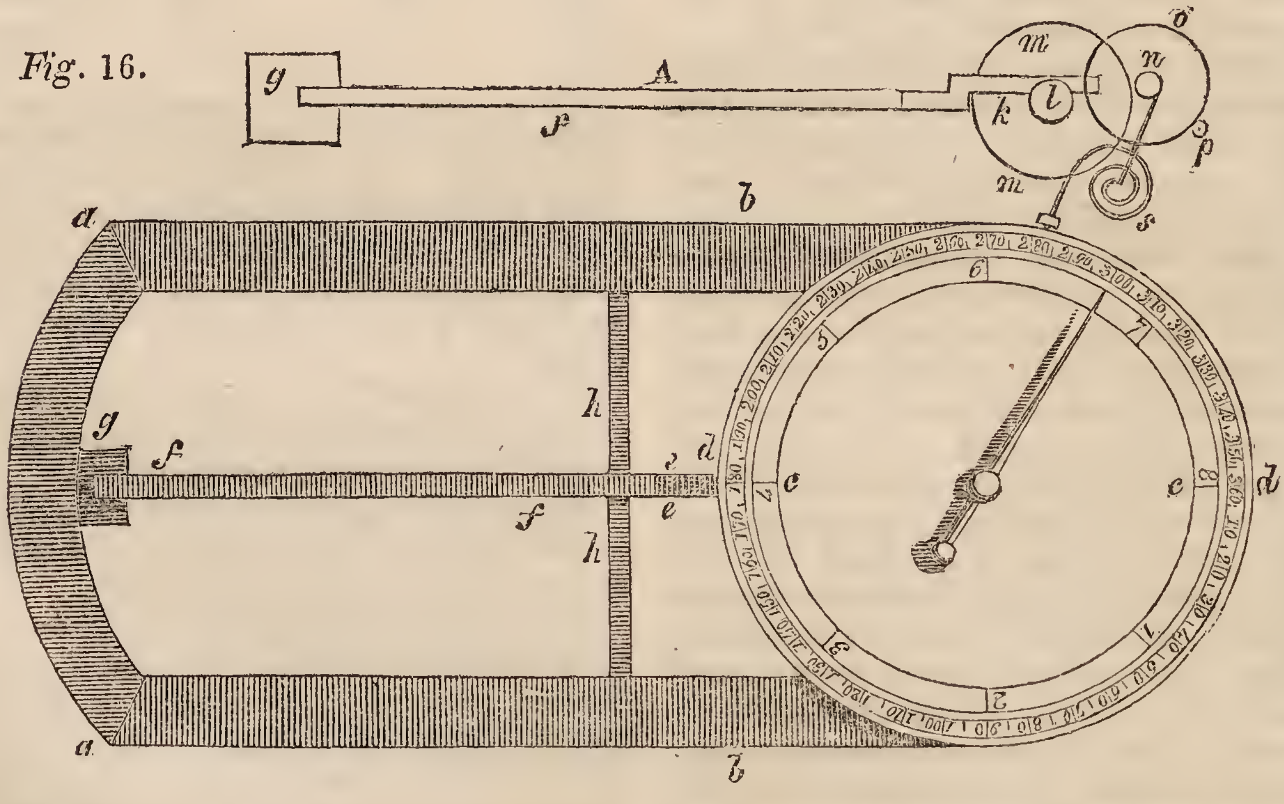
Fig. 15.

bar, and where it touches the index 20 times as long as the space between the point of the bar and its fulcrum; and the space between the end of the lever and the free end of the index is just 20 times the length of that between the point of the lever and the axis of the index; hence, when the bar expands $\frac{1}{400}$th of an inch, the point of the index will have moved over (20 × 20) = 400 times as much space, or one inch; or if the bar expand $\frac{1}{4000}$th of an inch, the index will move $\frac{1}{10}$th.

The scale is divided into inches and tenths; and the mere friction of the bar *f*, which is removable at pleasure, with a piece of flannel till it becomes sensibly warm, will be sufficient to show variations of the index. Ferguson states that it gave the following results:—with bars of iron and steel, 3; copper, 4½; brass, 5; tin, 6; lead, 7.

9. In the supplement to his lectures there is however a much more delicate pyrometer described, (*fig.* 16,) which will show the expansion of a bar of metal to the $\frac{1}{45000}$th of an inch, or even to the 90,000th.

Fig. 16.



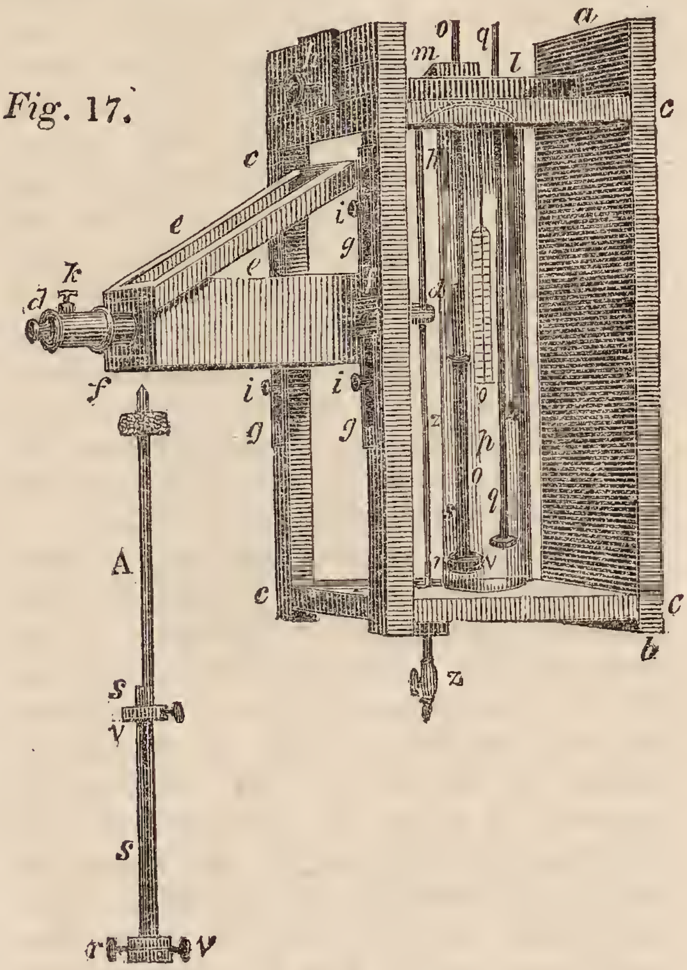
The frame *a b* is of mahogany, supported on short pillars, so as to admit a lamp under it for heating the bar *f;* one end of which lies in a cavity in the piece of metal *g*, and the other, after passing on a friction wheel over the cross-bar *h h*, presses against the short lever *e e.* The manner in which this short lever acts on the index is seen in the adjoining diagram A, where *k* is the short lever that moves under the dial *d* between friction wheels. On the side of *k* are 15 teeth in the space of one inch, which play in the twelve leaves of the pinion *l* on the axis of the wheel *m m*. This wheel has round its circumference 100 teeth, which work in the ten leaves of the pinion *n*, on the axis of the wheel *o* of 100 teeth, that gives motion to the pinion *p* of ten leaves, on the axis of which the index is fixed.

As the wheels *m* and *n* have each 100 teeth, and the pinions *n* and *p* ten leaves, it is obvious that when the wheel *m* has made one revolution the pinion *p*, and of course the index, will have made 100 revolutions; as the pinion *l* has twelve leaves, and the bar *i k* has fifteen teeth to one inch (equivalent to $12\frac{1}{4}$) it is obvious that while *i k* moves one inch the pinion *p* will have moved $100 + \frac{1}{4}$ or 125 times round, and the index would at the same time be carried 125 times round the circle *d d*. This circle is graduated into 360 degrees, and being eleven inches in diameter, it is subdivided into half degrees. Hence each degree of that circle will be equivalent to an expansion of $125 \times 360 = \frac{1}{45000}$th of an inch of expansion in the bar *f;* and as the half degrees can readily be distinguished on the dial, the instrument will show expansions only amounting to $\frac{1}{90000}$th part of an inch. A silk thread is several times wound round the axis of *n* and passes to the slender spring *s*, which keeps the teeth of the pinions and wheels in close contact, and pulls back the train of wheels when the cooling of the bar *f* allows the short bar *i k* to recede.

The inner circle of the dial is divided into eight parts, corresponding to so many thousandths of an inch in the expansion of the bar *f*, or $\frac{1}{1000}$th of an inch for each degree of the outer circle over which the index has moved. Bars of different metals laid in *g* for a given time, and exposed to the same lamp, afford an indication of their relative expansibility; and to ensure equality in the bars it is recommended to have them *wire drawn* through the same hole. There is, however, in this instrument no accurate measure of the temperature applied to each bar; and, notwithstanding the delicacy of the movement, it seems inferior to Ellicot's pyrometer, as it wants a constant and uniform standard by which to compare the expansions in each separate experiment.

10. A new method of ascertaining the expansibility of different substances was suggested by the late Mr. Jesse Ramsden, and on his hint it was attempted by the ingenious and indefatigable De Luc, whose researches on the barometer gave this subject an increased interest to his mind. The object in view was to determine the relative expansion of solids by observation with a microscope furnished with a micrometer. The *microscopic pyrometer* of De Luc is seen in *fig.* 17, where *a b* represents a strong board of even-grained deal; to which the frame *c c c c* is firmly joined, that when *a b* is suspended vertically from a strong post, the front of the instrument bearing the microscope *d d* is towards the operator.

Fig. 17.



The microscope is securely united to the frame by the braces *e e* and the cross-bar *f;* and the whole of this part of the apparatus can be moved up or down by the slides *g g*, which fit so tightly on their centres as to require slight blows with a hammer on the frame to cause them to move down, and may be further tightened by the screws *i i i*. The microscope is kept horizontal by the cross-pieces, and by an inner sliding frame not seen in the figure. The microscope is so adjusted that an object is distinctly seen when a full inch from its lens; and it is furnished with a micrometer, movable by *k*, for ascertaining the expansions of the rods subjected to experiment. A piece of thick deal *l* is seen at the top of the frame lying horizontally from a groove cut in the board *a b* to the front of the frame; this is movable by means of the screw *h*, and is perforated by a piece of cork *m* firmly driven into it until level with its lower surface. The cork is then pierced vertically to receive the glass rod *o o o*, which is thus suspended in a thin cylindrical glass jar *p*, 21 inches high, and 4 inches in diameter, filled with water. The glass rod is the standard of comparison, and to it is attached a rod *s s*, of the metal to be tried, by two connected rings *r r*, which are tightened on the rods by two screws. Another set of rings *v* is applied higher up; but through this the rod *s s* freely slides, while it firmly clips the glass rod by means of a screw. A delicate thermometer hangs in the centre of the jar *p* to note the temperature of the water, which is occasionally agitated to secure uniformity of temperature by the rod *q q*. A syphon *z* to draw off the water completes the apparatus.*

In using this pyrometer, warm water is poured into the jar, in order to heat the rods; the rods are adjusted to the focus of the microscope by the screw *n;* the thermometer gives the degree of heat employed; and, by means of marks on the bars, their relative expansion is given in divisions of the micrometer, the value of which is known by previous experiments. The connection of the rods is more distinctly seen at A; but it is unnecessary to give a more minute description of an instrument which has been superseded by the more accurate and more elegant contrivance of Ramsden, so elaborately detailed by General Roy, to which we shall presently advert.

11. From experiments with this instrument, De Luc ingeniously applied a correction to the scale of barometers for temperature, by what, in the same paper, he calls "*metallic thermometers*." The scale of the barometer was fixed on a bar of metal of known expansibility, so as to raise the scale in exact proportion to the expansion of the mercury; and thus the mere inspection of the barometric scale will give the true height, without the trouble of applying the equation or formula of correction for temperature, as in ordinary observations.

12. We come now to certainly the most complex, but the most perfect of all contrivances for determining the relative expansions of solids, the *microscopic pyrometer of Ramsden*, contrived by that eminent artist, for determining with the utmost possible precision, the expansibility of the rods employed by General Roy, in the geodesical operations that are the foundation of the great trigonometrical survey of Britain. Fig. 18 contains plans and sections of this beautiful contrivance, and although we do not propose to enter into a minute detail of the different parts of the instrument, a general description will show, to those who have not considered such subjects, the nice precautions which are necessary to accuracy in like operations, while it explains its construction. Ramsden's pyrometer is attached to a strong and well joined deal table, or frame 5 feet long, 28 inches broad, and 42 inches high; of which an end elevation is seen *fig.* 18, B; the plan of its top will be best understood from an inspection of A. *a b* and *c d* are troughs of deal, (firmly screwed to the table) 3 inches in diameter, and a little longer than the frame; *a b* projects a little over the table, but *c d* is in a line with the frame, as may be seen at B. Each trough contains a cast iron prism, 1¼ inch on each side, firmly fixed in the troughs, at the ends *a* and *c*, by means of brass collars embracing the prisms, and tightened by screws as at G, while the ends *b* and *d* pass freely through loose collars, without any shake, when their dimensions are altered by temperature. The prism *a b* is called the *eye prism;* because it carries at each end the eye-pieces of the microscopes *l m n*, and *o p r;* which are figured on a larger scale at F and E. The other prism *c d* is called the *mark prism;* because it carries at one end the mark I, and at the other cross wires H; *e f* is a copper boiler 2¾ inches wide, and 3½ deep, rather shorter than the wooden troughs. The centre of the boiler, or rather of the object lens standing perpendicular to it, is 5.81 inches from the cross wires of the mark in *c d*, and 20.33 inches from the wire of the micrometer attached to the corresponding eye-piece. The boiler rests on five small rollers, seen in the enlarged section D. The boiler, like the troughs, has a cock to the right hand; and in the plan A, it is represented with a bar in it, to show the position of the rods to be tried. The water in the boiler is heated by the 12 spirit lamps *g g g g*, standing on four

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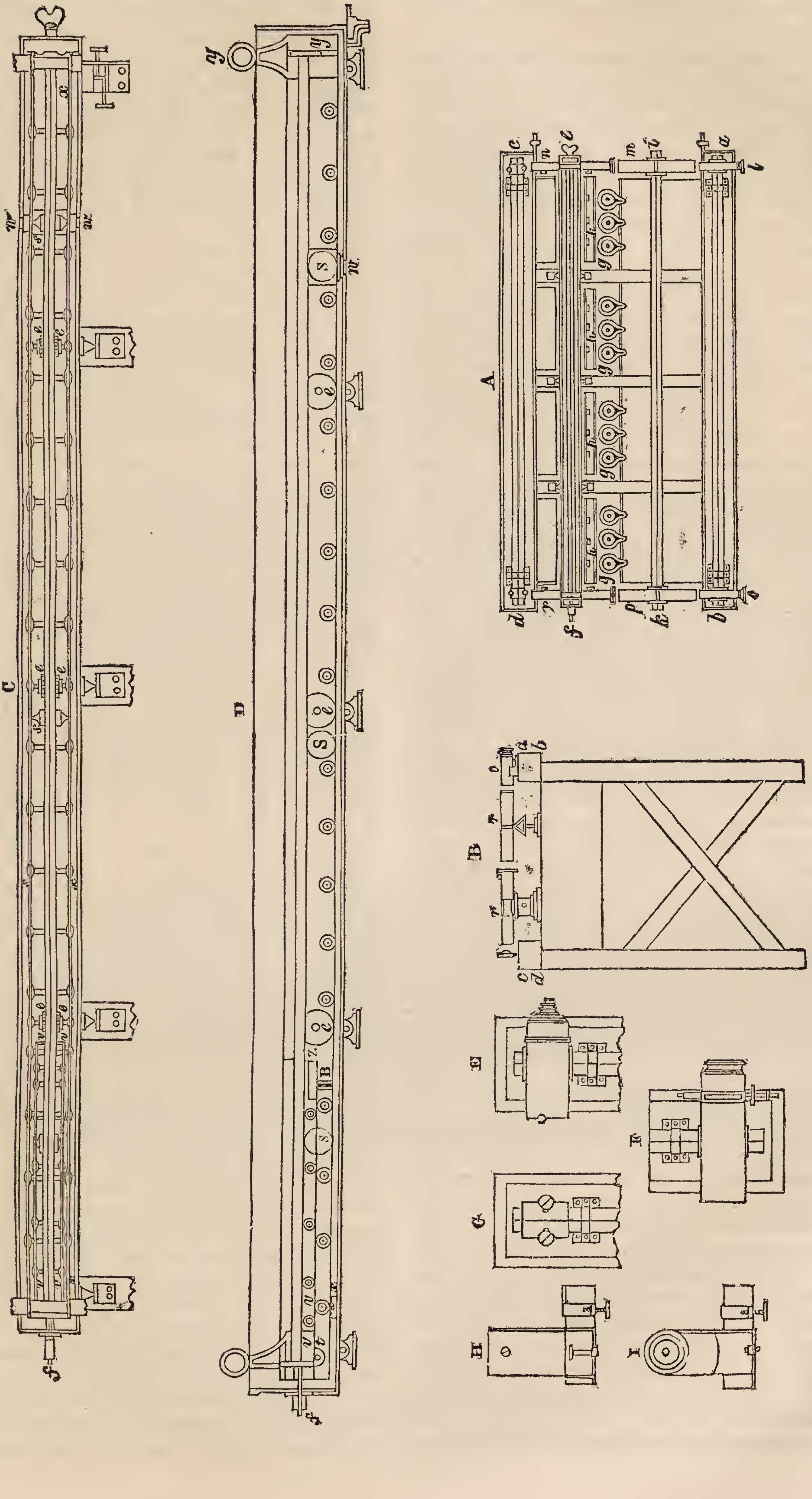
* Phil. Trans. vol. lxviii. part i. p. 437.

Fig. 18.



movable shelves, and showing only their handles *h h h h*, when under the boiler.

The boiler contains another essential part of the apparatus, *viz.* two brass slides, composed of two cheeks, kept at equal distances by cross bars as in C, where a prism or bar is represented as resting in the centre of the slides. The long slide reaches from microscope to microscope, and has its cheeks $1\frac{3}{4}$ inch deep. It is attached to the boiler only at the point *w*, and it rolls on the small roller *x*, near the left hand of D. The right hand end of the long slide is shut up by a piece of strong brass *y*, *y*, supporting two rings, for the part *n* of the fixed microscope. The short slide *v*, *v*, *v*, *v*, is only $14\frac{1}{2}$ inches long; its cheeks are $1\frac{1}{4}$ inch deep, kept parallel by braces, as seen in D. It moves within the long slide; and its outer end rests on the cylindrical surface of the last brace of the long slide, fitted to receive it, while a narrow longitudinal bar *z* moves freely in the notch of a bridge B, framed for it in the long slide. The outer end of the short slide is shut up by a similar piece of brass to that closing the opposite extremity of the long one. The bar or rod to be examined abuts against the piece of brass *y*, it rests on the three rollers *s s s*, 1 inch in diameter, and is kept in the centre of the slides by three milled nuts *e e e*, that screw up so as not to press too much on the sides of the bar. At *f* is a tube and wire moving through a collar of oiled leather, that by means of a helical spring presses a flat piece of metal attached to the wire, against the shut end of the short slide and rod to be measured, so as to keep the other extremity of the rod in contact with *y*. On the application of heat, the rod expands, and overcoming the slight resistance of the spring, carries before it the short slide, and with it the tube containing the object lens of the micrometer microscope *o*, *p*, *r*, a space proportional to the temperature applied; and it is this space, measured by the micrometer, that determines the numerical value of the expansion of the rod.

The microscope tubes are divided into three pieces, for the convenience of applying the instrument to measure rods shorter than five feet. For this purpose the central screening tube of the fixed microscope, supported on the mahogany prism *i k* by a collar, may be moved and *clamped* at any part of that prism; the eye-piece, in like manner, may be moved along the eye prism; but the object lens tube was left in the rings of the slide, and another lens of the same focus was clamped to the cheeks of the slide at suitable distances.

The standard prisms, during each experiment, were kept at the freezing temperature, by being surrounded with pounded ice. The microscopes were then accurately adjusted to the marks, by bringing the cross wires to bisect them, and until this was accomplished, the rod to be measured was also surrounded with ice. The lamps were then applied to the boiler, and the elongation of the rod, at the boiling heat, was ascertained by the micrometer attached to the microscope *o*, *p*, *r*. In these delicate investigations there were two observers, who simultaneously used both microscopes, lest any alteration had taken place in the fixed end of the rod; and to ensure accuracy, the experiments were twice at least repeated.

The value of the indications of the micrometer, on which so much depends, was previously thus ascertained:

The head of the micrometer screw = 0.9 inch in diameter, and was divided into fifty equal parts, each of which was reckoned two; and they were therefore numbered to 100. Fifty-five revolutions of the head were found equal to 0.77175 of an inch; it follows that there are 71.27 threads of the screw in one inch; and seven revolutions and *nearly* $\frac{12}{100}$ths move the micrometer wire $\frac{1}{10}$th of an inch; consequently $\frac{1}{100}$th of part of a revolution, or half a division of the head, will answer to a motion of something more than 0.00014th of an inch. Having found 7.13 revolutions equal to 0.1 inch at the wires, it is obvious that the number answering to 0.1 inch at the *mark* being also found and added to the former, their sum will give the measure of 0.1 inch at the object lens of the microscope *o*, *p*, *r*, or the space through which the free end of the rod has moved by the change of temperature. This last point was ascertained by experiment to be = 24.93 revolutions of the micrometer head; which being added to 7.13 = 32.06, "for the number of revolutions measuring a motion of 0.1 at the object lens, or an expansion of $\frac{1}{10}$th of an inch," or half a division of the micrometer head is equivalent to an expansion of the rod under examination of $\frac{1}{32000}$ of an inch; and $\frac{1}{4}$ of a division, which may readily be seen by the eye, $= \frac{1}{64000}$.

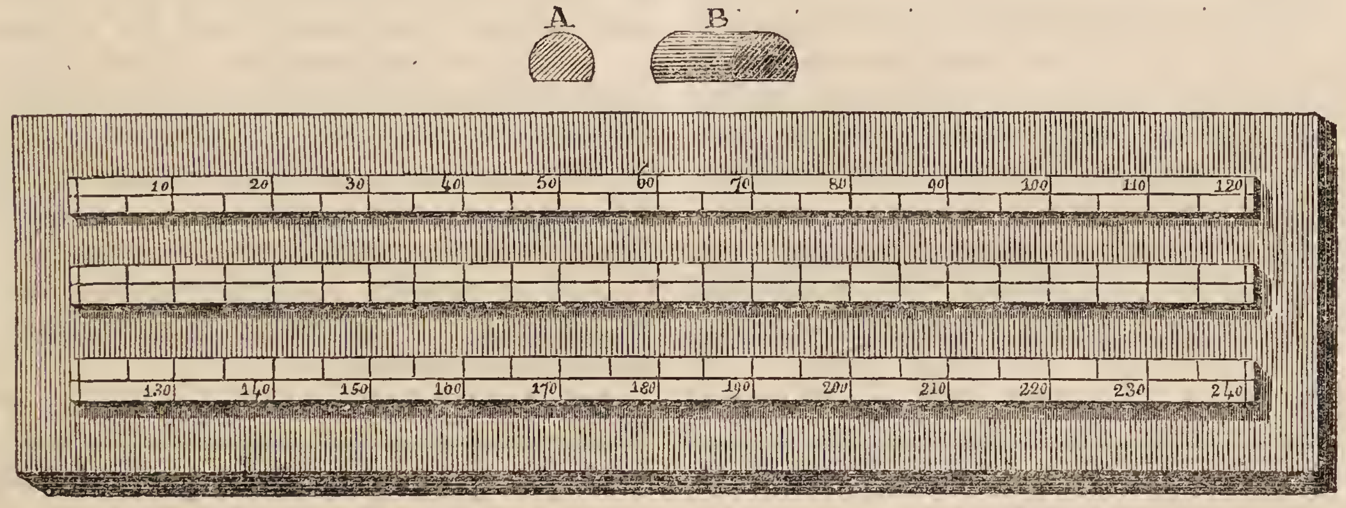
Such is the *microscopic pyrometer* of Ramsden; an instrument not indeed suitable for ordinary purposes, but admirably adapted for obtaining an accurate estimate of the comparative expansibility of different solids; an object of the highest importance, not only in bringing to perfection the delicate instruments required by the refinements of modern philosophical investigations, but essential to the perfection of different kinds of machinery in daily use, and even to a successful investigation of the laws and nature of heat itself. With this instrument Roy determined the expansion of the seven solids in the annexed table.

Expansion of

	By 180°. Revolutions. Parts.	By 1°. Parts.	Inch. on 5 feet.	Inch. on 1 foot.
Dutch brass	35.69 =	19 $\frac{83}{000}$ =	0.111323 =	0.0222646
English plate brass, a rod .	36.41 =	20 $\frac{23}{100}$ =	0.113568 =	0.0227136
Ditto, in the form of a trough	36.45 =	20 $\frac{25}{100}$ =	0.113693 =	0.0227386
Steel rod	22.02 =	12 $\frac{23}{100}$ =	0.068684 =	0.0137368
Cast iron prism	21.34 =	11 $\frac{86}{100}$ =	0.066563 =	0.0133126
Glass rod	15.54 =	8 $\frac{63}{100}$ =	0.048472 =	0.0096944
Ditto tube	14.93 =	8 $\frac{29}{100}$ =	0.046569 =	0.0046569

13. The instruments hitherto noticed are inapplicable to very high temperatures, or to ascertain the heat of closed fire-places; an object, in many processes in the arts, of the utmost importance. To supply this deficiency, our celebrated Wedgwood took advantage of the property which clay has of *contracting by heat*, and remaining afterwards in that state of contraction. This property is not, strictly speaking, an exception to the general law of expansion by increase of temperature: clay is not a homogeneous body, but a mechanical mixture of argil and silex, which by the influence of heat are brought into more intimate union, and therefore diminish in bulk; until a temperature sufficiently high to melt them, that is, to convert them into a homogeneous mass, is applied: after which the product obeys the general law of expansion by heat. Availing himself of this property, Mr. Wedgwood employed as *pyrometric pieces* cylinders of fine porcelain clay, slightly flattened on one side, as seen in A B, *fig.* 19,* formed by pressing the clay into an iron tube, and baked in a potter's furnace. It was found, after repeated trials, that the pieces of clay contracted more and more in an uniform ratio to the degree of heat communicated to them, and permanently retained this contraction; so that by applying them when cold to a scale, an indication of the degree of heat was obtained.

Fig. 19.



The scale employed by Wedgwood consisted of two brass rods ¼ inch square, and two feet in length, fixed on a brass plate *convergingly*, so that they were distant at one end just 0.5, and at the other 0.3 inch. For convenience the rods are usually divided and fixed as in the figure on the plate, forming two nearly parallel grooves.† With the above-stated convergence the whole

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[*] Phil. Trans. vol. lxxii. lxxiv. lxxvi.

[†] The degree of convergence being only one-tenth of an inch in a foot, is not perceptible in the figure.

groove is divided into inches and tenths, making 240 degrees in the whole scale; and the higher the temperature to which the pyrometric piece has been exposed, the further will it slide up the scale.

In order to compare his scale with Fahrenheit's mercurial thermometer, which cannot measure a temperature much beyond 600°, Mr. Wedgwood was compelled to make use of the *expansions* of a pyrometric piece of fine silver, applied to a gage on the same principle as that above described. By this, the expansions of the silver for 50° and 212° Fahrenheit were first noted; and then the silver and clay pyrometric pieces were compared at the same temperature. By such means Wedgwood estimated the value of each degree of his scale at 130° of Fahrenheit; and he reckoned that the 0° of his scale corresponded with the 1077°.5 of the common scale. On this principle comparative tables of the two thermometers have been constructed; but their accuracy depends on two circumstances which have not been determined to the satisfaction of the philosophic world. Clay being a heterogeneous mixture, it by no means follows that its contractions are equable at different temperatures; and even were this ascertained, there is great doubt how far the means employed by Wedgwood did accurately estimate the degree of Fahrenheit at which his scale commences.

There is still another serious objection to the general use of such an instrument. It occurred to the ingenious inventor, that different portions of clay would possess different degrees of contractibility; and he endeavoured to secure uniformity, to a certain extent, by laying in a large stock of Cornish clay, which he hoped would supply innumerable pyrometric pieces of the same quality. It was found, however, that spontaneous changes take place in such clay, which render its indications liable to variation at distant intervals; or pieces, now formed of the same clay, will not give the same indication with pieces baked several years ago. Attempts were made to remedy this inconvenience by forming a clay of uniform quality of fixed proportions of silex and alumine. Fine Cornish clay yielded, on analysis, two parts of silex and three of alumine; and such a mixture made into a paste with $\frac{2}{5}$ths their weight of water, has been recommended for the fabrication of pyrometric pieces. The method detailed by Wedgwood should then be followed in moulding them. The paste is first to be rammed into a metallic mould 0.6 inch wide, 0.4 deep, and 1 inch long: they should be dried in the air, and when quite desiccated, Wedgwood gaged them in another mould exactly 0.5 of an inch wide, and of the form given in the figure. Before they are baked they will, of course, just enter the widest end of the scale, resting at 0°. When contracted by baking to $\frac{1}{5}$th of their bulk, they will pass to the 120°; and when reduced to $\frac{2}{5}$ths, they would pass to the 240°, or the extremity of the scale; but Mr. Wedgwood never did obtain a higher temperature than 160°. From these proportions each degree of Wedgwood's scale is equivalent to a contraction of $\frac{1}{600}$th part of the pyrometric piece.

The difficulty of obtaining clay of an uniform quality, and not liable to spontaneous change, has lately given rise to a suggestion of employing pyrometric pieces formed of Chinese agalmatolite; a suggestion of Mr. Sivright of Meggetland, well worthy of attention.*

A more formidable objection was started by some foreign chemists to Wedgwood's scale; one, indeed, that would have overturned the theory of the instrument. It was alleged, that the effect of a long continued, or often repeated, exposure to even *inferior degrees of heat*, would cause contraction of the clay, after it had undergone the action of a higher temperature. This point has been examined with much care by Guyton de Morveau, who has shown, in his valuable essay,† the inaccuracy of this opinion; although he contends that Wedgwood has greatly erred in the attempts to convert his scale into degrees of Fahrenheit's thermometer, as we shall immediately notice.

On the whole, the pyrometer of Wedgwood is an instrument well adapted to the purposes of the potter, or to convey some idea of the relative heat of furnaces; but we cannot regard the determination of the celebrated inventor as giving even a tolerable approximation to relative degrees of high temperatures by other scales. As, however, Mr. Wedgwood's tables of temperature are often quoted, we shall here subjoin them, with the corresponding degrees of Fahrenheit, according to his calculation.

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* Edinb. Phil. Journal, vol. vi. p. 179.

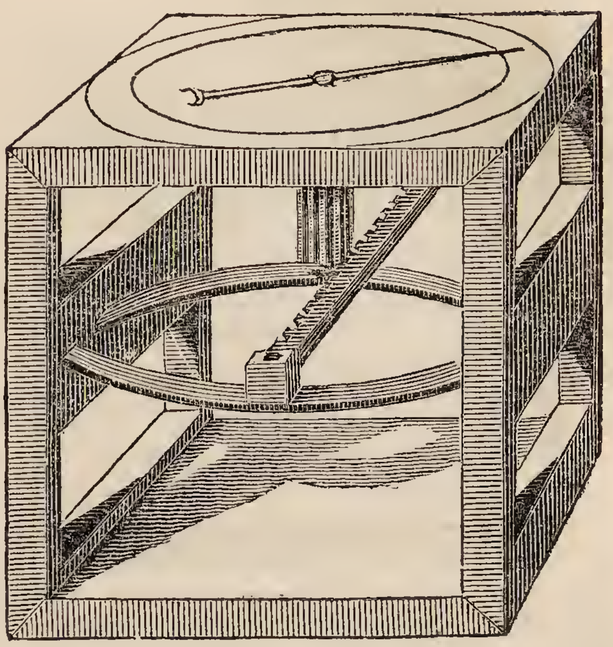
† Annales de Chimie, vol. lxxiv. lxxviii. xc.

	W.		F.
Red heat in full day-light	0°	=	1077°
Enamel heat	6	=	1857
Brass melts	21	=	3807
Swedish copper melts	27	=	4587
Fine silver melts	28	=	4717
Settling heat of flint glass	29	=	4847
Fine gold melts	32	=	5237
Delft ware baked	41	=	6407
Working heat of plate glass	57	=	8487
Flint glass furnace, low heat	70	=	10,177
Cream coloured ware baked	86	=	12,257
Welding heat of iron, least	90	=	12,777
Ditto ditto greatest	95	=	13,427
Stone ware, baked	102	=	14,337
Derby China vitrefies	112	=	15,637
Flint glass furnace, high heat	114	=	15,897
Inferior Chinese porcelain softened	120	=	16,677
Bow porcelain vitrified	121	=	16,807
Plate glass furnace, greatest heat	124	=	17,197
Smith's forge, greatest heat	125	=	17,327
Cast iron begins to melt	130	=	17,977
Bristol porcelain vitrifies	135	=	18,627
Hessian crucible melted	150	=	20,577
Cast iron thoroughly melted	150	=	20,577
Chinese porcelain, best sort softened	156	=	21,557
Greatest heat of an air furnace eight inches in diameter; deduct soften Nankeen porcelain at all	160	=	21,877
Extremity of Wedgwood's scale	240	=	32,277

These results are rendered doubtful by the causes already noticed; and the experiments of Morveau and Daniell with pyrometers of platina lead to very different results.

14. The metallic thermometer of Regnier is described in a report of the French Institute for 1798.* The inventor had remarked, that when a thin metallic rule, resting on a table, is raised by the middle, it forms a segmental arc of which the *versed sine*, that is a line perpendicular to the chord, drawn to the centre of the arc, is twelve times longer than the space through which the extremity of the bar has moved; and, on this principle, he proposed to construct an instrument for noting variations of atmospheric temperature. The small models which he exhibited answered perfectly; but his intention was, to apply his invention to instruments on a larger scale for public use.

Fig. 20.



The instrument consists of two plates of yellow copper, two mètres long, fixed in an iron frame, in a bent position, with their concave surfaces toward each other, as in the sketch, *fig.* 20. On one is fixed a pinion of eight leaves, on an axis, the end of which supports an index to mark the temperature. To the centre of the other plate is attached a toothed rack, in the position of the versed sine of the curve, playing in the leaves of the pinion. When the plates are cooled they approach each other, when heated their centres recede; and the only circumstances of consequence in the position of the bars or plates are, that they should be at some little distance from each other, and so bent that they cannot become parallel by any reduction of temperature to which they may be exposed. Regnier found, that two such bars, of two mètres in length, by a change of temperature equal to 60 centesimal degrees, changed the relative position of their centres, or *had a play* equal to 65 millimètres; but the correction for the expansion of the iron frame reduces this by $\frac{3}{5}$; so that there remains about 26 millimètres for the real play of the centres of the bars; and if the frames, in public instruments of this sort, are made of stone, that change, by diminishing the expansibility of the frame, will increase that of the bars. Regnier gave a radius of 659 millimètres to his index; so that it will traverse over a circle of 1.299 mètres in diameter. The pinion has 8 leaves in a diameter of 27 millimètres; and these proportions are such, that a temperature of 60 degrees centesimal will nearly cause a whole revolution of the index round a dial 4.085 mètres in circumference. Hence each degree would be about 68 millimètres in size, or rather more than $2\frac{1}{2}$ inches; and consequently might be distinctly seen at some distance.

15. The platina pyrometer of Guyton de Morveau, *fig.* 21, was laid before the French Institute in 1804, and was designed to measure the heat of open fireplaces and of furnaces.

Its basis is a small, yet solid plate *a*, *b* of highly baked porcelain, in which is a grove capable of containing a flat bar of platina *c*, 1.75 inch in length, 0.2 of an inch broad, and about 0.1 of an inch in thickness. One end of this bar rests or abuts against the bottom of the groove; the other presses against the short arm of a bended lever, the long arm of which, moving on a pivot, becomes the index of the instrument. The short arm of this lever is just one twentieth of the length of the long arm, which in the original instrument was equal to 1.8 inch; consequently the space moved over by the long arm will be twenty times as great as the motion caused in the short arm by the expansion of the bar.

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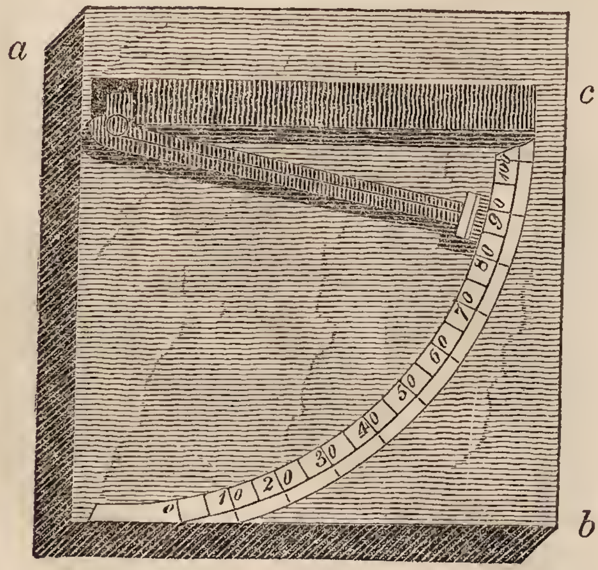
* Mémoires de l'Institut. Nationale, tom. ii. an. 7.

Fig. 21.



A finely graduated arc of a circle, of which the index is a radius, is fixed on the porcelain; and each degree of this arc is subdivided into ten parts by a vernier on the extremity of the index itself, and thus the instrument is capable of indicating an expansion of $\frac{1}{5730}$th part of the radius. All these parts are of platina.

With this instrument Guyton made many experiments, the general result of which proves, that Wedgwood has greatly erred in assigning too high a temperature for the degrees of his scale, a result confirmed by the later experiments of Daniell. Guyton ascribes Wedgwood's error to his estimating the fusing point of *silver* much too high. It was by means of a pyrometric piece of fine silver that Wedgwood connected his scale with that of Fahrenheit; and an error with respect to that metal must viciate all the results. According to Morveau, the fusing point of silver ought to have been at 22° W. instead of 28°: and each degree, instead of being equivalent to 130° of F., ought to have been no more than 62°.5; while the commencement of his scale should have been at 517° F., instead of at 1077°.5.

There is some reason, however, to believe, that Morveau has stated a red heat in day rather too low; for thermometers of mercury and of oil can sustain a temperature of 517° F. without any luminousness even in the dark.

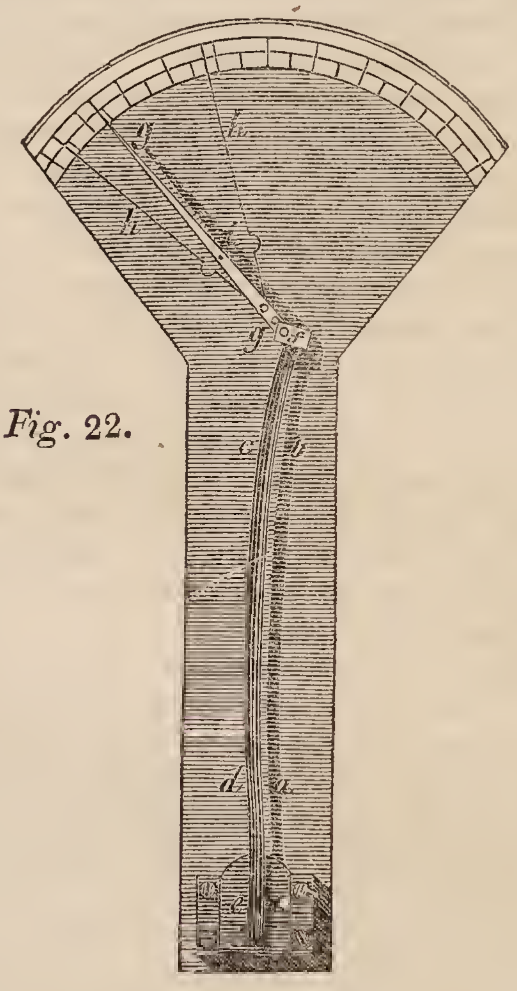
Morveau appears to have taken great pains to connect the scale of his pyrometer with the common thermometer; and he is probably nearer the truth than Wedgwood.*

His corrected table of Wedgwood's temperatures is as follows :—

	Wedg.		Fah.
Mercury boils	2°	=	642.75
Zinc melts	3	=	705.26
Antimony melts	7	=	955 23
Silver melts	22	=	1822.67
Copper melts	27	=	2205.18
Gold melts	32	=	2517.63
Iron welds	95	=	6508.88
Cast iron melts	130	=	8696.24
Porcelain melts	155	=	9633.68
Manganese melts	160	=	10517.12
Malleable iron melts	175	=	11454.56
Nickel melts	175	=	11454.56
Platina melts	175	=	11454.56

16. In 1803, Mr. James Crighton, of Glasgow, published a new "metallic thermometer," in which the unequal expansion of zinc and iron is the moving power. A bar is formed by uniting a plate of zinc, *fig.* 22, *c*, *d*, 8 inches long, 1 inch broad, and ¼ inch thick, to a plate of iron *a*, *b* of the same length. The lower extremity of the compound bar is firmly attached to a mahogany board at *e*, *e;* a pin *f* fixed to its upper end plays in the forked opening in the short arm of the index *g*, *g*. When the temperature is raised, the superior expan-

Fig. 22.



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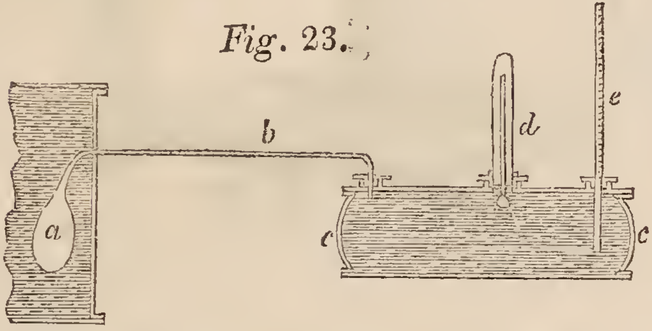
* Annales de Chimie, tom. lxxiv. lxxviii. xc.

sion of the zinc *c d* will bend the whole bar, as in the figure; and the index *g* will move along the graduated arc, from right to left, in proportion to the temperature. In order to convert it into a *register thermometer*, Crighton applied two slender hands *h*, *h* on the axis of the index: these lie below the index, and are pushed in opposite directions by the stud *i*, a contrivance seemingly borrowed from the instrument of Fitzgerald.

On the whole, the principle of this pyrometer is just; but it does not seem to possess any considerable advantages over several of those already noticed.

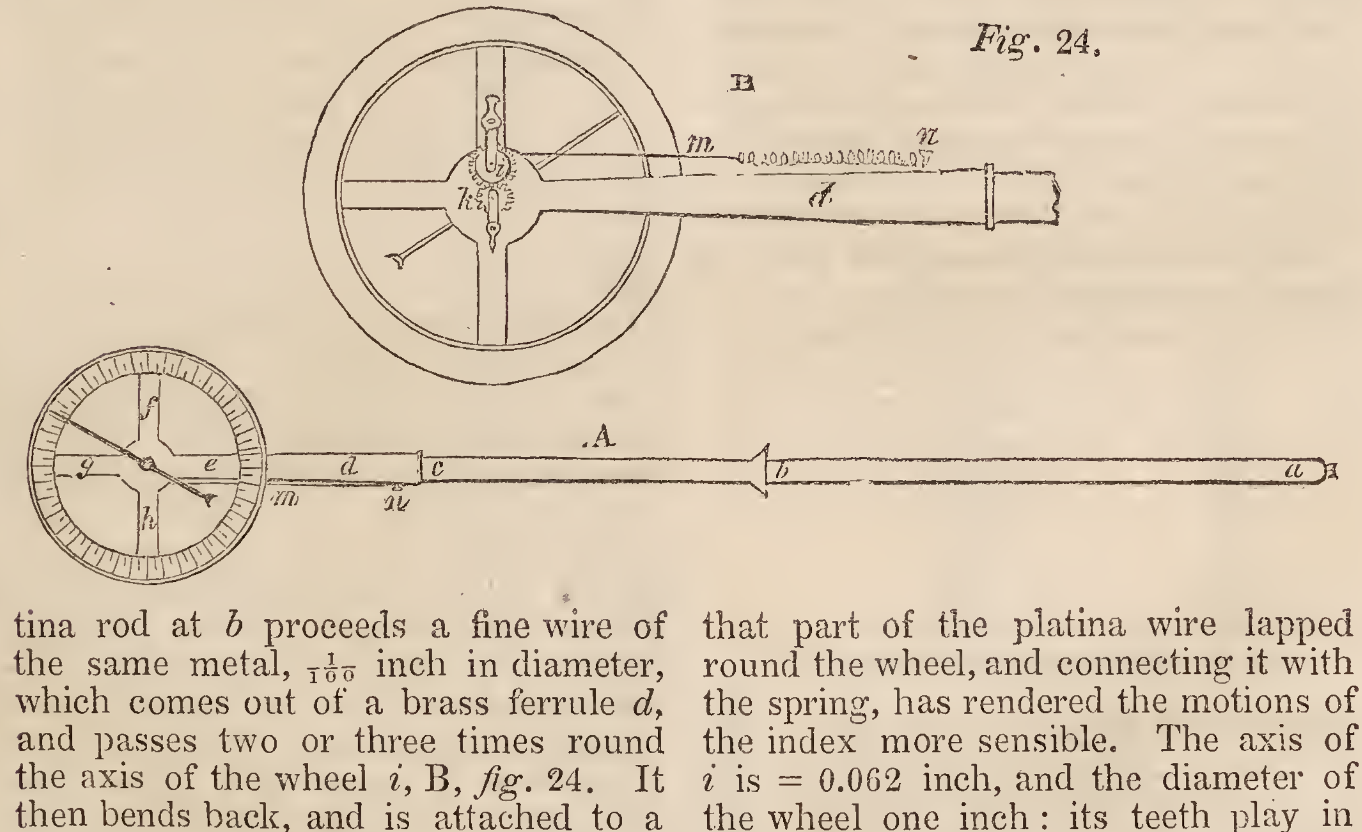
17. We have some doubts of the propriety of noticing a sort of *air pyrometer*, *fig.* 23, proposed by M. Schmidt of Jasy in Moldavia.* It is so evidently a mere theoretic proposal, and is, besides, an expensive, clumsy, and probably not very accurate mode of ascertaining high temperatures. It consists of a bottle *a*, and narrow tube *b* of platina, the former to receive the impression of the heat, and the latter to convey the expanded air into *c c*, an air-tight cistern partially filled with water. The cover of the cistern is perforated by three holes; in one of which the end of the platina tube is cemented; in the second is fixed a glass tube *d*, containing a common thermometer; and in the third, a slender graduated tube *e*, which dips into the water in the cistern. The thermometer is for ascertaining the temperature of the included air of the cistern before the experiment; the graduated tube for ascertaining the temperature communicated to the platina bottle by the ascent of this water raised by the pressure of the expanded air on the surface of the fluid in the cistern. Any further description would be superfluous.

Fig. 23.



The pyrometer of Mr. Daniell (*fig.* 24) was first described in Brande's *Quarterly Journal*.* The moving power is a rod or wire of platina 10.2 inches in length, and 0.14 inch in diameter, fixed in a tube of blacklead ware *a*, *b*, *c*, by a flanch within and a nut and screw without the tube at *a*. This tube has a shoulder moulded on it at *b*, for the convenience of always inserting it into the furnace, or muffle, to the same depth. From the extremity of the platina rod at *b* proceeds a fine wire of the same metal, $\frac{1}{100}$ inch in diameter, which comes out of a brass ferrule *d*, and passes two or three times round the axis of the wheel *i*, B, *fig.* 24. It then bends back, and is attached to a slender spring *m n*, which is fixed by one end to the pin at *n*, on the outside of the ferrule.

Fig. 24.



The substitution of a silk string for that part of the platina wire lapped round the wheel, and connecting it with the spring, has rendered the motions of the index more sensible. The axis of *i* is = 0.062 inch, and the diameter of the wheel one inch: its teeth play in the teeth of another wheel just one-third of its diameter, by which the wheel *k* has three times the movement of *i*; and the index on the axis of *k* moves therefore three times round for every revolution of *i*. The action of the spiral spring *m* draws round the wheel *i* and the index, when the expansion of the platina rod permits it to act. The dial is divided into 360 degrees. By experiment, Daniell ascertained that each degree of his scale = 7 degrees of Fahrenheit's: and he has published an account of some well conducted experiments on the fusing points of some of the metals with this instrument, which very widely differ from the results obtained by Wedgwood, but nearly agree with those of Morveau. Mr. Daniell found, that after being exposed to high temperatures, the pyrometer did not fall to the point from which it set out; a circumstance which he attributes, with justice, to changes in the form of the tube induced by a high temperature. This is certainly an imperfection in the principle of the instrument; but if the degrees of heat be marked by the ascending series, its indications seem tolerably correct, and, although perhaps little to be depended on in nice investigations, it may become an useful instrument to manufacturers who make use of high temperatures. The tube should not be exposed to a naked fire, except it be of wood charcoal; because the foreign ingredients of fossil coal will adhere or incorporate with the blacklead ware of the tube. On these grounds we should feel more inclined to recommend the pyrometer of Morveau, which, besides, is extremely portable; and, being wholly exposed to the heat, is less liable to be affected by extrinsic circumstances in its indications.

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* Nicholson's Journal, 8vo. series, vol. ii. 141.

* Vol. xi. p. 309.

The following table exhibits some of Daniell's comparative results.

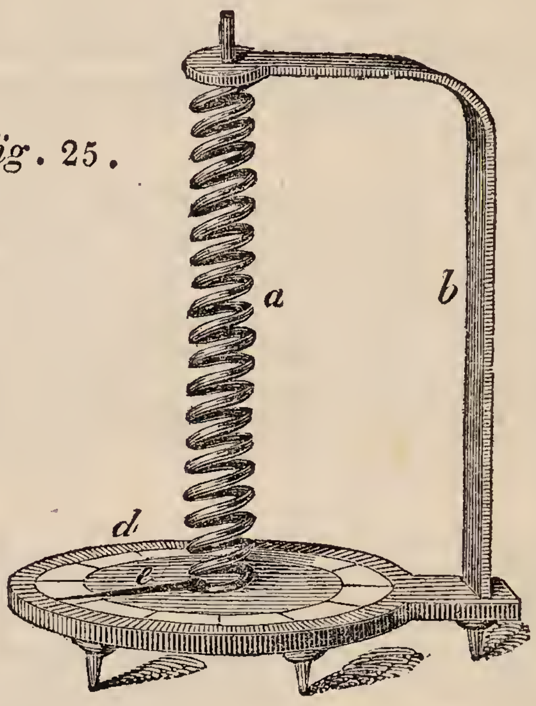
50° Fahrenheit = 7°.2′ Daniell.
100 = 14.0
150 = 22.5
200 = 30.5
250 = 38.5
300 = 45.4
350 = 51.5
400 = 58.5
450 = 66.9
500 = 73.5
550 = 77.0
580 = 84.0

600° by calculation he estimates at 86°.4.

	D.	F.
Melting point of tin	63	441
——— bismuth	66	462
——— lead	87	609
Boiling point of mercury	92	644
Melting point of zinc	94	658
Red heat in full daylight	140	930
Heat of a parlour fire	163	1141
Melting point of brass	267	1869
——— silver	319	2233
——— copper	364	2548
——— gold	370	2590
——— cast iron	497	3479

18. Messrs. Breguet, the celebrated chronometer-makers, have lately constructed a most elegant and delicate pyrometer, or metalline thermometer, of which we give a figure. (See *fig.* 25.)

Fig. 25.



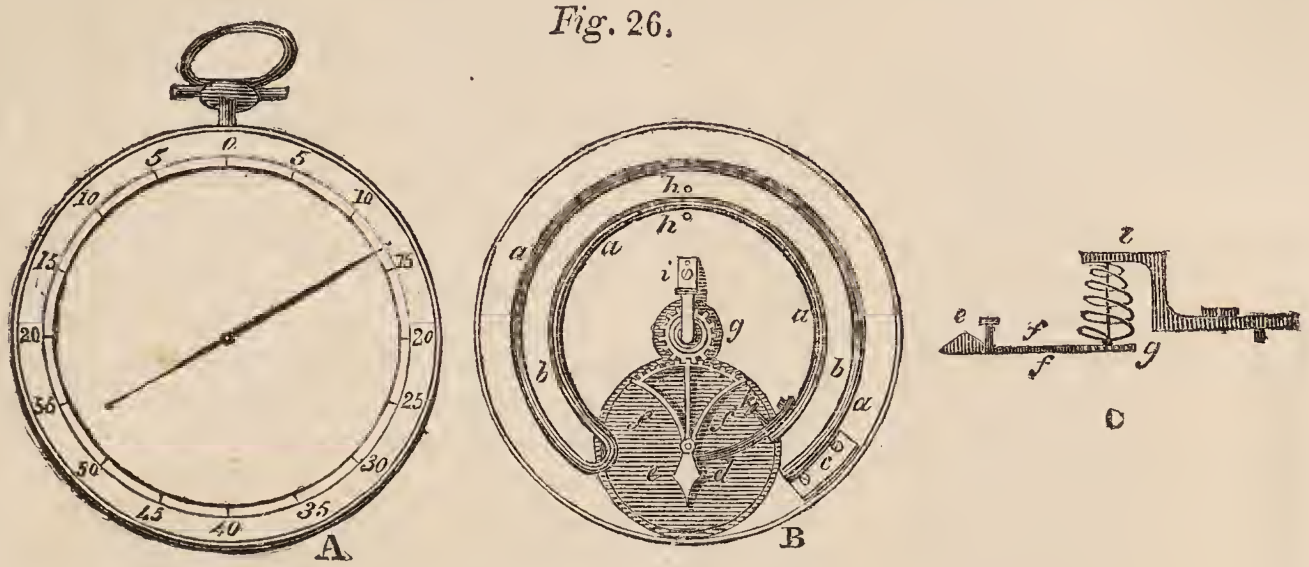
It consists of a helix formed of three metals of unequal expansibility. The exterior plate of this delicate helix is of silver, the interior of platina, and between them is one of gold. Two only are necessary to the perfect action of the instrument; but from the difference of expansibility between silver and platina, they would be liable to separate by sudden changes of temperature; and a thin plate of gold, which is of intermediate expansibility, is interposed. The whole form a single flat plate or wire about $\frac{1}{100}$th of an inch in thickness. The upper extremity of the helix is fixed to the brass support *b*, which by its form insulates the helix, and permits its coiling and uncoiling freely. To its lower extremity is attached a gold needle *e*, kept horizontal by a small counterpoise. This needle moves round a graduated circle representing degrees of the centigrade scale. When the ambient air is heated, the expansion of the metals carries round the needle in the direction of the coils of the helix, and a diminution of temperature moves it in the opposite direction by relaxing the coils. Experiment has proved that equal increments of temperature move the needle over equal spaces of the scale, so that it is comparable with other thermometers.

The sensibility of the instrument is represented as very great, when compared to a mercurial thermometer; and it is applicable to such purposes as ascertaining the temperature of a *vacuum*, which the mercurial thermometer is able to do less accurately, because of the dilatability of its bulb by the removal of pressure.

The height of the instrument, from which this description is drawn, is 3 inches, including the feet, which are half an inch; the diameter of the helix is rather less than $\frac{3}{10}$; and its length is $1\frac{1}{2}$; the diameter of the graduated circle is 2 inches inside, and its breadth $\frac{1}{4}$.

19. The instrument delineated in *fig.* 26, from one now before us, is a beautiful instrument of the same kind, the work of the Parisian artist, Frederick Houriet, which appears to be little known in this country. It is of the size of a thin ordinary watch, with a dial A divided according to the centigrade scale. The mechanism is covered by a thin plate of metal, which opens like a hunting watch; and the instrument is so delicate as to move, in less than a minute, after it is laid on the hand, at an ordinary temperature of 60° F.

Fig. 26.



The pyrometric piece is the bent compound bar *a*, *b*, *a*, *b*, composed of a plate of steel on the side *a*, and of another of brass on the side *b*, united together into one bar. The steel plate is $\frac{1}{50}$ inch in thickness, and the brass twice as much, forming a bar 9.5 inches in length, and about $\frac{1}{7}$ inch in depth. One extremity is firmly secured to the frame at *c*: the rest of it is free, bent up for the convenience of size, and secured against any accidental injury from rude handling by passing between two steel studs *h*, *h*. Its free extremity is terminated by a plate of steel *d* screwed to it, and projecting 0.3 inch beyond it, to press against the short arm *e* of the lever *f f*. The long arm of the lever ends in an arch-head with thirty teeth, that play in the teeth of a small wheel *g* with twenty-two teeth, which is fixed on the axis of the slender hand. The lengthening of the bar pushes the short arm of the lever, and the arch-head moves over a space proportional to the difference in length of the arms of the lever. How this motion is communicated in an increased ratio to the index is obvious from the construction. Under the cock *i*, which supports the common axis of the index, and *g*, is a spiral spring of flattened gold wire, intended to bring back the index when the contraction of the bar allows it, and to retain the piece *d* in contact with the short arm of the lever *e*. The instrument is adjusted by means of a steel screw *k* working in a small tube, which perforates the end piece *d*.

The whole instrument is most delicately made, and it accords in its indications with a mercurial centigrade thermometer, with which it has been carefully compared; forming one of the most elegant metalline thermometers hitherto described.

CHAPTER III.

History and Construction of Register Thermometers.

THE original suggestion of a thermometer which could register its own indications in the absence of the observer, is due to the celebrated John Bernoulli, who describes such an instrument in a letter to Leibnitz;* and an instrument on nearly the same principle was constructed by Kraft:* but their contrivances are inferior to several others of a later period, and do not require a detailed notice. We shall therefore proceed to describe the most approved register thermometers.

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* Leibnitzii et Bernoulli Commercium Philosoph. et Mathemat.

1. Lord Charles Cavendish communicated to the Royal Society different forms of thermometers, intended to register the *maximum* and *minimum* temperature in the absence of the observer.† His lordship's thermometer for showing the maximum is represented in *fig*. 27. It consists of a cylindrical bulb, and a stem terminating in an open capillary orifice, covered by a glass cap or ball *e* completely closing the thermometer. The bulb and part of the stem are filled with mercury, the rise and fall of which indicate the temperature in the usual way; above the mercury a portion of alcohol is introduced, sufficient to fill the rest of the tube and a small part of the cap. When the mercury rises it drives the spirit before it into the cap *e*, from which it cannot return while the instrument remains erect; and the deficiency of spirit in the tube, on the subsiding of the mercury, measured by a proper scale, will show how much the maximum rise of the thermometer exceeded its height at the time of the observation.

Fig. 29.

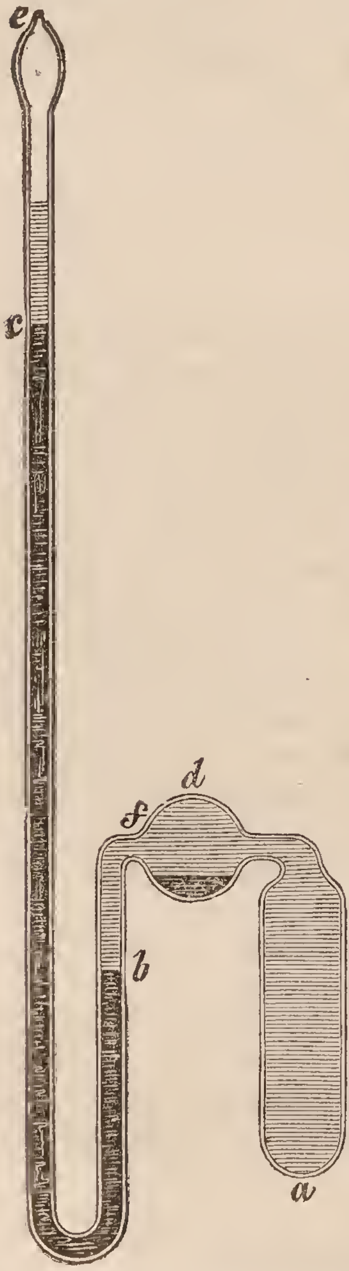


Fig. 28.

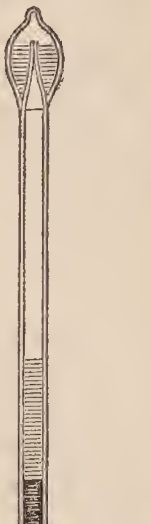
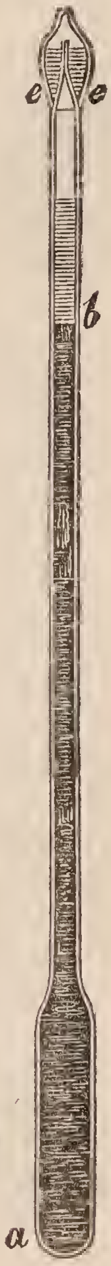


Fig. 27.



To prepare it for a fresh observation, the thermometer is to be heated by the hand until the spirit fills the whole tube, which is then to be inclined so that the spirit in the cap may cover the capillary orifice: as the ball cools, the spirit will drain into the tube thus inclined, and fill it as completely as before.

Fig. 28 is a construction of the same instrument, intended to obviate the inconvenience of so weighty a bulb as that filled with mercury must be.

Lord C. adds a correction which should be made on account of the difference of expansion of mercury and spirit, in computing the deficiency, if this be measured by the same scale as the ascent of the mercury: the degrees computed by the column of spirit will exceed those of the mercurial column by $\frac{1}{6}$ of a degree for every 10° of Fahrenheit of difference between them.

Fig. 29 is his lordship's minimum thermometer. Its bulb, $\frac{3}{4}$ of the ball *d*, and part of the leg *b*, are to be filled with spirit of wine; from *b* to *c* is occupied by a column of mercury, and about $\frac{1}{4}$ of *d* contains a portion of this fluid; a little alcohol is likewise introduced above the mercury before the orifice of the tube at *e* is closed in the usual manner. The mercury at *c* will, when furnished with a proper scale, indicate the present temperature in the ordinary way; but, when the spirit in the bulb contracts by cold, the mercury will rise in the short leg of the siphon from *b* into the ball *d*, from which it cannot get back into the tube *b*. This will therefore occasion a deficiency of mercury in that leg, which, measured by a proper scale attached to the short leg of the siphon, and subtracted from the present height of the mercury in the long leg, will show the lowest point to which the thermometer had fallen during the absence of the observer. To prevent the mercury falling in too large drops into the ball *d*, by which the delicacy of the instrument would be impaired, a solid but fine thread of glass passes through the short leg to the narrow neck *f* of the ball *d*, by which the passage is still further contracted, so that the mercury trickles through in most minute division. The instrument is prepared for a new observation by being inclined so as to bring the mercury in *d* to cover the orifice at *f*; the bulb is then heated, and the mercury is expelled from the ball into the short leg of the siphon, until it be filled with that fluid.

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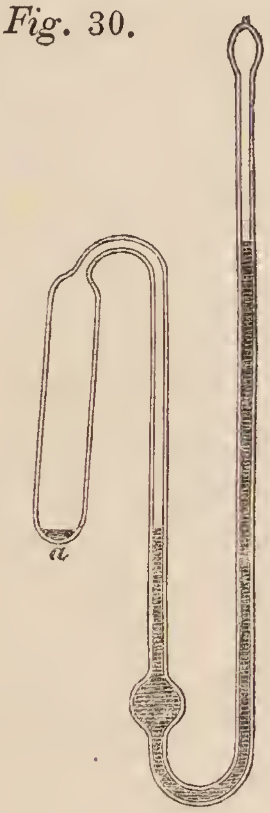
* Van Swinden, Comparaison des Thermomètres.

† Phil. Trans. vol. l. for 1757.

Fig. 30 is another form of the last instrument, which has the advantage of being more easily adjusted, and is less liable, from slight motion, to have the mercury which has passed into *a* brought back into the tube.

Fig. 30.

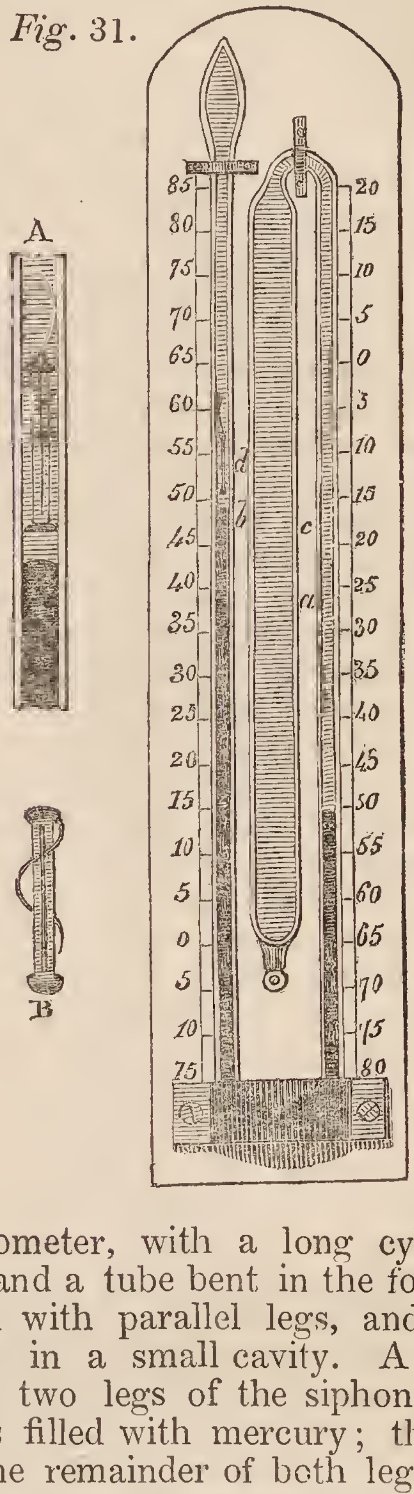


These instruments are extremely ingenious contrivances; but there are some practical difficulties in their construction; and the minimum thermometer is rather liable to be broken from the size of the bulb, and the several bendings of the tube. Hence Lord C. Cavendish's thermometers never appear to have come into general use, although very well adapted for certain purposes, as for ascertaining the temperature of the ocean at great depths. It is therefore unnecessary here to notice the corrections pointed out by Mr. Cavendish in its applications to various purposes.

2. Next in point of time is the contrivance of Fitzgerald; which has been already noticed, as well as that of Crighton; for rendering their metallic thermometers indicators of the maxima and minima of temperature during the absence of the observer.

3. The Register Thermometer invented by Mr. James Six, of Colchester, was first described in the *Philosophical Transactions*,* and is represented in *fig*. 31. It is, in fact, a spirit of wine thermometer, with a long cylindrical bulb, and a tube bent in the form of a siphon with parallel legs, and terminating in a small cavity. A portion of the two legs of the siphon from *a* to *b* is filled with mercury; the bulb, and the remainder of both legs of the siphon, as well as a small portion of the cavity, are filled with highly rectified alcohol. The double column of mercury is intended to give motion to the two indices *c*, *d*; the form of which is better seen at A. Each index consists of a bit of iron wire inclosed in a glass tube, which is capped at each extremity by a button of enamel. Their dimensions are such, that they would move freely in the tube, were it not for a thread of glass drawn from the upper cap of each, and inclined so as to press against one side of the tube, forming a delicate spring of sufficient power to

Fig. 31.



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* Vol. lxxii.

retain the attached index at any part of the tube, to which it is raised by the column of mercury. The action of the instrument will now be readily understood. When an increase of temperature expands the spirit in the bulb, it depresses the mercury in the limb *a*, and proportionally raises it in the limb *b* of the siphon: the mercurial column in the latter raises the index *d* before it; and when the mercury sinks in that leg, the bottom of the index *d*, retained at that height by the glass spring, will indicate how high the mercury had risen. When the spirit in the bulb contracts by cold, the mercury in the limb *b* descends, and the consequence is a proportional ascent of the column in the side *a;* which likewise carrying the index *c* before it, leaves its lower extremity at the point to which the column of that side had risen. In this manner the *maximum* and *minimum* temperatures are seen at any desired interval of time; and all that is necessary to prepare the instrument for a fresh observation is to bring down both indices to the surface of their respective columns by means of a magnet, which will act on the bit of iron wire included in the body of each index. From the above description, it is obvious, that there must be an *ascending* scale to measure the degrees of expansion in *b*, and a *descending* scale applied to *a* to mark the contraction of the spirit. Mr. Six graduated his thermometers by placing them in water at different temperatures, and marking on his scales the heights corresponding to every 5° of a standard mercurial thermometer immersed in the same liquid. This elegant invention has become a common instrument; and on account of the ease with which the glass spring of the index may be broken off, many instrument makers substitute a slender bristle, tied to the upper part of the index, and lapped round its body, as at B. This renders the spring less easily spoiled by the careless shifting of the index; but the hair, by being long steeped in spirit, is liable to have its elasticity destroyed; and a slender silver or platina wire would be preferable. The usual dimensions of the instrument are, a bulb from 6 to 16 inches in length, and from 0.2 to 0.3 inch in internal diameter; the siphon from the $\frac{1}{46}$ to the $\frac{1}{20}$ of an inch in width, and of a length proportioned to the size of the bulb; the indices about 1 inch long; the terminal expansion of the tube is, in most of the instruments now made, rather too small; in Six's original instrument, this part was a cylinder of 2 inches in length, by half an inch in diameter, to a bulb of 16 inches in length, and $\frac{5}{16}$ inch in internal diameter.

The chief defect of Six's thermometer arises, as in most other contrivances of this sort, from the unequal expansion of the spirit, and the introduction of two liquids of very different expansibility in the instrument; while, from the construction, it would be difficult to apply any general correction to its indications. It does not indicate the expansion of the spirit only, but also that of the mercurial column; which, where nice observation is required, would be of some moment; and the necessary friction of the indices will also tend to diminish the effect of expansion. Yet this instrument is a valuable addition to meteorology; and is probably the most convenient for ascertaining the temperature of the ocean, at great depths, of any hitherto given to the public.

4. The day and night thermometers of Dr. John Rutherford, from the simplicity of their construction, and low price, have in some measure superseded the register thermometer of Six. This ingenious and elegant device was first published in the Transactions of the Royal Society of Edinburgh,* and is represented in *fig.* 32; where A represents a spirit, and B a mercurial thermometer, each provided with its own scale, placed horizontally on the same piece of box wood or ivory. B contains, as an index, a bit of steel wire, which is pushed before the mercury, and is left in that situation to mark how high the temperature had been. A contains a glass index half an inch long, with a small knob at each end; it lies in the spirit, which can freely pass beyond it when expanded by heat; when contracted by cold, from the attraction between spirit and glass, the last film of the column of spirit is enabled to overcome the slight friction of the index on the inside of the tube, and to carry it back towards the bulb. This attraction is so considerable, that although the index will move freely up and down in the spirit, on inclining the instrument, it will rest on the last film, and require several smart concussions given to the thermometer, to make it

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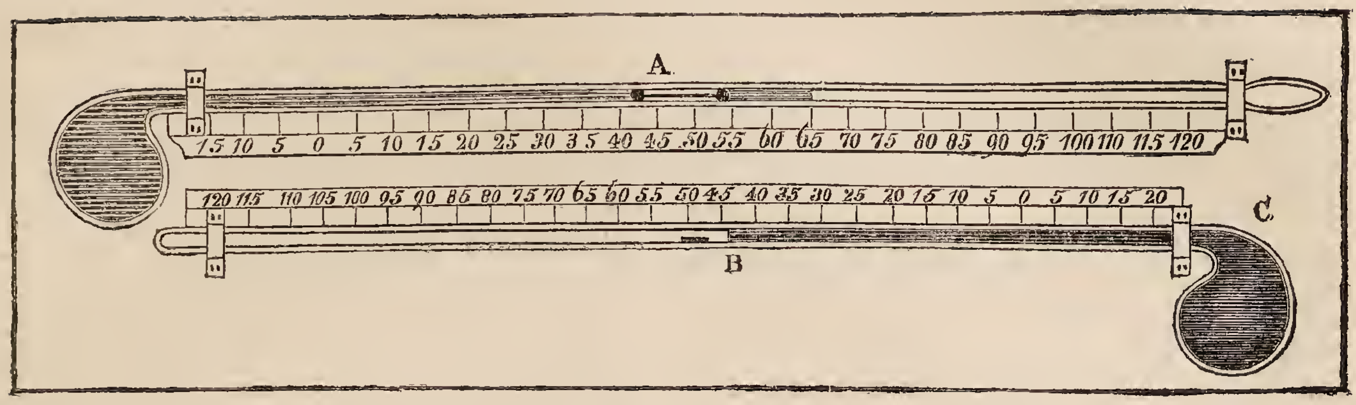
* Edin. Phil. Trans. vol. iii.

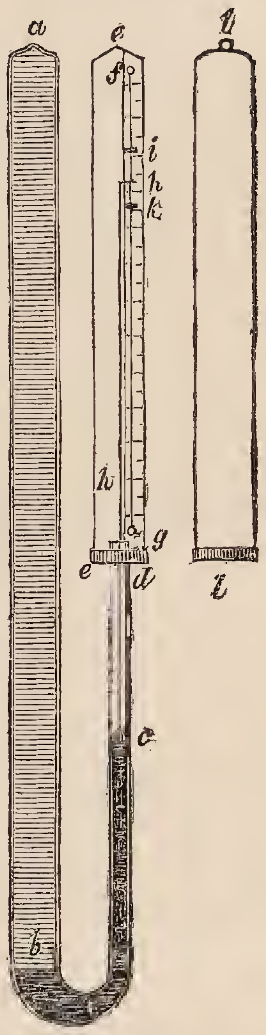
Fig. 32.



escape into the empty part of the tube. From the position of both thermometers it is obvious, that to bring both indices to the surface of the respective fluids, it is only necessary to incline the instrument toward C; and it is thus prepared for a fresh observation.

The accuracy of Rutherford's thermometers depends on the ease with which they are constructed, and the application of a due correction for the inequalities of expansion between them; this can be more readily accomplished than with Six's thermometer; because the indications of each fluid are independent of each other. The discrepancies between both thermometers have been carefully examined by De Luc,* and more lately by De Wildt, of Hanover;† the results will be given in the sixth chapter, from which the correction can be applied. Such a correction will render them applicable to the nicest meteorological observations of maxima and minima. For more ordinary purposes they are very convenient, as not being easily deranged, and being adjusted, for each observation, with the utmost facility.

Fig. 33.



5. In the *Transactions* of the same society,‡ we find another register thermometer, by Mr. Alexander Keith, a gentleman of great mechanical invention, and long an active member of that society. It is represented in our *fig.* 33; where a b is a glass tube, 14 inches long, and $\frac{3}{4}$ inch in calibre, sealed at the top, and below communicating with a bent tube b, d, 7 inches long, and 0.4 inches in diameter, open at the top, where it is cemented to a metallic plate e, which supports the ivory scale e, e, $6\frac{1}{2}$ inches long. From a to b, the tube is filled with highly rectified alcohol, and from b to c with mercury. At c is a conical float of ivory or glass, resting on the surface of the quicksilver, and supporting a kneed wire h, intended for moving two indices of black silk i, k, that slide along the fine gold wire g, f, as will be readily seen from the figure.

To prepare the instrument for observation, the indices are drawn, by means of a crooked wire prepared for the purpose, till they touch each side of the knee of the *float wire*. It is obvious that, as the heat alters the dimensions of the column of spirit in a, b, the mercury will rise or fall in the small tube, and the float swimming on the surface of the mercury will raise or depress the knee h, which will move the indices accordingly on the wire g, f. The instrument is defended from wind or rain by the glass case l, l, which, by means of its metal collar, fits tight on e, and is only removed to adjust the indices.

This instrument, it is true, is influenced by atmospheric pressure, when its cover is removed; but the effects of barometrical variations are scarcely appreciable except in an air thermometer; and when made on a large scale, is applicable, as Mr. Keith has shown, to the important purpose of marking the *periods* of the atmospheric changes of temperature. This he effected by making the large tube 40 inches long, but retaining the original width; while the small tube is increased in diameter, but not in length. The float *c* is enlarged, and the float wire carries, instead of the knee, a soft pencil, which is made to press lightly against a hollow vertical cylinder, 7 inches long and 5 in diameter, moved by clock-work, once round in 31 days. This cylinder is covered with smooth paper, ruled longitudinally into 31 columns, to correspond to the days of the month; and every column is subdivided into 6 equal parts, each corresponding to 4 hours. The cylinder is ruled across into 100 divisions, intended to correspond to the 100° of Fahrenheit marked on the ordinary scale of the instrument, which is unnecessary when the cylinder is applied. Thus, as the cylinder revolves, the point of the pencil will trace a line on the paper more or less deviating from a horizontal line, as the mercury rises and falls; and thus the paper will present a *chart* of the variations of the thermometer for a whole month, the value of which, in degrees of Fahrenheit, will be indicated by the numbers on the margin of the paper. Keith recommends the observer to have a copper plate for giving ruled impressions on smooth paper, to be applied monthly to the cylinder; and these, bound up together, will present tabular views of the fluctuations of the thermometer for every month. It is hardly necessary to state that a similar contrivance is applicable to the indications of the barometer.

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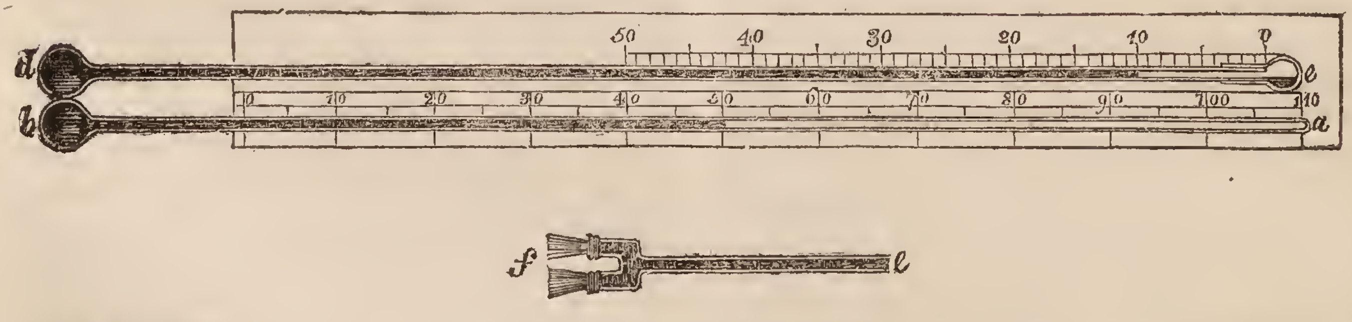
* Recherches sur les Mod. de l'Atmosphère.
† Jameson's Edin. Phil. Journal for October, 1826.
‡ Edin. Phil. Trans. vol. iv.

6. We are indebted to Mr. Henry Home Blackadder, for some very ingenious methods of ascertaining the temperature of the air, at any given hour, by a subsequent inspection of a thermometer. His first invention resembles one of Rutherford's thermometers, suspended on a pivot. If a spirit thermometer be preferred, it is to be hung vertically and inverted, so that the index may rest on the last film of the liquid. Suppose that we desire to know the temperature at 5 o'clock, A. M., a lever connected with a clock is applied, so as to bring the thermometer to a horizontal position at that hour; and, at the same time, the motion causes the bulb of the thermometer to approach some source of heat a little higher than that of the air; as, for instance, a small lamp, by which the spirit would rise beyond the now horizontal index, leaving it at the point to which the spirit had contracted before the reclination of the instrument. When a mercurial thermometer is employed, the instrument is also hung vertically, but with its bulb lowermost; and the index, therefore, resting on the mercury. It is brought to a horizontal position by the same means as the other thermometer; and then its bulb, coming into contact with a camel-hair pencil, kept continually moist with water, is cooled so as to cause the mercury to shrink, and leave the index at the height of the column while the instrument was in the upright position.

By a subsequent improvement he has contrived to dispense with the index altogether. This modification is seen in *fig.* 34, where two thermometers are placed parallel on the same piece of box wood or ivory: *a b* is a common mercurial thermometer; *c d* is of the same size, but is not hermetically sealed. The end of its stem is ground flat, and is introduced into the neck of a small ball at *e*; which, as well as the stem, contains some mercury. The stem is pushed up until it just reaches the ball, to which it is cemented by colourless varnish.

Fig. 34.

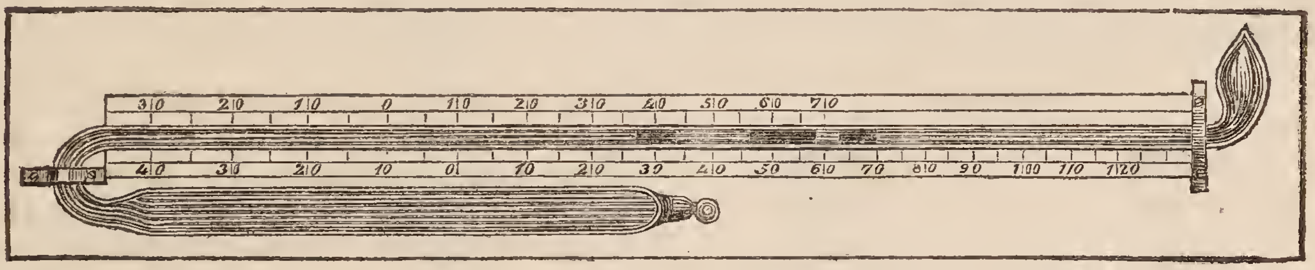


When these thermometers are in the upright position, the globule of mercury in *e* covers the orifice of the tube; and on applying the heat of the hand to *d*, the mercury in the stem joins that in the ball; and the stem will remain filled with the mercury while the instrument is vertical: but when it is brought into a horizontal position by the machinery above mentioned, and the bulbs approximated to the pencils *e f*, suspended over them for that purpose, and supplied with liquid through the channels in them, the mercury in the ball will leave the orifice of the stem, and that in the latter will descend, as represented in the figure; and its subsequent contraction is marked by an inverted scale; which, with the indications of the other thermometer, will enable us to ascertain the temperature at the moment of the reclination of the instrument. Thus, as both instruments are equal, if the same diminution of temperature has sunk *a b* to 50°, and has produced a contraction of 10° in *e d;* it is evident the sum of both numbers will show the degree at which the common thermometer stood at the moment of the change of position; which in this case has been 60°.

This idea is most ingenious, and is said in practice to work exceedingly well. It promises to be useful in meteorological investigations, although not so complete as the register thermometer of Keith, which continues to note its own indications for a whole month; while that requires to be readjusted for each observation.

7. We shall conclude this chapter by a notice of another register thermometer invented by Dr. Traill, and seen in *fig.* 35. It is a single spirit thermometer, in which a column of mercury $\frac{1}{3}$ of an inch in length is introduced: at each end of this column lies an index of fine steel wire, gilded by means of a galvanic circuit, to prevent oxidation in the spirit. An inspection of the figure will show how the variations of bulk of the spirit in the bulb will move the column of mercury; and by this the indices are pushed in opposite directions, but will remain at the lowest and highest points to which they are driven by the mercury. The difference between the two scales will be the length of the mercurial column. The indices are brought in contact with the mercury by a magnet.

Fig. 35.



This thermometer has the advantage of giving the *maxima* and *minima* by the changes in volume of a single fluid; for the expansion of so short a column of mercury is quite inappreciable. The defect of this construction is the liability of the mercury to separate by sudden motions of the instrument. This is least likely to happen when the mercurial column is short and the calibre of the tube is minute; and it is to admit of a fine tube that gilded steel wire is preferred to an index coated with glass.

Chapter IV.

Differential Thermometers, and their Modifications.

Thermometers of this kind are not affected by general changes of temperature in the surrounding medium; but are delicate indicators of partial changes affecting one of their balls. Some of the forms of the air thermometer described by Van Helmont, bear a general resemblance to the instrument known by the name of *differential thermometer:* but they were rudely constructed, without a fixed scale, and unsusceptible of accuracy, or of application to the delicate investigations required by modern experimental philosophy. We are indebted to the ingenuity of Professor Leslie of Edinburgh for a perfect differential thermometer, and its application to some very important purposes.

In January 1800, he published a description of a new *hygrometer* and *photometer*,* of which the principle depends on the difference in the volume of air contained in two equal balls of glass, connected by a tube bent in the form of a letter U, when the balls are unequally heated. This difference is measured by the motion of a coloured liquid contained in the bent tube. "In ordinary cases," says Leslie, "the intermediate liquor would continue stationary; for the air in both balls having the same temperature, and, consequently, the same elasticity, the opposite pressures would exactly counteract each other;" but if one ball becomes colder than the other, "it is manifest that the liquor would be pushed towards it by the superior elasticity of the air included in the other." This is the principle which suggested to him the

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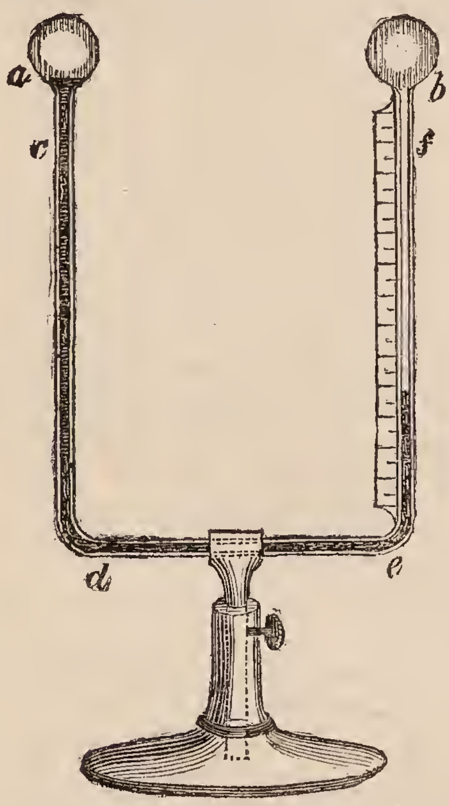
* Nicholson's Journal, 4to, vol. iii. p. 461.

1. *Differential Thermometer*, used with so much skill and ingenuity in those delicate investigations *on heat*, with which he was occupied from the above period, until the publication of his work early in 1804. The differential thermometer in its most usual form is represented in (*fig.* 36,) where *a*, *b*, are two equal glass spherules, connected by the tube *c*, *d*, *e*, *f*, slightly dilated just below the ball *a*, and at *e*, and partially filled with a coloured liquid, as represented in the figure. The dilatation below *a*, is intended as a reservoir of liquid; and that at *e*, for the more easy adjustment of the liquor to the commencement of the scale, by passing bubbles of air from one ball to the other. The liquid recommended by Leslie,* after many trials, is strong sulphuric acid, tinged by carmine. The scale he adopts, is *millesimal*, from the freezing to the boiling point of water; or 10 degrees of it are equal to one of the scale of Celsius. The instrument is cemented to a wooden foot, either immediately, or is furnished with a sliding stem to adapt it to different heights. Each leg of the instrument is usually from 3 to 6 inches in length, and the balls are from 2 to 4 inches apart. The calibre of the stem *e, f*, is from the 1-50th to 1-60th of an inch; that of the rest of the syphon a little larger.

Fig 36.



When exposed in a room, or in the open air, the differential thermometer remains stationary at 0°, whatever may be the temperature of the ambient air; but if one of its balls be more heated than the other, the unequal expansion of the included air puts the coloured fluid in motion. In employing this instrument in experiments on the radiation of caloric, its principal use, the ball *a* is that to which the heat is applied; or is, as Leslie calls it, the *sentient* ball; and the mounting of the liquid in the other stem indicates the *difference* of elasticity of the air in both balls, and hence the name of the instrument. It owes to its insensibility to general changes of temperature, its peculiar fitness for measuring the influence of radiation.

The theory of the instrument supposes that *gases expand uniformly with equal increments of temperature:* this is, perhaps, not strictly true; yet, as Leslie remarks, it is so nearly correct, that, in the limited range of the instrument, the irregularity from that cause is quite inappreciable.

It is worthy of remark, that, a few weeks after the publication of Mr. Leslie's "Inquiry," a part of the "Philosophical Transactions" of London appeared, which contained a set of experiments almost similar to many of his, and a description of an instrument, in principle precisely similar to his differential thermometer. This was

2. *Rumford's Thermoscope.*

It consisted also of two horizontal balls, united by a syphon; and the only difference from Leslie's thermometer is, that the scale is attached to the horizontal part of the tube (which is the longest portion); and the coloured liquid is a bubble moving to and fro, when the balls are unequally heated, in the horizontal part of the tube. Rumford professes to have borrowed the idea of the thermoscope from Leslie's hygrometer; but the latter has roundly charged him with a more direct plagiarism from the *differential thermometer;* to which accusation we do not recollect that any satisfactory answer has been published.

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* Experimental Inquiry, p. 417, 1804.

The differential thermometer has undergone several alterations of form to adapt it to particular purposes as an air thermometer. One of the most common is seen at *fig.* 37, where the ball *b* is cemented to the tube, after the introduction of the liquid, as in the old air thermometer, but this form has been rendered more elegant and convenient by the modification of *Dr. De Butts of Baltimore,* (See *fig.* 38,) in which no cement is necessary; for the stem and both balls are united by the blow-pipe. These instruments are to be either fixed perpendicularly on a stand, or suspended. The liquid is contained in the lower ball, and the heat is applied to the upper one; so that the stem is provided with a *descending scale.*

Fig. 37.

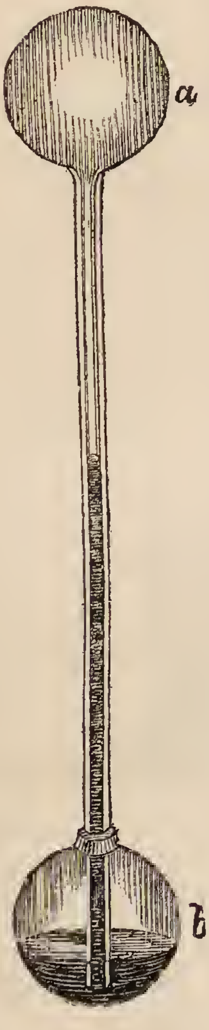
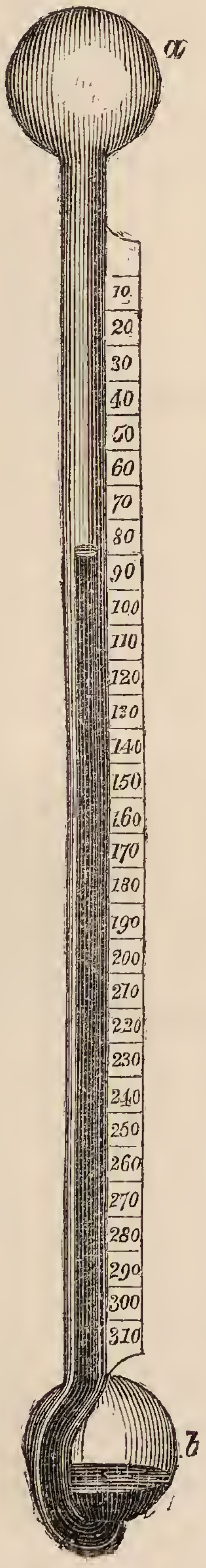


Fig. 38.

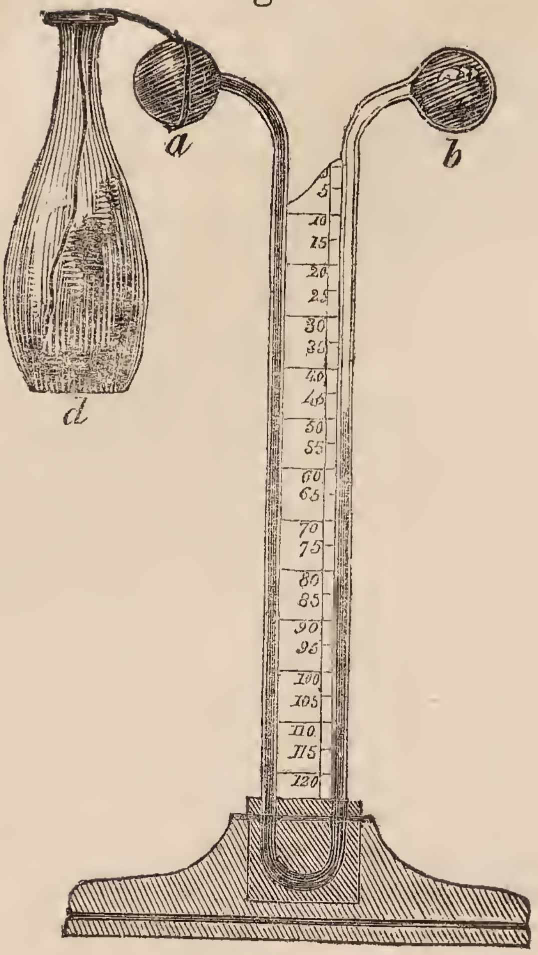


3. *Leslie's, or the Thermometric Hygrometer.*

When the ball of the differential thermometer containing the supply of coloured liquid is coated with several folds of tissue paper, and kept moist with distilled water, the instrument becomes an *Hygrometer:* for the descent of the coloured liquid in the other stem will mark the diminution of temperature caused by the evaporation of the water from the humid surface; and as this effect is proportional to the relative dryness of the ambient air, it will give an indication of the comparative quantity of water suspended in the atmosphere, at the different times of observation. In most cases, two minutes are sufficient to produce the full effect on the instrument; and the included liquid then becomes stationary, until the whole moisture is exhaled from the ball. The drier the ambient air is, the more rapidly will the evaporation go on; and the cold produced will be greater. When the air is nearly saturated with moisture, the evaporation goes on slowly; the cold produced is moderate, because the ball regains a large portion of its lost heat from surrounding bodies; and the degree of refrigeration of the ball is an index of the dryness of the air. Could we ascertain with precision the capacity of air for moisture, at different temperatures, this hygrometer would likewise afford a measure of the absolute quantity of water suspended in the air. The most approved form of the instrument, according to Leslie, is seen in *fig.* 39. The balls are parallel, and bent from each other; *a* is covered smoothly with several folds of tissue paper, which is to be kept continually moistened with pure water, drawn from the vase *d*, by the capillary attraction of a few fibres of silk. In order to obviate any inequality from the disturbing effect of light, the ball *b* is formed of pale blue glass; and the *papered* ball is covered with thin Persian silk of the same hue.

Should the water become frozen on the ball, this hygrometer will still act; for evaporation goes on from the sur-

Fig. 39.



face of ice, in proportion to the dryness of the air. Mr. Leslie estimates, that when the ball is moist, air, at the temperature of the ball, will take up moisture equal to the sixteen thousandth part of its weight, for each degree of his hygrometer; and as ice in melting, requires 1-7th of the caloric consumed, in converting water into vapour, when the papered ball is frozen, the hygrometer will sink more than when wet by 1° in 7°; and hence in the frozen state, we must increase the value of the degrees 1-7th: so that each of them will correspond to an absorption of moisture, equal to one-fourteen thousandth part of the weight of the air.

When this hygrometer stands at 15°, the air feels damp; from 30° to 40°, we reckon it dry; from 50° to 60°, very dry; and from 70° upwards, we should call it intensely dry. A room will feel uncomfortable, and would probably be unwholesome, if the instrument in it did not reach 30°.* In thick fogs it keeps almost at the beginning of the scale. In winter, in our climate, it ranges from 5° to 15°; in summer often from 15° to 55°; and sometimes attains to 80° or 90°. The greatest degree of dryness ever noticed by Leslie, was at Paris in the month of September, when the hygrometer indicated 120°.

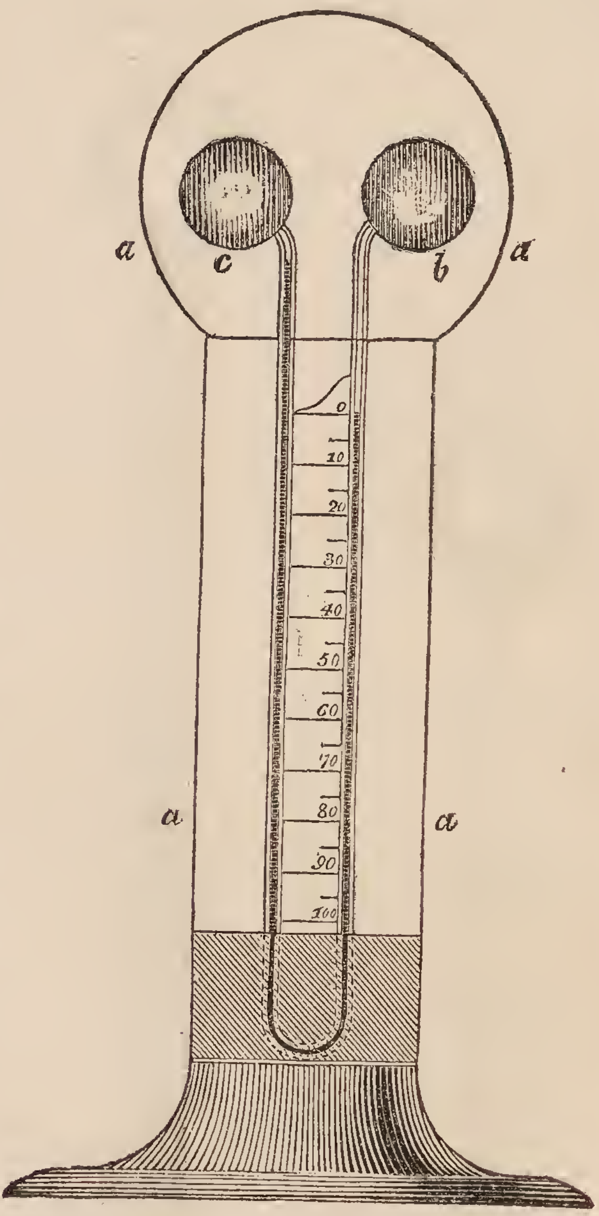
The thermometric hygrometer is of two forms; the *stationary*, (*fig.* 39,) and the *portable*, which resembles the instrument delineated in (*fig.* 41), without its glass shade. This last form is defended by a wooden case which screws over it, to fit it for the pocket.

4. *Leslie's Photometer.*

This elegant instrument is the differential thermometer, covered by a case of transparent glass, and having one of its balls either painted black, or, what is better, formed of black glass enamel.

The *Stationary* Photometer (*fig.* 40) has both its balls at the same height, and covered by a spherical shell of the most transparent glass; which, with the annexed glass tube, defend the balls from the disturbing influence of currents of air.

Fig. 40



The *Portable* Photometer (*fig.* 41) has the balls in the same vertical line, in order to admit a turned tube of wood A, of the same form as its cover *a*, *a*, to screw on the brass collar *d*, as a defence to the instrument when in the pocket; and for further convenience

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* Leslie "On the Relations of Air, Heat, and Moisture," p. 70.

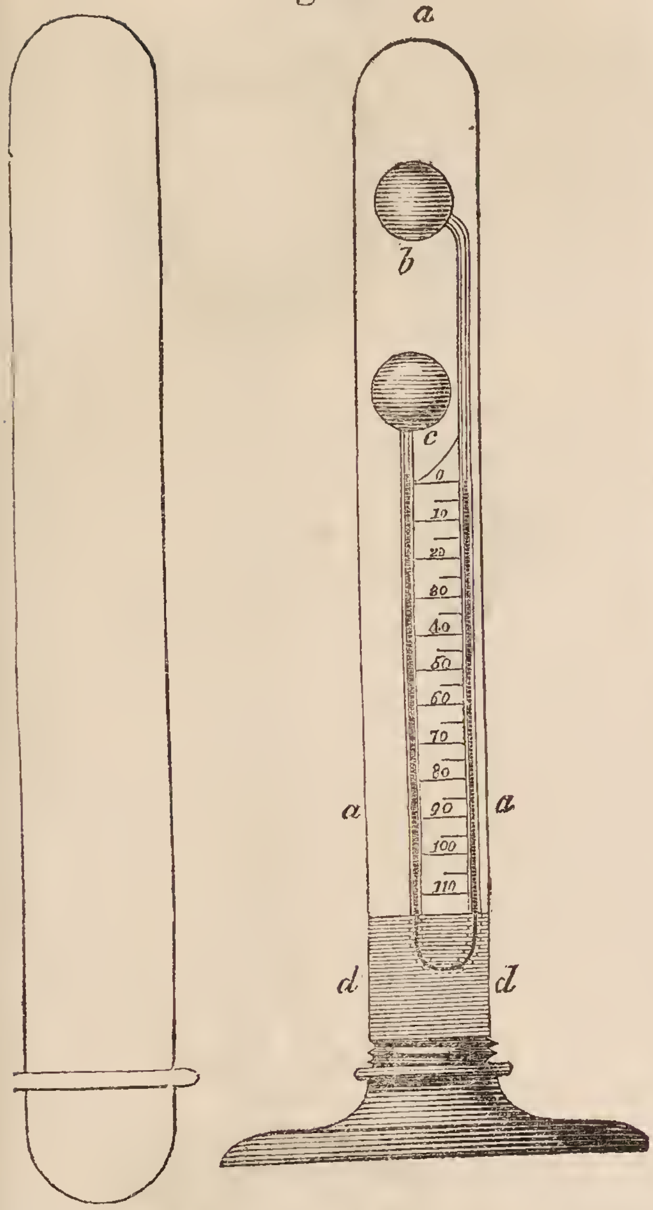

Fig. 41.

the socket is made to unscrew from the sole of the instrument. The ball, *b*, is of black, or deep reddish-brown enamel, while *c* is as diaphanous as possible. The graduation, and other parts of the photometer, are on the same scale and construction as in the differential thermometer.

Dr. Franklin, and others, had remarked the superior power which dark colours possessed of absorbing the calorific influence of the sun's rays; and Dr. Watson, afterwards Bishop of Llandaff, had, in 1773, observed, that when a thermometer, having its ball blackened, was exposed to the sun's light, it rose 10° higher than it had previously done in a similar situation. The researches of Mr. Leslie put this fact in a more striking point of view, and led to the invention of this instrument.

The theory of the photometer hangs on the supposition, that the intensity of light emitted from any body, is always proportional to the temperature excited by its incidence on the blackened ball. This is probably true with regard to the undecomposed rays of the sun, in which the caloric and the light, if different kinds of matter, are intimately blended; but there is strong reason to suspect, that light emitted by terrestrial bodies is not always proportional to the concomitant temperature. Thus the intense splendour of phosphorus burning in oxygen gas, gives out far less heat than the comparatively dull combustion of hydrogen in the same gas; and we have found this photometer often more affected by the emanations from a fire so dull, that not a single letter could be discerned in a well-printed page, than by the degree of daylight, by which we could read the same print with pleasure and facility. It is differently affected too by light of different colours, where their illuminating property appears the same; and the experiments of Herschel, Englefield, and others, show that the maximum of heat in the solar beam, decomposed by the prism, by no means corresponds with the illumination, but is even altogether beyond the margin of the spectrum.*

As a measure, however, of the intensity of undecomposed solar light, it appears to support the character it receives from the inventor.—" The photometer," says he, " exhibits distinctly the progress of illumination from the morning's dawn to the full vigour of noon, and thence its gradual decline till evening spreads her sober mantle. It marks the growth of light from the winter solstice to the height of summer, and its subsequent decay through the dusky shades of autumn; and also enables us to compare, with numerical accuracy, the brightness of different countries—the brilliant sky of Italy, for instance, with the murky air of Holland."

The direct impression of the sun's rays at noon, about the summer solstice, in this country, equals from 90° to 100° of this instrument; and at midwinter, the force of the solar beams is from 25° to 28°. The indirect light, from a summer's sky, at noon, is from 30° to 40°; in winter, it is from 10° to 15°. In the most gloomy weather, in summer, the photometer rarely indicates less than 10° at noon; but in winter it sometimes barely exceeds a single degree.

The observations on the light of day with this instrument should always be made in the open air; and the direct effect of the sun's rays noticed, as well as the indirect reflection from the sky.

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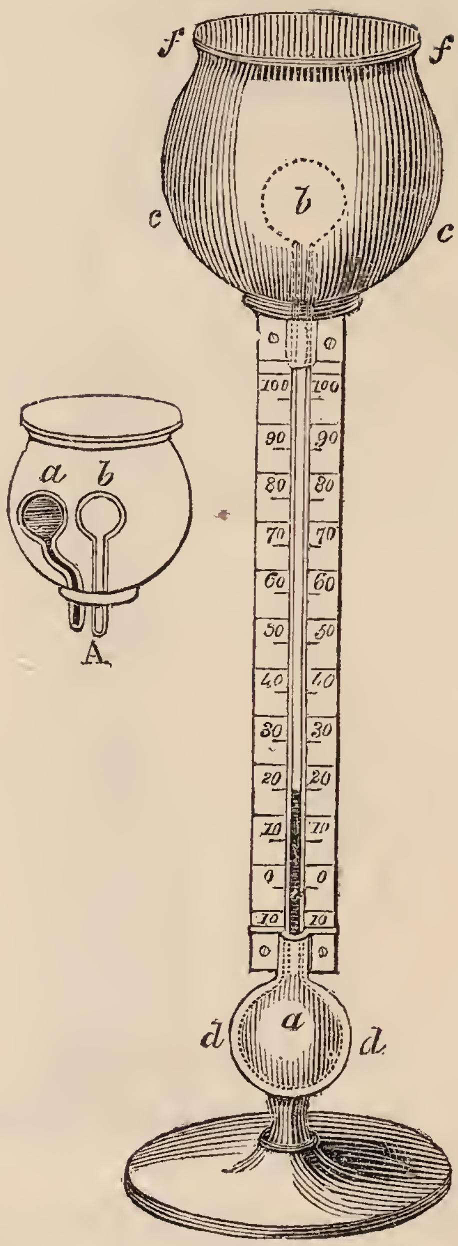
* Herschel, Phil. Trans. 1800; Englefield, Journ. Roy. Institution, vol. i.

5. *Pyroscope.*

When one ball of the differential thermometer is smoothly covered with thick silver leaf, or inclosed in a polished sphere of silver, and the other ball is naked, it forms the *pyroscope ;* an instrument intended by its inventor, Mr. Leslie, to measure the intensity of heat radiating from a fire into a room, or the frigorific influence from a cold body. A figure is unnecessary, as the instrument is usually made either like the differential thermometer, like that represented in (*fig.* 37,) or the hygrometer, (*fig.* 39.) The theory of its construction and application is, that all the rays incident on the metallic surface, are returned from it ; while those that reach the transparent ball expand the air within it, and depress the coloured liquid in the stem. In this way the comparative radiation from various bodies may be ascertained ; and it is so delicate an instrument, that in a warm room it will be visibly affected by a pitcher of cold water, at the distance of a few inches.

6. The *Æthrioscope of Leslie* is another modification of the differential thermometer which we shall here notice. One of its most usual forms is given in *fig.* 42 ; and is what the inventor calls the *Pendant Æthrioscope.* The ball *a* of the thermometer is inclosed within a brass sphere, *d, d,* without touching it ; and for the convenience of adjustment, this sphere may be unscrewed in the middle. The other ball, *b*, which is about one half the diameter of the first, is in the centre of an oblong spheroidal cup, *c, c,* which may be covered by a top that fits on at *f, f.* The coloured liquid in the stem is supported by capillary attraction in the dilated extremity of the tube, where it joins the ball *a*. The brass work is highly polished, and the inside of the spheroidal cup is well gilt.*

Fig. 42.



This very elegant instrument is intended, in the language of Mr. Leslie, " to indicate the *cold pulses* emanating from the sky ;" or, in other words, to give a comparative idea of the radiation proceeding from the surface of the earth toward the region of perpetual congelation in the atmosphere. The brass coverings defend both balls from the influence of the sun's rays, or other adventitious sources of heat ; and when the ball *b* is cooled by radiation toward the heavens, the air within it contracts, and the elasticity of that within *a*, forces up the liquid in the stem, the height of which marks the intensity of the radiation.

When the cover is on, the liquid remains at 0° ; but when it is removed, and the instrument presented to a clear sky, either by night or by day, it instantly begins to rise, and continues to mount until the ball *b* has sustained the greatest diminution of temperature, which radiation at that time can produce.†

The circumstances which favour radiation from the surface of the earth toward the sky, namely, a clear and calm atmosphere, are admirably pointed out by Dr. Wells, in his excellent Essay on *Dew :* and this instrument becomes a valuable indicator of the state of the air favourable to the deposition of that interesting meteor. Such is its extreme delicacy, that, when rising, its progress is checked by the smallest cloud sailing over it; and it may be kept in a state of oscillation, by being alternately stretched beyond the edge of a parasol, and drawn within its shade, when the sky is clear and serene.

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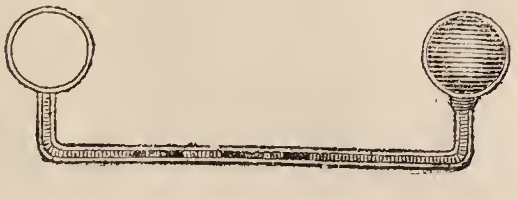
* Edin. Phil. Trans., vol. viii.

† Leslie appears to have been led to this invention by some of his own experiments on radiant caloric ; but it is proper to state, that Dr. Wollaston had shown, that when a delicate thermometer, in the focus of a concave metallic mirror, is presented to the sky, cold is indicated.

Besides the forms given above, Leslie describes two others, the *Standard* and the *Sectorial.* The first has the stem bent up as in the hygrometer, and one of the balls covered with silver leaf or gilt, near the side of the cup *c, c,* with the naked ball in the centre, as the balls *a, b,* in A. The second has the cup, *c, c,* formed with a notch in its bottom, for admitting a partial vertical motion round the ball *b.* The motion is given by means of a toothed sector and pinion; from which the name is derived. This form is applicable to ascertain the radiation from the earth, when we ascend a mountain, or rise in a balloon.

7. The differential hygrometer and photometer would have come into more general use had they indicated the maximum and minimum between any two times of observation. In their present construction they only show the state of the atmosphere at the moment of observation, and, therefore, require an attention which few have leisure or inclination to bestow on meteorological observations. To render them more extensively useful the following alteration is suggested, by which they are brought nearly to the thermoscope of Rumford in form. The tubes connecting the balls have the upright part of their stems shortened, and the horizontal part extended: instead of a coloured fluid filling the stems, there is a short column of mercury introduced into the horizontal part of the stem; the motion of which, towards either ball, carries before it a piece of steel wire, which constitutes the index of maximum and of minimum change during the absence of the observer. This construction will be readily understood from the figure 43, which represents the register hygrometer. A double scale lies along the horizontal part of the instrument. When the indices are adjusted for a fresh observation, they are brought by a magnet to each extremity of the little column of mercury.*

Fig. 43.



Chapter V.

Of some peculiar Applications of the Thermometer.

There are a few applications of the thermometer to certain useful purposes, which ought to find a place in the history of the instrument; I allude particularly to the Statical Thermometer of Dr. Cumming, the Balance Thermometer of Mr. Kewley, the Hygrometrical Instrument of Mr. Daniell, and the Barometrical Thermometer of Mr. Wollaston.

1. The *Statical Thermometer* of Dr. Cumming, now of Chester, was contrived by that gentleman, in 1808, and intended by him as a mode of opening and closing windows and ventilators in apartments, by the variations in temperature of the included air. This ingenious application of statical principles was shown to numerous friends at different times, in his residence at Denbigh, and was afterwards, for a considerable time, exhibited in the Denbigh Dispensary. The general form of the instrument is represented in *fig.* 44, where *a b* is a glass matrass, or a ball and tube of iron; the globular termination of which is capable of containing four or five pints of air, and the tube is about twenty-five inches in length, and from one to two inches in diameter. A portion of the tube is filled with mercury; and in this state it is inverted, and its extremity plunged in a cylindrical jar for containing the same fluid. The ball is covered by a net of strong cord, or of wire, which forms a ring at the top, for the suspension of the ball and tube. From this ring passes a cord over the pulley, *d;* and it may either pass upward under the pulley, *e,* to be attached to the frame of a swing window, as shown at *g;* or downwards over the pulley *f,* to be fixed to the ventilator *h.* When the heat of the apartment expands the air in the ball, it depresses the mercurial column in the tube, *b;* by which the whole instrument

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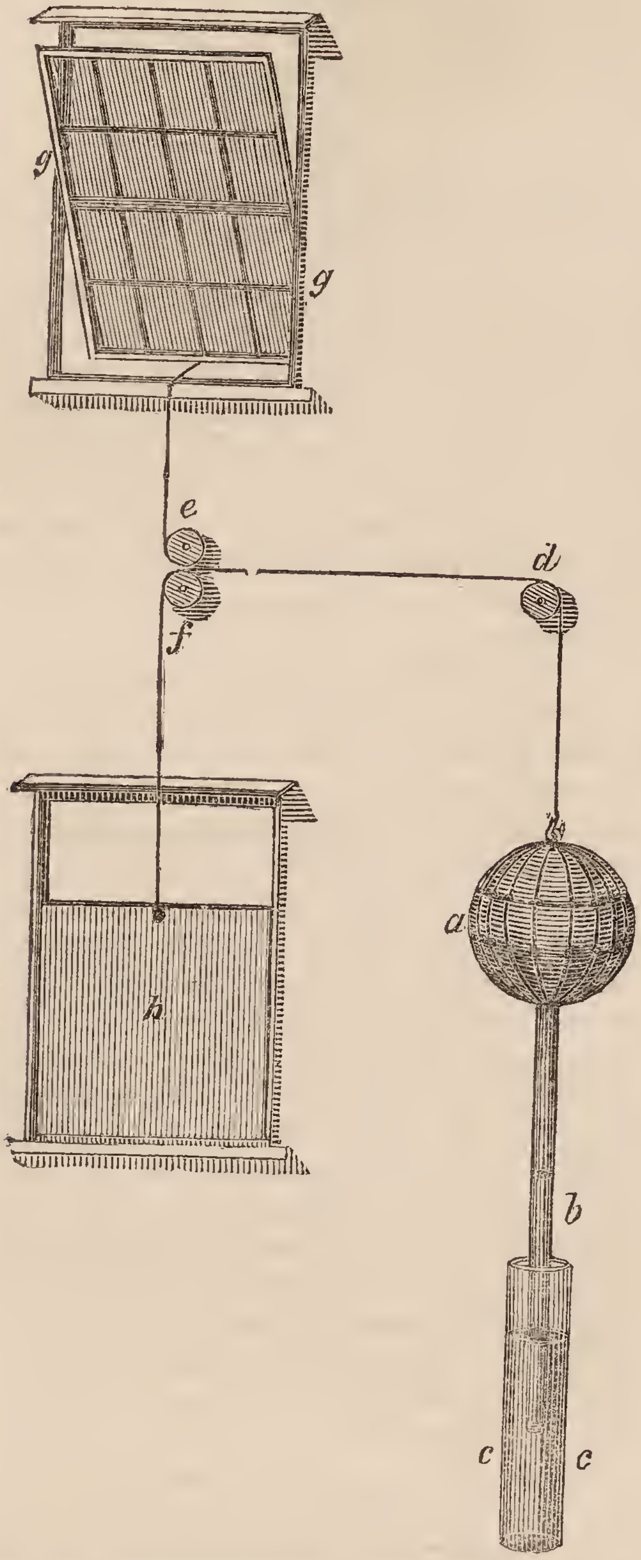
* This instrument acts rather slowly, and the motion of the indices is not quite smooth; but it appears capable of supplying a desideratum in meteorological observations—a register hygrometer.

becomes as much lighter as the weight of the mercury expelled from the tube, and the weight at the top of the window, or of the ventilator, opens these apertures which were kept shut by the weight of the statical tube and ball. On the other hand, when the cooling of the air in the chamber causes the contraction of the air included in the ball, the pressure of the atmosphere forces the mercury into the tube, which thus becomes so much heavier; and as it descends, it drags with it the window frame, or ventilator, attached to it.

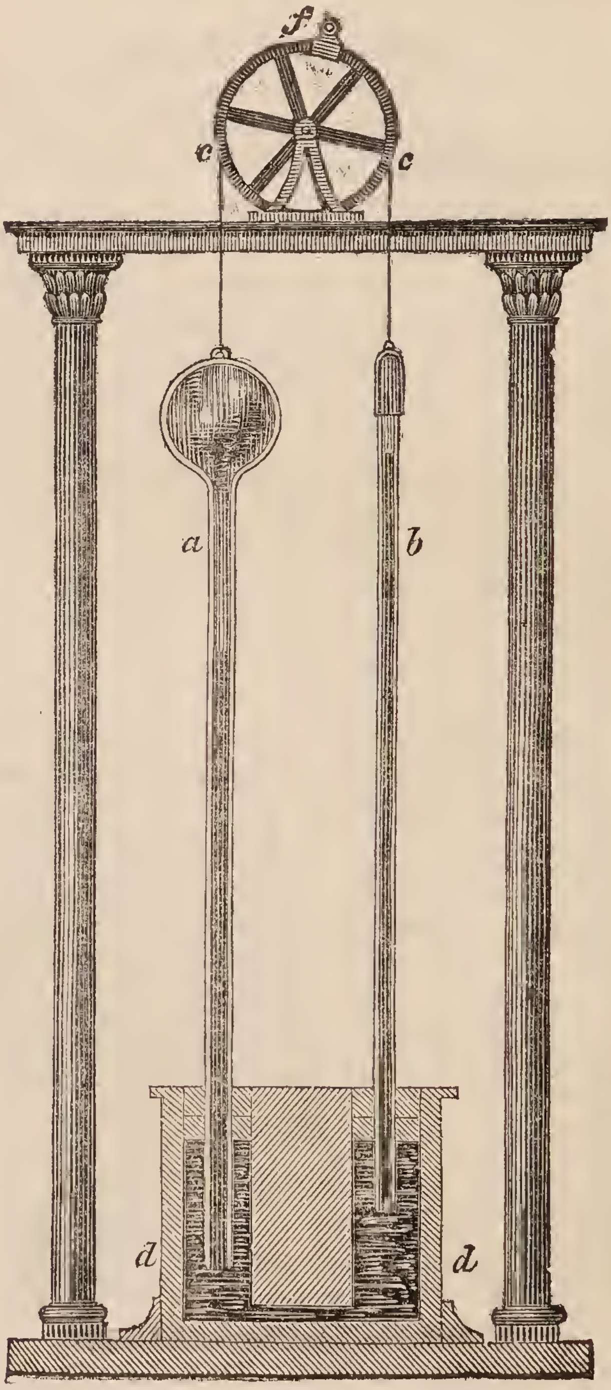
Fig. 44.



This simple and very ingenious contrivance is applicable to hot-houses, rooms, and apartments of every description, that are liable to considerable changes of temperature; and it possesses considerable powers: for in a tube two inches in diameter, every inch in the rise or fall of the mercury is equivalent to a moving power of about one pound. It is liable to be slightly altered also by changes in atmospheric pressure.

Dr. Cummings's attention was drawn to the importance of regulated temperature in the treatment of disease, whether in public institutions or in private practice, and the contrivance above noticed was the method by which he endeavoured to obtain this important object: but he soon perceived that the principle was applicable to various meteorological purposes; and the instrument has, in his hands, undergone successive modifications and improvements, until it has become the basis of a thermometer, hygrometer, and photometer, capable of registering their own indications, by the aid of clock-work, at any given time. Details of these different contrivances would lead us into too

Fig. 45.

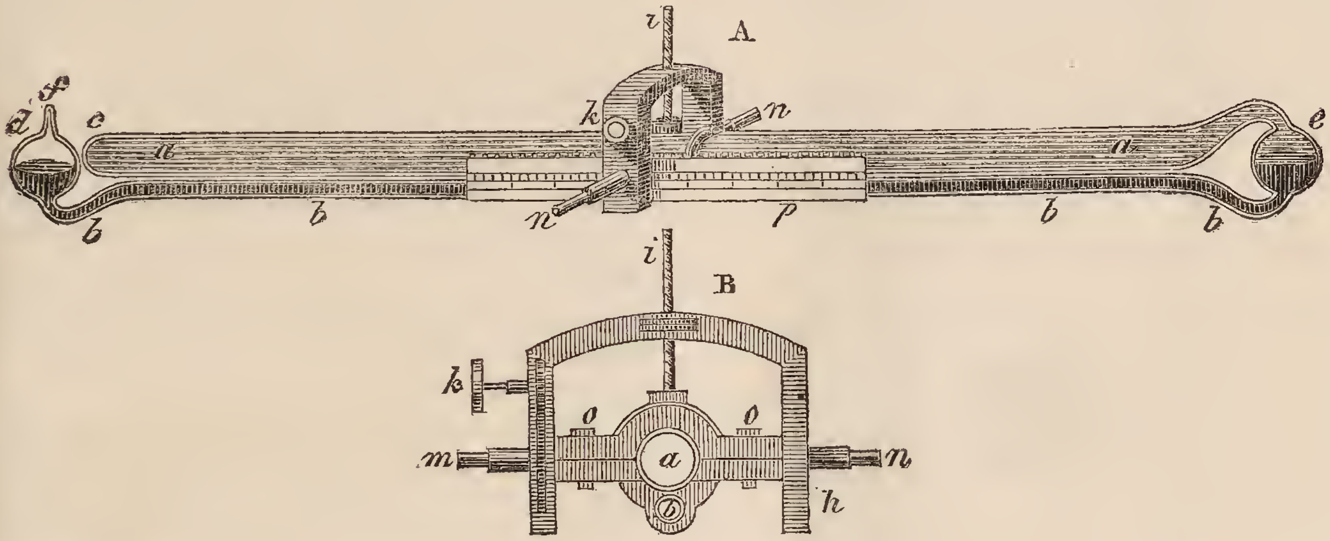


wide a field; but it may be proper to state, that finished drawings of them all have been in the possession of a distinguished member of the Meteorological Society of London for upwards of four years. The principle of them will be readily understood from the preceding figure, (*fig.* 45,) in which *a* represents an air thermometer; *b* a barometer, suspended from the opposite side of the wheel *c c*, to compensate the influence of variations in atmospheric pressure on the instrument; *d d* is a siphon-cistern, in both sides of which the mercury will always remain on the same level; *f* is an index, to which a pencil may be fixed, for tracing the variations of the instrument on a plate revolving by means of clock-work.

The portions of the tubes which dip into the mercury should be of equal substance; and the tube of the air-thermometer should be a cylinder capable of containing twice as much mercury as the corresponding portion of the barometer which counterpoises it. A small correction may be required for the varying immersion of the tubes, produced by the oscillations of the instrument. This must be determined by experiment, and allowed for in the graduation of the scale.

2. The *Balance Thermometer* of Mr. Kewley is a contrivance for a similar purpose, and is represented in *fig.* 46, A.

Fig. 46.



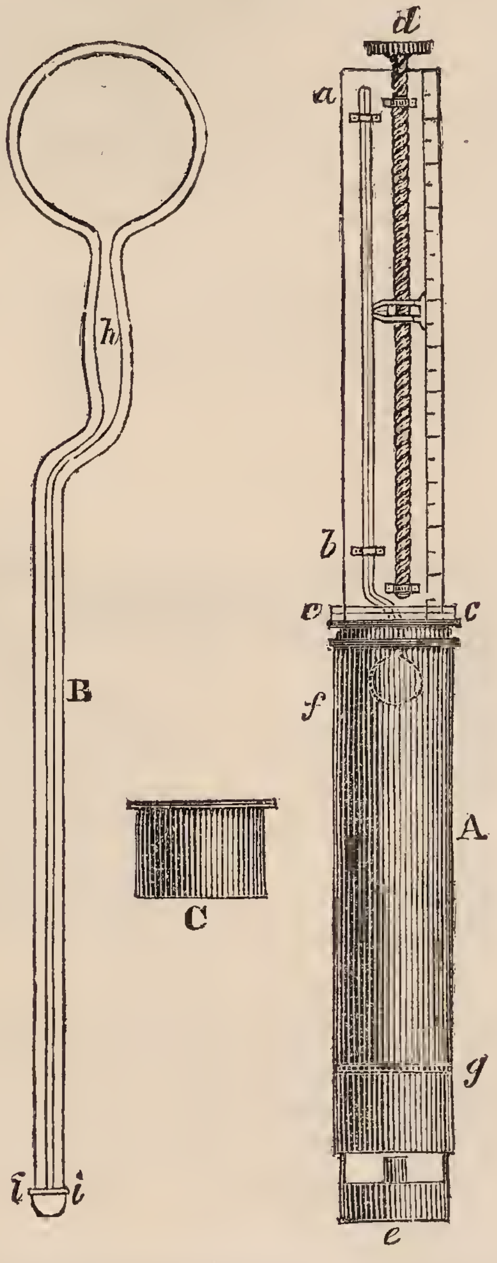
This instrument is the subject of a patent, the date of which is 1816. It consists of a tube of glass, *a*, *a*, closed at *c*, and terminating at *e* in a ball, which communicates with another tube of smaller diameter, which also terminates in a ball at *d*, having a communication with the external air at *f*. The tube *a*, *a*, and one half of the ball, *e*, are filled with spirit, or any light easily expansible fluid. The other tube, from *e* to *d*, is filled with mercury. The whole is suspended in the iron frame, *h*, *i*, *k*, *m*, B, by means of two clamping pieces, which are adjusted to the tubes by the screws, *o*, *o*. The centre of gravity is suitably adjusted by means of the milled nut, sunk in the transverse part of the frame, and receiving the screw, *i*; in order that the whole may librate on the knife edges, *m*, *n*, destined to rest on surfaces resembling the suspension frame of a common balance. A brass scale, *p*, is moved by the nut, *k*, on the arbor of which is a pinion playing in the *teeth* of the plate, *p*.

It is obvious, that by adjusting the mercury in each arm of this balance, it will be in *æquilibrio;* but when the spirit in *a* is expanded by heat, it will force some more of the mercury into the ball, *d*, and that arm of the instrument will preponderate; when it again contracts, the atmospheric pressure will cause the mercury to resume its original situation.

The instrument may be used as a thermometer, by ascertaining at what temperature it is in equilibrio, and when either end preponderates, finding how much is necessary to restore the balance by the motion of the brass plate, *p;* but its chief value arises from its applicability to shut and open doors or windows, according to the temperature of the apartment; in which case, a lever, or tooth-wheel, is fixed on one of its centres of oscillation. It is almost needless to remark, that the whole may be constructed of iron, and of any convenient size. In point of simplicity and cheapness, however, it is inferior to that proposed by Dr. Cumming, which preceded it in date; although until now no detailed account of that invention has been published.

3. The *Barometrical Thermometer* of the Rev. F. Wollaston is seen in *fig.* 47, A. It is a thermometer with a large bulb, devised for the measurement of altitudes, by observing the temperature at which liquids boil on different elevations;* on the principle first pointed out by Fahrenheit,† that the boiling of a fluid varies with the pressure of the atmosphere. Cavallo first applied this principle to the measurement of heights,‡ and the instrument proposed by Mr. Wollaston is intended to facilitate this method.

Fig. 47.



The bulb of the mercurial thermometer he proposed to use, is one inch in diameter, with a dilatation *h*, as seen in B, and ending in a capillary tube, five inches long, which is not closed in the usual manner, but, after being broken off smoothly, is sealed by a little cap of glass, as at *i*, *i*. The scale is 4.15 inches long, divided into 100 parts, and may be subdivided by a vernier into 1000 parts; giving 241 parts to each inch of the scale; and to facilitate observation, these are read off by a small lens jointed to the index, but not represented in our figure. The index is moved by a micrometer screw, *d*. The thermometer is supported by means of stuffing between two circular plates of metal, *c*, *c*, through which it passes, and which are tightened by screwing them together. They form a metallic collar, that may be screwed by either end into the top of the copper boiler, *f*, *g*, which becomes the case of the thermometer on inverting it; and then the bulb is protected by a copper cap C, which also serves as a measure of the due quantity of water to be used in the experiment. The portion of the copper tube below the bottom, *g*, of the boiler, is capable of holding the lamp, *e*, which is attached to it by two sliding wires. Thus the instrument becomes very portable. The boiler is 5.5 inches deep and 1.2 in diameter, with an aperture at the top to permit the escape of the steam, by which the heat is applied to the bulb.

When we have to determine an altitude, the boiling point is noted at the bottom of the eminence, and again when we have ascended; and the value of the difference between those points on the scale having been ascertained by experiment, we can estimate the height ascended, provided no change has, in the mean time, taken place in the barometrical pressure; or these points may be simultaneously found by two observers. The only correction required is, for the specific gravity of air at different temperatures, which may be found by General Roy's tables. The use of the dilatation, *h*, is to receive the expanded mercury, before it arrives at the boiling point: and the small cap, *i*, *i*, is intended to receive a globule of mercury, to be detached occasionally from the column in the stem, when it is wished to alter the range of the scale to suit various altitudes. The method of separating this globule is, to elevate the mercury, by heating the bulb until the thread of metal may be shaken over the flat end of the capillary tube; and when we wish to join the globule again to the thread, the two portions of mercury are brought into contact by heat, and then as the instrument, held in the vertical position, cools, the globule will follow the thread into the stem.

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* Phil. Trans. for 1817.

† Phil. Trans. vol. xxxii.

‡ Phil. Trans. vol. lxvi.

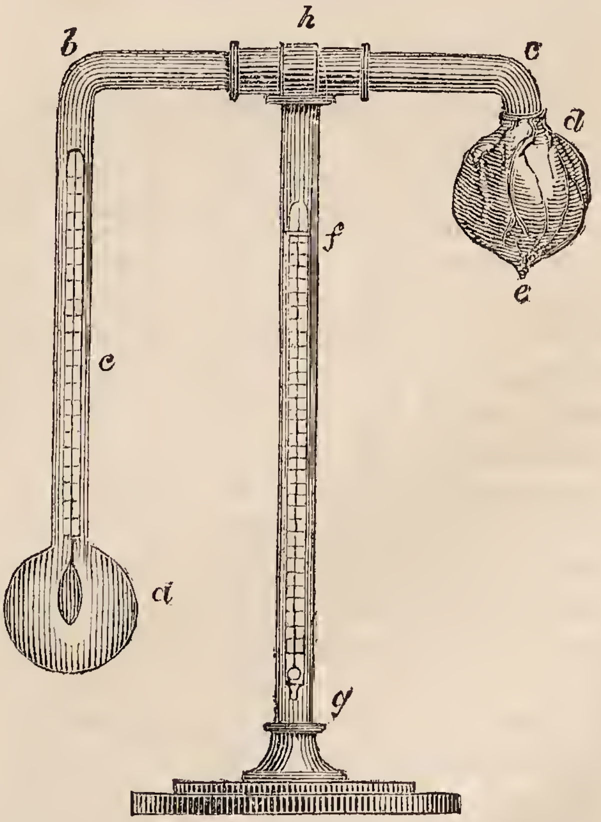
This instrument is of great delicacy; being capable of showing a difference of altitude of not more than three feet; but, unfortunately, although not very bulky, it is not very portable, from the liability of the stem to be broken by the weight of the bulb, even from the usual jolting of a carriage; an accident which happened thrice to the writer of this, within one month. From this circumstance, and its price, it is not likely to supersede the barometer in geological surveys.

4. M. Le Roi was the first who suggested the temperature at which dew begins to be deposited as a method of ascertaining the moisture of the air. De Luc has the merit of having proved that the quantity and force of vapour in a vacuum of any given dimensions, are equal to its force and quantity, in an equal volume of air, at the same temperature; or that the force and quantity of vapour in the air are dependant on its temperature.* This was confirmed by Mr. J. Dalton,† who investigated the force of vapour, at every temperature, from 0° to above 212° Fahrenheit, and expressed this force by the height of the mercurial column, which it could support in a Torricellian tube. These results are given in a tabular form, and are thus easily applied to hygrometric purposes. Dalton finds the *dew point*, like Le Roi, by pouring *cold* water into a glass, and marking the temperature at which it just ceases to cause the deposition of dew on the sides of the glass, in the open air. This is the point at which, in an air of that temperature, dew would just begin to be formed. From this fact he is able to infer, not only the force exerted by the vapour, but its quantity in a perpendicular column of the whole atmosphere, and the force of evaporation at the time of observation.

Thus, if the *dew point* be 45°, the force of vapour in Dalton's table = 0.316 of an inch of the mercurial column, or the one-ninety fifth of the whole atmospheric pressure; or, if the specific gravity of steam be 0.70, the weight of the steam or vapour in a given volume of air will be the one hundred and thirty-sixth part of the whole. Now, as the force of a *whole atmosphere of steam*, at the surface of the earth, would be the weight of a perpendicular column of it, and as in a mixed atmosphere of steam and air, the force exerted by each is as their relative weights, it follows, that when the dew point is 45°, the whole superincumbent column of vapour in the atmosphere, being equal to the one-ninety-fifth of the whole atmospheric pressure, will be equivalent to a pressure of 4.30 inches of water; or the vapour, if condensed, would afford that depth of water. From these data, Dalton has shown how we can find the force of evaporation at a given time: for the quantity of water evaporated from a given surface is proportional to the maximum force of vapour at the temperature of that surface; it being understood that the vapour is still in contact with a surface of water. Hence, if we have the dew point 45°, while the temperature of the air is 50°, by subtracting the force of vapour at 45° from that of 50°, we shall obtain the force of the evaporation at that time—thus, .375—.316 = .059, the force of evaporation.

5. It is on this principle that *Daniell's Hygrometer* is constructed; the invention of a gentleman distinguished for his meritorious labours in meteorology. It was published in 1820, along with a meteorological table, and seems to have been suggested by the cryophorus of Wollaston.‡ The form of the instrument is seen, as last improved by Mr. Daniell, in (*fig.* 48.) The ball *a* is of

Fig. 48.



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[*] Recherches sur les Modifications de l'Atmosphère.

[†] Manchester Memoirs, vol. v. 535.—vol. i. new series, p. 252.

[‡] Quarterly Journal of Science, vol. viii. 299—see also vol. ix. &c.

black glass, about one and a quarter inch in diameter, and is connected with a ball *d* of the same size, by a bent tube one-eighth of an inch in diameter. A portion of sulphuric æther, sufficient to fill three-fourths of the ball *a*, is introduced; a small mercurial thermometer, with a pyriform bulb, is fixed in the limb *a b*, the atmospheric air is expelled as completely as possible; and the whole is sealed at *e*. The ball *d* is covered with muslin; the whole is supported on a brass stand *f*, *g*, on which is another delicate mercurial thermometer. The tube can be removed from the spring tube *h;* and the whole, together with a phial of æther, packed neatly in a box, that goes easily into the pocket. The method in which the dew point is indicated by this instrument, is as follows:—The æther is all brought into the ball *a*, by inclining the tube; the balls are placed perpendicularly; the temperature of the ambient air is now noted; æther is poured from a dropping tube that fits the mouth of the small phial, on the muslin cover of *d;* and, the cold produced by its evaporation, causing a condensation of the elastic æthereal vapour within the ball, produces a rapid evaporation from *a*, by which the temperature of the thermometer in it sinks; and when the black ball is thus cooled to the dew point, a film of condensed vapour, like a ring, surrounds the ball. If the thermometer be at that instant noticed, we obtain the true dew point of air at the temperature indicated by the other thermometer.

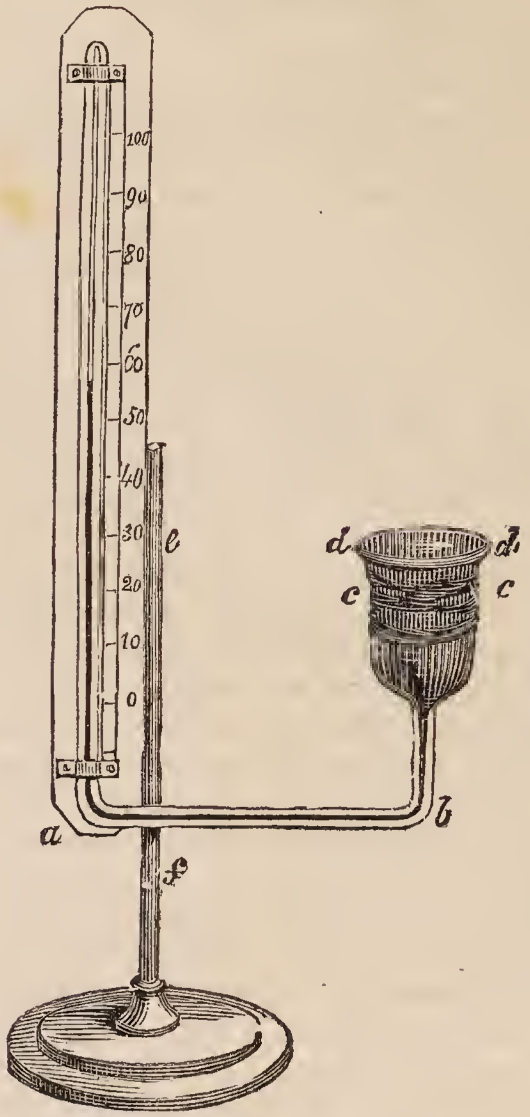
The observation is made in a very short period; and much of the labour required in the method of Le Roi is saved. There seems but one objection to this very ingenious instrument, and it is one, which, even with much practice, is not easily obviated. The surface on which the dew condenses, is small, and requires a peculiar light to be well seen; while the attention of the observer, distracted between the close inspection of the surface of the black ball, and the included thermometer, is not always able to fix with absolute precision the dew point.

6. This instrument has been modified in

Mr. Thos. Jones's Hygrometer;

an instrument on exactly the same principle as the original invention of Daniell, but simpler in construction, more compact, and less expensive. It is seen in *fig.* 49, consisting of a delicate mercurial thermometer, with its tube at *a*, *b*, bent so as to bring its cylindrical ball *c*, parallel with, and at a little distance from, its stem. The bulb is one inch long, and is terminated by a flattened surface *d*, of black glass, which projects a little beyond the sides of the bulb. The bulb below the flattened surface is covered with black silk. The instrument is supported on the wire *e f*, which is attached to the scale by a pivot, that allows the black surface to be inclined to the light, and the whole, with a phial of æther, are contained in a small case.[*]

Fig. 49.



When used, the temperature of the air is first noted; then æther is poured on the silk cover of the bulb; and the condensation of the dew is seen on the black extremity of the bulb.

The difficulty of marking the incipient condensation on these instruments is the same; and it has produced various modifications of the instrument.

7. Dr. Cumming, of Chester, finds that the dew point is most conspicuously shown, by inclosing the bulb of a delicate thermometer, covered by a sponge, in a tube of planished tinned iron, silver, or platina. When the sponge is moist-

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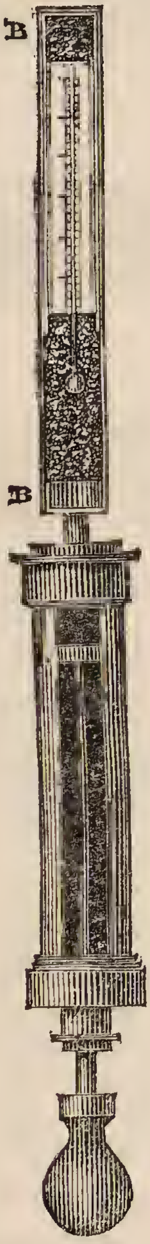
[*] Phil. Trans. for 1826, part ii. p. 23.

ened with any very evaporable fluid, such as æther or alcohol, and a stream of air blown through the tube, a more rapid and more conspicuous deposition of dew takes place on the surface of the metallic tube, than we ever recollect to have observed in similar experiments.

Fig. 50 is Dr. Cumming's hygrometer fitted to a portable air-syringe, by which a current of air is produced through the tube B B. The bulb of the delicate thermometer within it is surrounded with fine sponge, to retain the evaporable fluid; the tube B B is of highly polished metal, with an aperture in its upper part covered with a glass tube, for the inspection of the thermometer, as represented in the figure.

Fig. 50.



Chapter VI.

On the Imperfections common to all Instruments for the Indication of Heat.

1. The terms *thermometer* and *pyrometer* might lead to the supposition, that the instruments so designated were actual indicators of the quantity of *caloric* contained in those bodies to which they are applied; but a single experiment is sufficient to show that this view is erroneous. If we place equal quantities of water and of snow, both at temperature 32°, in a room at 60°, the temperature of the water will, as indicated by the thermometer, after some time, rise considerably; but the effect of the heat on the ice will only be to melt it partially, while its temperature remains steadily at 32°. Here we have *caloric* received by the ice which does not affect the thermometer.

The principle upon which the thermometer and the pyrometer act is, the tendency which heat or *caloric* has to diffuse itself among contiguous bodies. When applied to a hot body, they acquire a portion of the heat from that body; and when applied to a cold one, they communicate to that body a portion of their own caloric. These changes in the quantity of its own caloric are indicated by changes in the bulk of the thermometric fluid, or pyrometric piece; and such instruments, therefore, do no more than show a certain excess of heat given out by the hottest to the coldest body. On this ground the names of *thermoscope* and *pyroscope* are more suitable for such instruments than their more common designations.

That different bodies, in equal quantities, whether measured by weight or volume, contain unequal quantities of caloric, has been established by the investigations of Boerhaave, Black, Wilcke, Irvine, Crawford, Lavoisier, &c. It does not belong to this place to enter into this subject, but it is sufficient to mention the grounds for this important conclusion.

If we mix one pound of water at 212°, and as much water at 32°, when due precautions are employed to mix them without loss of heat or the addition of extraneous temperature, the thermometer plunged in the mixture will indicate very nearly 122°, or the arithmetical mean between the extremes; which proves that equal quantities of the *same body* contain quantities of caloric proportional to their temperature. If, however, we mix a pound of mercury with a pound of water, at different temperatures, when the mercury is the hottest, the temperature of the mixture will be greatly *below* the mean; and when the water is the hottest body, the mixture will be greatly *above* the mean temperature. A series of such experiments have shown that the same quantity of caloric which can raise the temperature of water only 4°, will raise that of mercury 112°. If this be the case at every temperature, which is most probable, the *quantity of caloric* in water is to that in an equal weight of mercury, at the same temperature, as 112 : 4 = 28 : 1.

Besides this method of finding out the comparative quantity of caloric in bodies, there is another founded on the fact, that ice in melting absorbs an uniform quantity of caloric. Professor Wilcke, of Copenhagen, first conceived the idea of employing the melting of ice or snow, for the purpose of ascertaining the comparative quantity of caloric in different bodies; and this method was improved in the hands of Lavoisier and Laplace, by the invention of the *Calorimeter.*

It consists of two vessels of tinned iron, and a wire cage, which are fitted so that one may be inserted within the other, leaving a cavity between the sides of each. (See *fig.* 51.) The wire-cage, *a*, is the innermost; and is destined to receive the heated body, the subject of experiment. The space between it and the second vessel, *b*, is to be filled with pounded ice, or snow, as well as the perforated cover, *f*, of the cage *a*. It is the melting of this snow which affords the indication of the comparative quantity of caloric in the bodies submitted to experiment; it rests on a wire sieve, at the bottom of the cylinder *b*, and is received at the orifice of the pipe *d*. To guard against the effects of external temperature, the cavity between the vessel *b*, and the exterior one, is filled also with pounded ice or snow; the general lid *h*, of the whole being also covered with snow, and its edges resting in a groove *e*, lined with the same material, the interior of the instrument is defended from all direct access of external temperature. The two tubes, *g*, in the lid, are for the introduction of thermometers; but during experiments those tubes are shut up, to prevent the access of currents of air through the calorimeter. The water collected between the outer and second vessel may be drawn off by the pipe *i*. Experiments of this kind should be made in a room at a temperature of 32°. Before commencing the experiment, the snow is saturated with moisture by its melting, to obviate as much as possible the error from not collecting the whole of the water.

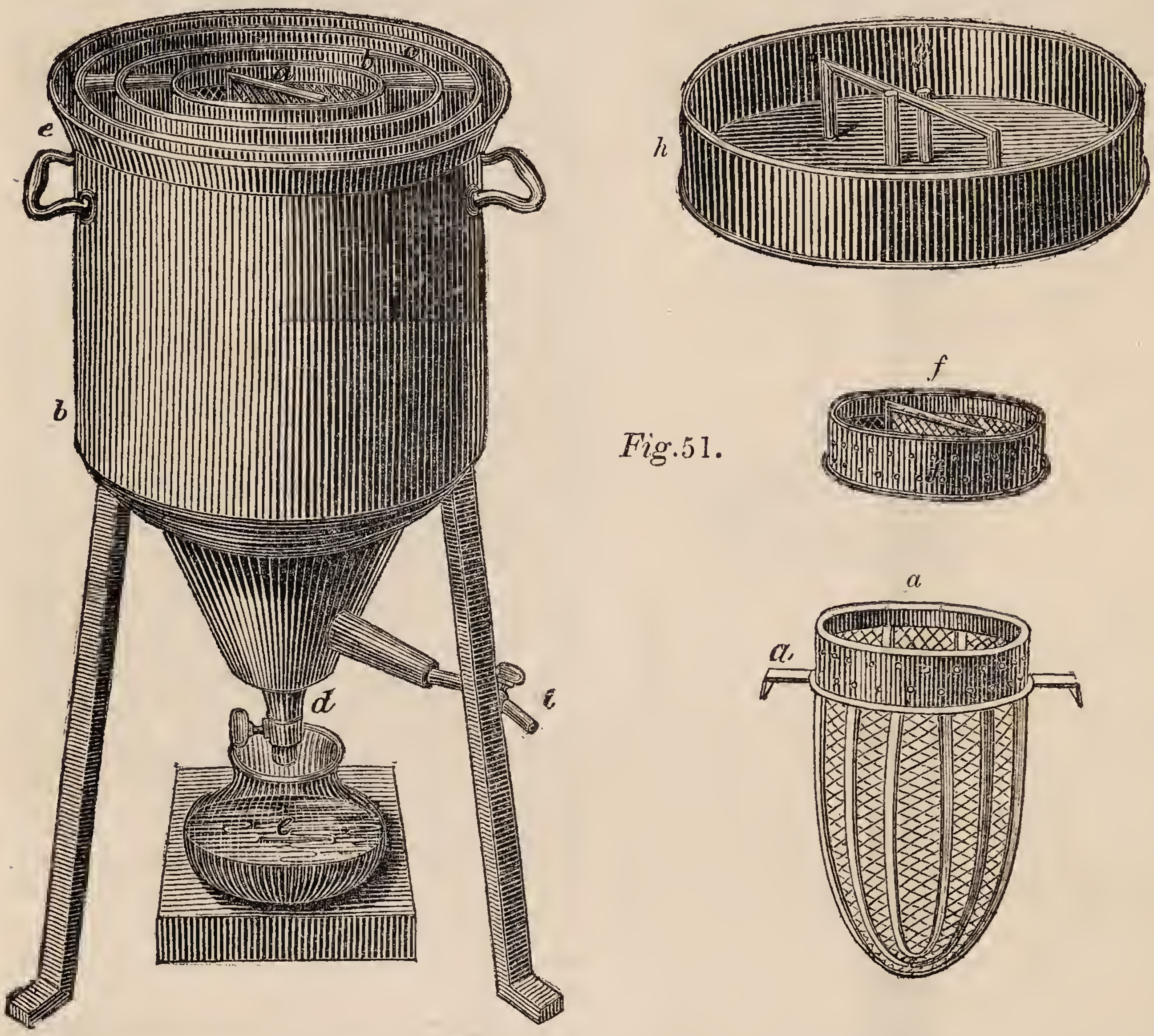


*Fig.*51.

The indications of this instrument,

from the cause just alluded to, and the impossibility of altogether obviating the effects of currents of air through it, during experiments, are not susceptible of such accuracy as the more simple method of mixture; although this also is liable to lead to erroneous conclusions, from the difficulty of obtaining the true result of the effect of the different mixtures.

2. There is an obvious source of error in the indications of all instruments employed to measure temperature, in as much as the apparent changes in the volume of the thermometric substance are not the real augmentation or diminution of bulk it undergoes. The glass of the thermometer, and the frame of the pyrometer, are also expanded and contracted by changes of temperature; so that our instruments only show the *excess of the expansions* of the thermometric fluid, or pyrometric bar over those of the glass and frame; by which the true indications are diminished.

From the extreme nicety of some of the investigations in which the mercurial thermometer is employed, a comparative ratio of the expansions of mercury and glass has been most diligently sought after by De Luc, Ramsden, Roy, and others. From their investigations it has been ascertained that all solids and liquids vary in their rate of expansion; but that the expansibility of glass depends so much on the manufacture of that article, and varies so much in the different kinds, that no general equation as a correction for this source of error can be of practical utility. Even the form of the glass rod is material. Roy gives the expansion of a glass tube = 0.0046569th; of a solid glass rod = 0.0096944th, in passing from the freezing to the boiling point of water.*

An important series of experiments by Lavoisier and Laplace have been published by Biot,† from which it appears that of twenty-three solids tried, glass was the least expansible of them all.

The length of different glass rods, which at 32° Fahrenheit = 1.00000000, at 212° Fahrenheit is augmented as follows:

Glass of St. Gobain	1.00089089	$=\frac{1}{1122}$
Glass tube, without lead	1.00087572	$=\frac{1}{1142}$
Ditto	1.00089760	$=\frac{1}{1114}$
Ditto	1.00091751	$=\frac{1}{1096}$
French glass, with lead ...	1.00087199	$=\frac{1}{1147}$
English flint glass	1.00081166	$=\frac{1}{1248}$

This will show the impossibility of any general correction being applied; and the adoption of any formulæ for this purpose would be an affectation of accuracy, of which, unfortunately, the subject is incapable. If, however, such formula is considered desirable, it may easily be constructed from the experiments of De Luc on glass tubes ‡ at different temperatures, which, reduced to the scale of Fahrenheit, are,

Temp.		Bulk.	Temp.		Bulk.
32°	=	100000	150°	=	100044
50	=	100006	167	=	100056
70	=	100014	190	=	100069
100	=	100023	212	=	100083
120	=	100032			

3. Another error of some magnitude is produced by the inequalities of the expansions of the same substances by equal increments of temperature.

If we could consider expansion simply as the effect of the application of heat, equal increments of temperature should produce equal rates of increase of volume; but the expansion is the *resultant*, in solids and in liquids, of two opposite forces—of the repulsive energy of caloric opposed by the cohesion of the particles of matter; and, accordingly, it not only differs in the different kinds of solid and liquid matter, but in the same body at different temperatures. As might be expected from this view, it must be in an increasing ratio with the temperature; because the force of cohesion must diminish with the distance of the particles of matter.

In aëriform bodies, the force of cohesion does not exist; and we might infer that equal increments of heat would produce equal expansions, in all gases, at all temperatures. In gases, the ratio might even be expected to decrease, in a minute but inappreciable degree, with the temperature; because the increased distance of the particles will tend to diminish the repulsive energy.

Experiment in these particulars accords with theory. The ratio of expansion in solids and liquids is found to be an *increasing* one, as the temperature is augmented, and is very different in each substance; while in the gases, it is not only equable in the same gas, but equal in all.

The manner in which an increasing

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* Phil. Trans., vol. lxxv.

† Traité de Physique, t. ii. 158.

‡ Recherches, t. i.

ratio of expansion must affect thermometers and pyrometers, graduated on the principle of equal degrees between two fixed points, is obvious; and it early became an object of solicitude. Drs. Halley and Brooke Taylor devised the method of investigation—namely, by mixing together equal weights of water at different ascertained temperatures, and finding how much the thermometer, plunged in that mixture, differed from the mean temperature. From not sufficiently attending to the various requisites to obviate error, they did not arrive at the true conclusion. We owe to De Luc the more successful investigation of this problem; by a train of nice experiments, in which he endeavoured to guard against the sources of error arising from the cooling effect of the vessel containing the mixture, and of the escape of vapour, he proved that the different thermometric fluids do not expand in a uniform ratio to the quantities of caloric applied; but follow an increasing rate as their temperature is raised. Mercury he found to be the most regular in its expansions; yet it also showed very sensible deviations. When equal weights of water at 32° and 212° were mixed, the mercurial thermometer did not indicate the mean temperature 122°, but only 119°; an oil thermometer, in the same experiment, stood no higher than 117°, and one of spirit of wine at 108°; while, with a thermometer filled with water, the temperature of the mixture appeared only to be 75°. His experiments showed the great superiority of the mercurial over the alcoholic thermometer; but this superiority, it probably owes, as Mr. John Dalton has remarked, in a great measure to the distance of the ordinary range of temperature, from the freezing and boiling point of mercury; for the experiments of De Luc show that the irregularities of all fluids are much augmented about the points of their consolidation and passing into vapour.

From the usual method of graduating the mercurial thermometer from two fixed points only, the error from inequality of expansion will be greatest at the mean between the two points; when, according to De Luc, it amounts to about 3° Fahrenheit below the real temperature. In the following table are given the result of De Luc's researches, on the two fluids chiefly used for thermometers —mercury and alcohol. In the first column are the indications of the mercurial thermometer, according to De Luc's or Reaumur's scale; in the second, the indications of the alcoholic thermometer at the corresponding temperatures; and in the third, are, what ought to be, the real temperatures as discovered by experiment.

Mercurial Ther.	Alcohol Ther.	Real Tem.
80	80.0	80.0
75	73.8	75.28
70	67.8	70.56
65	61.9	65.77
60	56.2	60.96
55	50.7	56.15
50	47.3	51.26
45	40.2	46.37
40	35.1	41.40
35	30.3	36.40
30	25.6	31.32
25	21.0	26.22
20	16.5	21.12
15	12.2	15.94
10	7.9	10.74
5	3.9	5.43
0	0.0	0.0

There is reason to believe, however, that De Luc states the irregularity of the mercurial thermometer too high. Dr. Crawford investigated this point with great care, and concluded, that when the difference of temperature of the two portions of fluid did not exceed 100° F., the average deviations of the mercurial thermometer were not above 0.25 of a degree. De Luc himself allows, that the result of the mixtures must be inaccurate, if the *capacity* of the water operated on is changed during the experiment. In mixing together hot and cold water, the probability is, that the diminished volume of the mixture causes a diminution of capacity; and, consequently, an *increase* of temperature, beyond what is due to the heat of the two portions mixed together. By exposing a mercurial thermometer in a vessel, in which the included air was exposed to the frigorific influence of melting snow, and the heat of watery vapour at 212°, he found that it indicated 121°, or only a single degree less than the arithmetical mean.* From comparison with air thermometers, Gay Lussac inferred that the mercurial thermometer was equable in its expansions, between the freezing and the boiling point of water.[*] Petit and Dulong have investigated the subject, by a comparison with the air thermometer, and with the expansions of a pyrometer of very infusible metals (platina and copper) which appear, by the experiments of Lavoisier and Laplace, to be very equable in their expansions, below the boiling point of water. They found that the irregularity in mercury increases with the temperature; and would even appear to be greater than the rise of the mercurial thermometer indicates, were not the increasing expansion of the mercury diminished by the increasing ratio of the dilatation of the glass itself.[†] Hence the source of error in thermometers arising from the expansion of the glass, is rather advantageous than detrimental to their accuracy.

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* Experiments on Animal Heat.

The difference between the indications of the alcoholic and mercurial thermometers, as lately ascertained by Dr. De Wildt, do not materially differ from the determinations of De Luc; and having been obtained for every 5° of Reaumur's scale, apparently with much care, we give the result as a table of correction for Rutherford's thermometers; for which purpose they were intended by the author.

Mercury.	Spirit.	Mercury.	Spirit
−45° =	−28°.50	+20°	+16°.48
40	25 .92	25	20 .97
35	23 .19	30	25 .60
30	20 .32	35	30 .38
25	17 .30	40	35 .31
20	14 .13	45	40 .38
15	10 .82	50	45 .60
10	7 .36	55	50 .97
5	3 .75	60	56 .48
0	0 .00	65	62 .14
+5	+ 3 .90	70	67 .95
10	7 .95	75	73 .90
15	12 .14	80	80[‡].00

The indications of air thermometers were at one time supposed liable to uncertainty, from the inequalities of their expansion. Guyton and Prieur imagined that they progressively expanded in a greater ratio than the temperature,[§] but this has been proved to be erroneous; and the mistake probably arose from their neglecting the effect of hygrometric water in the gases. General Roy[*] found that their expansion followed a ratio, *decreasing* with the elevation of temperature; and the same result was obtained by Mr. Dalton,[†] Dr. Murray,[‡] Gay Lussac,[§] and Petit and Dulong;[||] but there is reason to conclude that this apparently decreasing ratio in the expansion of gases is owing to the error caused by the unequable expansions of the mercury in the thermometer, and the dilatation of the bulb of that instrument; and philosophers now agree to consider the expansions of gases equable and equal, as before stated; especially as the decreasing ratio disappears, if we apply De Luc's correction of the real mean between 32° and 212°.

From the foregoing observations, we may conclude, that the air thermometer requires no correction of its indications; that the accuracy of the mercurial thermometer is not materially affected by the inequalities of the expansions of the mercury, in ordinary ranges of temperature; that the expansion of alcohol is pretty uniform, until about 30° R. or 100° F.: above that point its expansions become more irregular; but it has the advantage over every other liquid, of marking the lowest degrees of natural or artificial cold hitherto observed.

The irregularities affecting pyrometers, except from alteration in the size of the substances supporting the bars, are extremely minute. The experiments of De Luc and Roy would lead to the conclusion, that the expansibility of solids is not quite equable. Roy thinks that this slight irregularity may be apparent rather than real; but Lavoisier and Laplace state, that the expansions of solids keep pace with those of the mercurial thermometer, from the freezing to the boiling point of water; and Petit and Dulong assert, that the expansion of metals is progressive above 212° F.

These irregularities, if they exist, are so minute, as not, in any ordinary practical purpose, to affect the indications of pyrometers; and at high temperatures extreme accuracy is seldom of much consequence.

The differences arising from two pyrometers of different materials may be corrected by Table IV. in the appendix of the results obtained in 1782, by Lavoisier and Laplace, lately recovered by Biot. Each substance at 32° F. = 1.00000000.

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[*] Annales de Chimie, t. xliii.

[†] Annales de Chimie et Physique, t. ii. p. 240.

[‡] Jameson's Edin. Phil. Journal, Oct. 1826, and Kastner Archiv fur die Gesammte Natural, Decem. 1825.

[§] Journal de l'Ecole Polytechnique.

[*] Phil. Trans. vol. lxvii.

[†] Manchester Memoirs, vol. v. p. 599.

[‡] Edin. Phil. Trans. [§] Ann. de Chim. t. xliii.

[||] Annales de Chim. et Phys. t. ii.

There is one precaution in graduating thermometers, which will render any irregularities of little consequence; that is, in forming the scale, not to rest satisfied with only two fixed points, and *equally dividing* the intervening space; but to obtain intermediate points by means of mixtures, or by comparison with a standard instrument so formed; and by the shortness of the intervals adapting the scale to inequalities in the bore of the tube, or to the less important irregularities just now considered.

APPENDIX.

No. I.—TABLES OF CORRESPONDENCE OF THE DIFFERENT THERMOMETRICAL SCALES.

(From Dr. Murray's *System of Chemistry*).

TABLE FOR THE CENTIGRADE THERMOMETER.

Centigrade.	Reaumur's.	Fahrenheit's.	Centigrade.	Reaumur's.	Fahrenheit's.	Centigrade.	Reaumur's.	Fahrenheit's.
100	80.	212.	53	42.4	127.4	6	4.8	42.8
99	79.2	210.2	52	41.6	125.6	5	4.	41.
98	78.4	208.4	51	40.8	123.8	4	3.2	39.2
97	77.6	206.6	50	40.	122.	3	2.4	37.4
96	76.8	204.8	49	39.2	120.2	2	1.6	35.6
95	76.	203.	48	38.4	118.4	1	0.8	33.8
94	75.2	201.2	47	37.6	116.6	0	0.	32.
93	74.4	199.4	46	36.8	114.8	−1	−0.8	30.2
92	73.6	197.6	45	36.	113.	−2	−1.6	28.4
91	72.8	195.8	44	35.2	111.2	−3	−2.4	26.6
90	72.	194.	43	34.4	109.4	−4	−3.2	24.8
89	71.2	192.2	42	33.6	107.6	−5	−4.	23.
88	70.4	190.4	41	32.8	105.8	−6	−4.8	21.2
87	69.6	188.6	40	32.	104.	−7	−5.6	19.4
86	68.8	186.8	39	31.2	102.2	−8	−6.4	17.6
85	68.	185.	38	30.4	100.4	−9	−7.2	15.8
84	67.2	183.2	37	29.6	98.6	−10	−8.	14.
83	66.4	181.4	36	28.8	96.8	−11	−8.8	12.2
82	65.6	179.6	35	28.	95.	−12	9.6	10.4
81	64.8	177.8	34	27.2	93.2	−13	−10.4	8.6
80	64.	176.	33	26.4	91.4	−14	−11.2	6.8
79	63.2	174.2	32	25.6	89.6	−15	−12.	5.
78	62.4	172.4	31	24.8	87.8	−16	−12.8	3.2
77	61.6	170.6	30	24.	86.	−17	−13.6	1.4
76	60.8	168.8	29	23.2	84.2	−18	−14.4	−0.4
75	60.	167.	28	22.4	82.4	−19	−15.2	−2.2
74	59.2	165.2	27	21.6	80.6	−20	−16.	−4.
73	58.4	163.4	26	20.8	78.8	−21	−16.8	−5.8
72	57.6	161.6	25	20.	77.	−22	−17.6	−7.6
71	56.8	159.8	24	19.2	75.2	−23	−18.4	−9.4
70	56.	158.	23	18.4	73.4	−24	−19.2	−11.2
69	55.2	156.2	22	17.6	71.6	−25	−20.	−13.
68	54.4	154.4	21	16.8	69.8	−26	−20.8	−14.8
67	53.6	152.6	20	16.	68.	−27	−21.6	−16.6
66	52.8	150.8	19	15.2	66.2	−28	−22.4	−18.4
65	52.	149.	18	14.4	64.4	−29	−23.2	−20.2
64	51.2	147.2	17	13.6	62.6	−30	−24.	−22.
63	50.4	145.4	16	12.8	60.8	−31	−24.8	−23.8
62	49.6	143.6	15	12.	59.	−32	−25.6	−25.6
61	48.8	141.8	14	11.2	57.2	−33	−26.4	−27.4
60	48.	140.	13	10.4	55.4	−34	−27.2	−29.2
59	47.2	138.2	12	9.6	53.6	−35	−28.	−31.
58	46.4	136.4	11	8.8	51.8	−36	−28.8	−32.8
57	45.6	134.6	10	8.	50.	−37	−29.6	−34.6
56	44.8	132.8	9	7.2	48.2	−38	−30.4	−36.4
55	44.	131.	8	6.4	46.4	−39	−31.2	−38.2
54	43.2	129.2	7	5.6	44.6	−40	−32.	−40.

TABLE FOR REAUMUR'S THERMOMETER.

Reaumur's	Centigrade.	Fahrenheit's.	Reaumur's	Centigrade.	Fahrenheit's.	Reaumur's.	Centigrade.	Fahrenheit's.
80	100.	212.	42	52.5	126.5	4	5.	41.
79	98.75	209.75	41	51.25	124.25	3	3.75	38.75
78	97.5	207.5	40	50.	122.	2	2.5	36.5
77	96.25	205.25	39	48.75	119.75	1	1.25	34.25
76	95.	203.	38	47.5	117.5	-0	0	32.
75	93.75	200.75	37	46.25	115.25	-1	-1.25	29.75
74	92.5	198.5	36	45.	113.	-2	-2.5	27.5
73	91.25	196.25	35	43.75	110.75	-3	-3.75	25.25
72	90.	194.	34	42.5	108.5	-4	-5.	23.
71	88.75	191.75	33	41.25	106.25	-5	-6.25	20.75
70	87.5	189.5	32	40.	104.	-6	-7.5	18.5
69	86.25	187.25	31	38.75	101.75	-7	-8.75	16.25
68	85.	185.	30	37.5	99.5	-8	-10.	14.
67	83.75	182.75	29	36.25	97.25	-9	-11.25	11.75
66	82.5	180.5	28	35.	95.	-10	-12.5	9.5
65	81.25	178.25	27	33.75	92.75	-11	-13.75	7.25
64	80.	176.	26	32.5	90.5	-12	-15.	5.
63	78.75	173.75	25	31.25	88.25	-13	-16.25	2.75
62	77.5	171.5	24	30.	86.	-14	-17.5	0.5
61	76.25	169.25	23	28.75	83.75	-15	-18.75	-1.75
60	75.	167.	22	27.5	81.5	-16	-20.	-4.
59	73.75	164.75	21	26.25	79.25	-17	-21.25	-6.25
58	72.5	162.5	20	25.	77.	-18	-22.5	-8.5
57	71.25	160.25	19	23.75	74.75	-19	-23.75	-10.75
56	70.	158.	18	22.5	72.5	-20	-25.	-13.
55	68.75	155.75	17	21.25	70.25	-21	-26.25	-15.25
54	67.5	153.5	16	20.	68.	-22	-27.5	-17.5
53	66.25	151.25	15	18.75	65.75	-23	-28.75	-19.75
52	65.	149.	14	17.5	63.5	-24	-30.	-22.
51	63.75	146.75	13	16.25	61.25	-25	-31.25	-24.25
50	62.5	144.5	12	15.	59.	-26	-32.5	-26.5
49	61.25	142.25	11	13.75	56.75	-27	-33.75	-28.75
48	60.	140.	10	12.5	54.5	-28	-35.	-31.
47	58.75	137.75	9	11.25	52.25	-29	-36.25	-33.25
46	57.5	135.5	8	10.	50.	-30	-37.5	-35.5
45	56.25	133.25	7	8.75	47.75	-31	-38.75	-37.75
44	55.	131.	6	7.5	45.5	-32	-40.	-40.
43	53.75	128.75	5	6.25	43.25	-33	-41.25	-42.25

TABLE FOR FAHRENHEIT'S THERMOMETER.*

Fahrenheit's.	Reaumur's.	Centigrade.	Fahrenheit's.	Reaumur's.	Centigrade.	Fahrenheit's.	Reaumur's.	Centigrade.
212	80.00	100.00	170	61.33	76.66	128	42.66	53.33
211	79.55	99.44	169	60.88	76.11	127	42.22	52.77
210	79.11	98.88	168	60.44	75.55	126	41.77	52.22
209	78.66	98.33	167	60.00	75.00	125	41.33	51.66
208	78.22	97.77	166	59.55	74.44	124	40.88	51.11
207	77.77	97.22	165	59.11	73.88	123	40.44	50.55
206	77.33	96.66	164	58.66	73.33	122	40.00	50.00
205	76.88	96.11	163	58.22	72.22	121	39.55	49.44
204	76.44	95.55	162	57.77	72.77	120	39.11	48.88
203	76.00	95.00	161	57.33	71.66	119	38.66	48.33
202	75.55	94.44	160	56.88	71.11	118	38.22	47.77
201	75.11	93.88	159	56.44	70.55	117	37.77	47.22
200	74.66	93.33	158	56.00	70.00	116	37.33	46.66
199	74.22	92.77	157	55.55	69.44	115	36.88	46.11
198	73.77	92.22	156	55.11	68.88	114	36.44	45.55
197	73.33	91.66	155	54.66	68.33	113	36.00	45.00
196	72.88	91.11	154	54.22	67.77	112	35.55	44.44
195	72.44	90.55	153	53.77	67.22	111	35.11	43.88
194	72.00	90.00	152	53.33	66.66	110	34.66	43.33
193	71.55	89.44	151	52.88	66.11	109	34.22	42.77
192	71.11	88.88	150	52.44	65.55	108	33.77	42.22
191	70.66	88.33	149	52.00	65.00	107	33.33	41.66
190	70.22	87.77	148	51.55	64.44	106	32.88	41.11
189	69.77	87.22	147	51.11	63.88	105	32.44	40.55
188	69.33	86.66	146	50.66	63.33	104	32.00	40.00
187	68.88	86.11	145	50.22	62.77	103	31.55	39.44
186	68.44	85.55	144	49.77	62.22	102	31.11	38.88
185	68.00	85.00	143	49.33	61.66	101	30.66	38.33
184	67.55	84.44	142	48.88	61.11	100	30.22	37.77
183	67.11	83.88	141	48.44	60.55	99	29.77	37.22
182	66.66	83.33	140	48.00	60.00	98	29.33	36.66
181	66.22	82.77	139	47.55	59.44	97	28.88	36.11
180	65.77	82.22	138	47.11	58.88	96	28.44	35.55
179	65.33	81.66	137	46.66	58.33	95	28.00	35.00
178	64.88	81.11	136	46.22	57.77	94	27.55	34.44
177	64.44	80.55	135	45.77	57.22	93	27.11	33.88
176	64.00	80.00	134	45.33	56.66	92	26.66	33.33
175	63.55	79.44	133	44.44	56.11	91	26.22	32.77
174	62.11	78.88	132	44.55	55.55	90	25.77	32.22
173	62.66	78.33	131	44.00	55.00	89	25.33	31.66
172	62.22	77.77	130	43.55	54.44	88	24.88	31.11
171	61.77	77.22	129	43.11	53.88	87	24.44	30.55

* All the decimals in this Table are circulating decimals.

TABLE FOR FAHRENHEIT'S THERMOMETER CONTINUED.

Fahrenheit's.	Reaumur's.	Centigrade.	Fahrenheit's.	Reaumur's.	Centigrade.	Fahrenheit's.	Reaumur's.	Centigrade.
86	24.00	30.00	43	4.88	6.11	0	−14.22	−17.77
85	23.55	29.44	42	4.44	5.55	−1	−14.66	−18.33
84	23.11	28.88	41	4.00	5.00	−2	−15.11	−18.88
83	22.66	28.33	40	3.55	4.44	−3	−15.55	−19.44
82	22.22	27.77	39	3.11	3.88	−4	−16.00	−20.00
81	21.77	27.22	38	2.66	3.33	−5	−16.44	−20.55
80	21.33	26.66	37	2.22	2.77	−6	−16.88	−21.11
79	20.88	26.11	36	1.77	2.22	−7	−17.33	−21.66
78	20.44	25.55	35	1.33	1.66	−8	−17.77	−22.22
77	20.00	25.00	34	0.88	1.11	−9	−18.22	−22.77
76	19.55	24.44	33	0.44	0.55	−10	−18.66	−23.33
75	19.11	23.88	32	0.	0.	−11	−19.11	−23.88
74	18.66	23.33	31	−0.44	−0.55	−12	−19.55	−24.44
73	18.22	22.77	30	−0.88	−1.11	−13	−20.00	−25.00
72	17.77	22.22	29	−1.33	−1.66	−14	−20.44	−25.55
71	17.33	21.66	28	−1.77	−2.22	−15	−20.88	−26.11
70	16.88	21.11	27	−2.22	−2.77	−16	−21.33	−26.66
69	16.44	20.55	26	−2.66	−3.33	−17	−21.77	−27.22
68	16.00	20.00	25	−3.11	−3.88	−18	−22.22	−27.77
67	15.55	19.44	24	−3.55	−4.44	−19	−22.66	−28.33
66	15.11	18.88	23	−4.00	−5.00	−20	−23.11	−28.88
65	14.66	18.33	22	−4.44	−5.55	−21	−23.55	−29.44
64	14.22	17.77	21	−4.88	−6.11	−22	−24.00	−30.00
63	13.77	17.22	20	−5.33	−6.66	−23	−24.44	30.55
62	13.33	16.66	19	−5.77	−7.22	−24	−24.88	−31.11
61	12.88	16.11	18	−6.22	−7.77	−25	−25.33	−31.66
60	12.44	15.55	17	−6.66	−8.33	−26	−25.77	−32.22
59	12.00	15.00	16	−7.11	−8.88	−27	−26.22	32.77
58	11.55	14.44	15	−7.55	−9.44	−28	−26.66	33.33
57	11.11	13.88	14	−8.00	−10.00	−29	−27.11	−33.88
56	10.66	13.33	13	−8.44	−10.55	−30	−27.55	−34.44
55	10.22	12.77	12	−8.88	−11.11	−31	−28.00	−35.00
54	9.77	12.22	11	−9.33	−11.66	−32	−28.44	−35.55
53	9.33	11.66	10	−9.77	−12.22	−33	−28.88	−36.11
52	8.88	11.11	9	−10.22	−12.77	−34	−29.33	−36.66
51	8.44	10.55	8	−10.66	−13.33	−35	−29.77	−37.22
50	8.00	10.00	7	−11.11	−13.88	−36	−30.22	−37.77
49	7.55	9.44	6	−11.55	−14.44	−37	−30.66	−38.33
48	7.11	8.88	5	−12.00	−15.00	−38	−31.11	−38.88
47	6.66	8.33	4	−12.44	−15.55	−39	−31.55	39.44
46	6.22	7.77	3	−12.88	−16.11	−40	−32.00	−40.00
45	5.77	7.22	2	−13.33	−16.66			
44	5.33	6.66	1	−13.77	−17.22			

No. II.—TABLES OF EXPANSIONS OF BODIES BY HEAT.

I. TABLE OF EXPANSIONS ACCORDING TO SMEATON.

SUBSTANCES.	DILATATION FOR A LENGTH EQUAL TO UNITY.	
	In decimal fractions.	In vulgar fractions.
Blistered steel	0.00115000	$\frac{1}{870}$
Tempered steel	0.00122500	$\frac{1}{816}$
Bismuth	0.00139167	$\frac{1}{719}$
Copper hammered	0.00170000	$\frac{1}{588}$
Copper 8 parts with 1 of tin	0.00181667	$\frac{1}{550}$
Cast brass	0.00187500	$\frac{1}{533}$
Brass, 16 parts with 1 of tin	0.00190833	$\frac{1}{524}$
Fine pewter	0.00228333	$\frac{1}{438}$
Grain tin	0.00248333	$\frac{1}{403}$
Iron	0.00125833	$\frac{1}{795}$
Brass wire	0.00193333	$\frac{1}{517}$
Speculum metal	0.00193333	$\frac{1}{517}$
Lead	0.00286667	$\frac{1}{349}$
Antimony	0.00108333	$\frac{1}{923}$
Lead 2 parts with 1 of tin	0.00250533	$\frac{1}{390}$
Copper 2 parts with 1 of zinc	0.00205833	$\frac{1}{486}$
White glass (barometer tube,)	0.00083333	$\frac{1}{1175}$
Zinc	0.00294167	$\frac{1}{340}$
Zinc hammered	0.00310833	$\frac{1}{322}$
Zinc 8 parts with 1 of tin	0.00269167	$\frac{1}{372}$

II. TABLE OF EXPANSIONS ACCORDING TO ROY.

SUBSTANCES.	FOR A LENGTH EQUAL TO UNITY.	
	In decimal fractions.	In vulgar fractions.
Steel rod	0.00114450	$\frac{1}{874}$
Brass scale, Hamburgh	0.00185550	$\frac{1}{589}$
Brass plate rod, English	0.00189296	$\frac{1}{528}$
Brass plate trough, English	0.00189450	$\frac{1}{528}$
Cast Iron prism	0.00111000	$\frac{1}{901}$
Glass tube	0.00077550	$\frac{1}{1289}$
Glass rod	0.00080833	$\frac{1}{1237}$

III. TABLE OF EXPANSIONS ACCORDING TO TROUGHTON.

SUBSTANCES.	LINEAR DILATATIONS FROM THE TEMPERATURE OF FREEZING TO BOILING WATER.	
	In decimal fractions.	In vulgar fractions.
Steel	0.0011899	$\frac{1}{848}$
Silver	0.0020826	$\frac{1}{480}$
Copper	0.0019188	$\frac{1}{521}$
Iron Wire	0.0014401	$\frac{1}{694}$
Platina.	0.00099218	$\frac{1}{1008}$
Palladium (according to Wollaston)	0.0010000	$\frac{1}{1000}$

IV. TABLE OF THE LINEAR DILATATION OF DIFFERENT SUBSTANCES FROM THE TEMPERATURE OF FREEZING TO THAT OF BOILING WATER, ACCORDING TO THE EXPERIMENTS OF LAVOISIER AND LAPLACE.

SUBSTANCES.	DILATATION FOR A LENGTH EQUAL TO UNITY.	
	In decimal fractions.	In vulgar fractions.
	At 212°	
Glass of St. Gobain	0.00089089	$\frac{1}{1122}$
Glass tube without lead	0.00087572	$\frac{1}{1142}$
Ditto	0.00089760	$\frac{1}{1114}$
Ditto	0.00091751	$\frac{1}{1090}$
English Flint Glass	0.00081166	$\frac{1}{1248}$
French glass with lead	0.00087199	$\frac{1}{1147}$
Copper	0.00172244	$\frac{1}{581}$
Ditto	0.00171222	$\frac{1}{584}$
Brass	0.00186671	$\frac{1}{535}$
Ditto	0.00188971	$\frac{1}{529}$
Hammered Iron	0.00122045	$\frac{1}{819}$
Iron Wire	0.00123504	$\frac{1}{812}$
Hard Steel	0.00107875	$\frac{1}{927}$
Soft Steel	0.00107956	$\frac{1}{926}$
Tempered Steel	0.00123956	$\frac{1}{807}$
Lead	0.00284836	$\frac{1}{351}$
Malacca Tin	0.00193765	$\frac{1}{516}$
Cornish Tin	0.00217298	$\frac{1}{462}$
Cupelled Silver	0.00192974	$\frac{1}{518}$
Parisian Standard Silver	0.00190868	$\frac{1}{524}$
Pure Gold	0.00146606	$\frac{1}{682}$
Parisian Standard Ditto not softened	0.00155155	$\frac{1}{645}$
Ditto, softened	0.00151361	$\frac{1}{611}$

V. TABLE OF THE EXPANSIONS OF LIQUIDS.

The expansions in this table were determined by Mr. Dalton. They are equal to what would be produced by an elevation of temperature from the freezing to the boiling point of water; the volume at the former being 1.

Mercury	.0200	= $\frac{1}{50}$
Water	.0466	= $\frac{1}{21{\cdot}5}$
Water saturated with salt	.0500	= $\frac{1}{20}$
Sulphuric acid	.0600	= $\frac{1}{17}$
Muriatic acid	.0600	= $\frac{1}{17}$
Oil of turpentine	.0700	= $\frac{1}{14}$
Æther	.0700	= $\frac{1}{14}$
Fixed oils	.0800	= $\frac{1}{12{\cdot}5}$
Alcohol	.0110	= $\frac{1}{9}$
Nitric acid	.0110	= $\frac{1}{9}$

No. III.

TABLE OF REMARKABLE TEMPERATURES ACCORDING TO FAHRENHEIT'S SCALE.

	°
Iron red hot in the twilight	884
Heat of a common fire (Irvine)	790
Iron bright red in the dark	752
Zinc melts	700
Quicksilver boils (Irvine)	672
——————— (Dalton)	660
——————— (Crichton)	655
Linseed oil boils	600
Lead melts (Guyton, Irvine)	594
Sulphuric Acid boils (Dalton)	590
The surface of polished Steel acquires a deep blue colour	580
Oil of Turpentine boils	560
Phosphorus boils	554
Bismuth melts (Irvine)	476
The surface of polished steel acquires a pale straw colour	460
Tin melts (Crichton, Irvine)	442
A Compound of equal parts of Tin and Bismuth melts	283
Nitric Acid boils	242
Sulphur melts	226
A saturated Solution of Salt boils	218
Water boils, (the barometer being at 30 inches); also a Compound of 5 of Bismuth, 3 of Tin, and 2 of Lead, melts	212
A Compound of 3 of Tin, 5 of Lead, and 8 of Bismuth, melts	210
Alcohol boils	174
Bees' Wax melts	142
Spermaceti melts	133
Phosphorus melts	100
Æther boils	98
Medium Temperature of the Globe	50
Ice melts	32
Milk freezes	30
Vinegar freezes at about	28
Strong Wine freezes at about	20
A Mixture of 1 part of Alcohol, and 3 parts of Water, freezes	7
A Mixture of Alcohol and Water in equal parts, freezes	7
A Mixture of 2 parts of Alcohol and 1 of Water, freezes	11
Melting point of Quicksilver (Cavendish)	39
Liquid Ammonia crystallizes (Vauquelin)	42
Nitric Acid, spec. gr. about 1.42, freezes (Cavendish)	45
Sulphuric Æther congeals (Vauquelin)	47
Natural Temperature observed at Hudson's Bay	50
Ammoniacal Gas condenses into a liquid (Guyton)	54
Nitrous Acid freezes (Vauquelin)	56
Cold produced from diluted Sulphuric Acid and Snow, the materials being at the temperature of 57	78½
Greatest Artificial Cold yet measured (Walker)	91

ADDENDA ET CORRIGENDA.

Page 6. The description of the *fig.* 7, in the text, is taken from the Memoirs of the Academy of Sciences. It is probable that in the construction of the instrument, Amontons employed a tube of narrow calibre, to enable the included air to support the mercurial column. The proportion between the tube and ball may be inferred from what is stated of the expansion of the whole, by the heat of boiling water.

P. 10. In the beginning of the paragraph, just after the first set of formulæ, there is an error which requires correction. Instead of *these formulæ applying to all degrees above the freezing point*, read, "these formulæ apply to all degrees above the zero of each scale;"

P. 14. The notation employed by De Luc requires some explanation. y denotes the height of the barometer *in sixteenths of a Parisian line ;* T the height of a thermometer, plunged in boiling water, above the melting point of ice, in hundredths of a degree of De Luc's scale; and a the constant quantity 10387, which Horsley thinks, from some of De Luc's experiments, should have been 10369; but in his investigations he retains the first number, as probably adopted on good grounds.

The logarithms used by De Luc are the tables of Briggs, in which the seven figures of the tables, as well as the indices, are reckoned integers; or he considers the *eighth* figure in the place of units: but it is most convenient to reckon all the figures after the index as decimals, and the formulæ will be $\frac{99 \times 100}{2} Log.\ y - a = \mathrm{T}$; or $\frac{}{99 \times 50}\ Log.\ y - a = \mathrm{T}$.

P. 15. In the second column of the table, 29. should be opposite to .95; *instead of* .39.

The terms *lower* and *higher*, in the third column of the same table, are so in the original paper; but the directions will be more plain, if the reader will recollect, that where *lower* is indicated, the correction is "to be subtracted;" and when *higher* is used, the correction is "to be added."

P. 23, *Col.* 1. The action of the pinions in Ferguson's Pyrometer is ill expressed in the text; and the reader is requested to substitute for the member of the sentence commencing with "*and the bar, &c.*" in the 14th line, the following—"and as the bar k has fifteen teeth to one inch, it is obvious that when k moves one inch, the pinion l will have made one revolution and a quarter, and the pinion p will have moved $100 \times 1\frac{1}{4}$, or 125 times round."

P. 29. In Wedgwood's table, *Col.* 1, line 27, for *deduct*, read, "did not."

In the same page, *Col.* 2, the radius of Regnier's instrument should have been 649 millimètres, its diameter 1.298 mètres, and its circumference 4.079 mètres.

(5)

ROGET, Peter Mark

GALVANISM.

Chapter I.

Origin of Galvanism.

(1.) The term Galvanism is employed to designate a peculiar form of electric agency, elicited under particular circumstances, and capable of producing certain effects on bodies, not usually resulting from the ordinary modes of excitation. The first notice that we find of any phenomenon referable to this branch of electricity, occurs in a metaphysical work, published in 1767, and entitled, *The General Theory of Pleasures*, by a German writer of the name of Sulzer, who observed, that by applying two metals, one above, and the other below the tongue, and then bringing them into contact, a peculiar taste was perceived. He ascribed this sensation to some vibratory motion, excited by the contact of the metals, and communicated to the nerves of the tongue. Content with this loose and fanciful explanation, Sulzer appears to have pursued the inquiry no farther; and the curious fact he had announced remained for many years unnoticed, until the attention of the philosophic world was drawn to the subject, by the discovery of Galvani. Important discoveries in science seem often to arise from accident; but, on closer examination, it is found that they always imply the exercise of profound thought. As the fertility of the soil is essential to the germination and growth of the seed which the wind may have scattered on its surface, so it is principally from the qualities of mind in the observer that an observation derives its value, and may be made eventually to expand into an important branch of science. This has been remarkably exemplified in the origin of galvanism. Its founder, Galvani, was professor of anatomy at Bologna, and had early distinguished himself by his attainments and his zeal in his profession, and especially by the ardour with which he cultivated comparative anatomy. It happened, in the year 1790, that his wife, being consumptive, was advised to take, as a nutritive article of diet, some soup made of the flesh of frogs. Several of these animals, recently skinned for that purpose, were lying on a table in the laboratory, close to an electrical machine, with which a pupil of the professor was amusing himself in trying experiments. While the machine was in action, he chanced to touch the bare nerve of the leg of one of the frogs with the blade of the knife that he held in his hand; when suddenly the whole limb was thrown into violent convulsions. Galvani was not present when this occurred, but received the account from his lady, who had witnessed, and had been struck with the singularity of the appearance. He lost no time in repeating the experiment, in examining minutely all the circumstances connected with it, and in determining those on which its success depended. He ascertained that the convulsions took place only at the moment when a spark was drawn from the prime conductor, and the knife was at the same time in contact with the nerve of the frog. He next found that other metallic bodies might be substituted for the knife; and very justly inferred that they owed this property of exciting muscular contractions to their being good conductors of electricity.

(2.) Far from being satisfied with having arrived at this conclusion, it only served to stimulate him to the further investigation of this curious subject; and his perseverance was at length rewarded by the discovery, that similar convulsions might be produced in a frog, independently of the electrical machine, by forming a chain of conducting substances between the outside of the muscles of the leg, and the crural nerve. Galvani had previously entertained the idea that the contractions of the muscles of animals were in some way dependent on electricity; and as these new experiments appeared strongly to favour this hypothesis, he with great ingenuity applied it to explain them. He compared the muscle of a living animal to a Leyden phial, charged by the accumulation of electricity on its surface; while he conceived that the nerve belonging to it performed the function of the wire communicating with the interior of the phial, which would, of course, be charged negatively. In this state, whenever a communication was made, by means of a substance of high conducting power, between the surface of the muscle and the nerve, the equilibrium would be instantly restored, and a sudden contraction of the fibres would be the consequence.

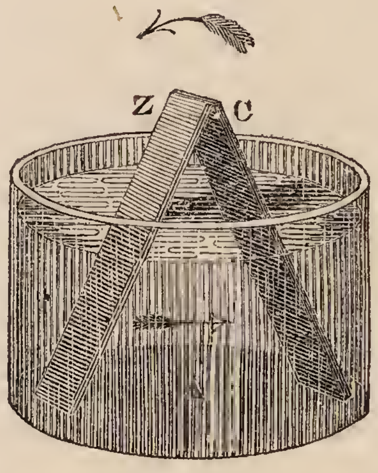
(3.) The discoveries of Galvani were no sooner made known to the scientific world, than they excited very general interest; and philosophers in every country in Europe vied with each other in repeating his experiments, in varying them in all possible ways, and in inventing all kinds of hypotheses to account for the phenomena. Some regarded them as the effects of a new and unknown agent, differing altogether from electricity; while others, adopting the views of Galvani, recognised them to be electrical, but attributed them to a peculiar modification of that power, residing in the animal system only, and which they accordingly distinguished by the name of *Animal Electricity*. But the discovery of new facts contributed more and more to multiply and strengthen the analogies between galvanism and electricity: till at length all doubt of the identity of the agent concerned in all these phenomena was removed by the discovery of the *Galvanic*, or *Voltaic Pile*. Whatever share accident may have had in the original discovery of Galvani, it is certain that the invention of the pile, an instrument which has most materially contributed to the extension of our knowledge in this branch of physical science, was purely the result of reasoning. Professor Volta, of Pavia, a name already familiar to electricians,* was led to the discovery of its properties by deep meditation on the developement of electricity at the surface of contact of different metals.† We may justly regard this discovery as forming an important epoch in the history of galvanism: and indeed, since that period, the terms, *Voltaism*, or *Voltaic Electricity*, have often, in honour of this illustrious philosopher, been used to designate that particular form of electrical agency, which is the subject of the present treatise.

Previously to our entering into a detailed exposition of the facts relating to this science, and of the theories which have been proposed for their explanation, it will be necessary to direct our attention to the nature of those arrangements of bodies, which are the sources of galvanic power.

Chapter II.

Simple Galvanic Circles.

Fig. 1.



(4.) The process usually adopted for obtaining galvanic electricity is to interpose between two plates of different kinds of metal a fluid capable of exerting some chemical action on one of the plates, while it has no action, or at least a different one, on the other plate: and then to establish a communication between the plates at some other part, either by their direct contact with one another, or by the intervention of conducting substances. Let us take, for example, a plate of zinc, Z, and another of copper, C, (*fig.* 1.) and immerse them, to a certain depth only, in diluted sulphuric acid, A, contained in a glass vessel, keeping their lower edges at a little distance from one another: then, inclining them towards each other, let us bring their upper edges, which are out of the fluid, into contact, as represented in the figure. The arrangement we have thus formed constitutes what is called a *galvanic circle*, in its simplest form, of which the three parts, or elements, are zinc, acid, and copper; each of these bodies being in contact with the two others. Under these circumstances it is found that a quantity of electricity is set in motion; a continued current of electric fluid passing from the zinc to the acid—from the acid to the copper—from the copper back again to the zinc—and so on, in a perpetual circuit. Such at least must be the explanation of the phenomena on the hypothesis of Franklin, implying the singleness of the electric fluid. But if the theory of Du Fay, which recognises two different fluids, be adopted, what has just been stated must be understood to refer exclusively to the current of vitreous electricity. Now, according to that theory, every such transfer of electricity consists of an interchange of the two fluids: the current of vitreous electricity just mentioned, must, therefore, necessarily be accompanied by an opposite current of resinous electricity; that is, of one flowing from the zinc to the copper; from the copper to the acid; and from the acid to the zinc. Hence in our future explanations of the phenomena of galvanism, it will be sufficient to express the former of these currents only; provided we bear in mind that the transfer of any quantity of vitreous electricity in a given direction, implies the transfer of an equal quantity of resinous electricity in the opposite direction.

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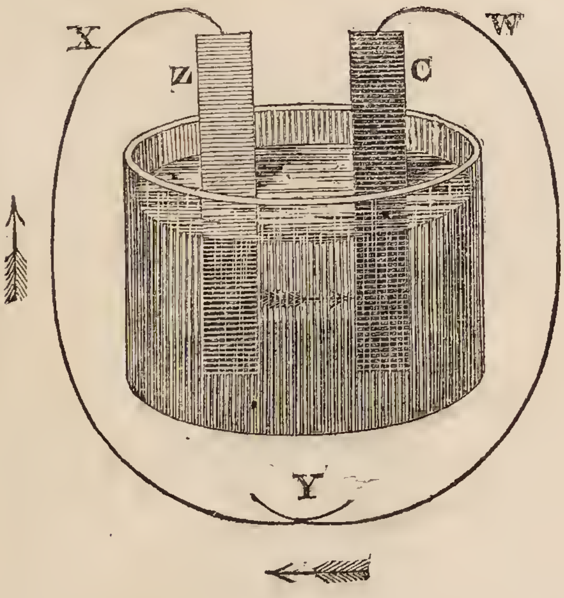
* See *Treatise on Electricity*, § 192, 193.

† Ibid. § 203.

(5.) The same effects will take place, if, instead of allowing the metallic plates to come in direct contact, the communication betwen them be effected by wires, (as shewn in *fig*. 2.) extending from the one to the other. The circuit of electricity will thus be lengthened, but the currents will move in the same direction as before; that of the positive electricity being denoted in the figure by the position of the arrows; namely, in the fluid, from the zinc towards the copper; and along the wires, from the copper to the zinc. The completion of the circuit by means of wires, enables us to direct the electric current through such bodies as we may wish to subject to its operation, and at the same time gives us the power of interrupting or renewing at pleasure the communication between the two metallic plates, by merely separating or joining together their remote extremities at Y. When united, the wire W, which proceeds from the copper-plate C, is imparting electricity to the wire X, which touches the zinc plate Z; hence, the former is considered as being in a positive, and the latter in a negative state.

Fig 2.

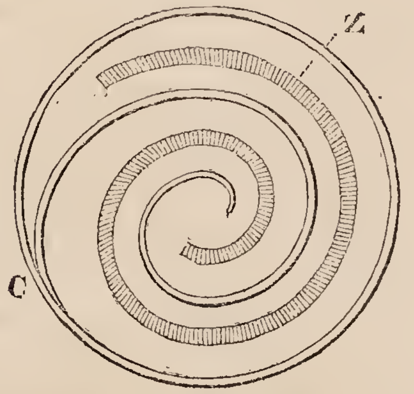


(6.) The electrical effects of the simple apparatus just described are, in general, too feeble to be perceived, unless by very delicate tests. The fact mentioned by Sulzer, and the experiments of Galvani on the muscles of frogs, in their original form, afford, however, examples of the operation of simple galvanic circles. When the tongue is interposed between zinc and copper, the saliva in contact with the metals performs the part of the acid in the experiment above mentioned, and the stream of electricity in its passage from the zinc to the copper, through the substance of the tongue, affects the nerves of that organ, so as to give rise to sensations of taste. In Galvani's experiment, muscular contractions were produced by forming a connection between two different metals, one of which was applied to the nerve, and the other to the muscles of a frog's leg. It is evident that such an arrangement composes a galvanic circle, deriving its activity from the chemical properties of the fluids in those parts of the frog that are in contact with the metals. Although the quantity of electricity set in motion by this slight action, must be supposed to be exceedingly minute, it is yet sufficient, when passing over the exquisitely sensible nerves of the tongue, or through the highly irritable fibres of a frog, to produce a very considerable impression.

(7.) It has even been found possible, by means of a very small galvanic circle of the same simple kind as that which we have described, to produce some of the more energetic effects of galvanism, such as raising the temperature of the wire which conducts it to a red heat. We are indebted to the ingenuity of Dr. Wollaston for the contrivance of an apparatus, which he calls an *elementary galvanic battery*, capable of exhibiting this effect.[*] He found that a single plate of zinc, of the size of a square inch, when properly mounted, and suspended in dilute sulphuric acid, between two copper plates of similar dimensions, was more than sufficient to ignite a wire of platina, one three-thousandth of an inch in diameter, which formed part of the connection between the two metals.

(8.) It will readily be conceived, that by enlarging the size of the plates, their power will be proportionally increased. The first battery of this kind, on a very large scale, was that constructed by Dr. Hare, professor of chemistry in the university of Philadelphia, and called by him a *Calorimotor*, from its remarkable power of producing heat.[†] It consisted of sheets of zinc, and of copper, formed into coils, so as to encircle each other, separated only by interstices of a quarter of an inch in width. This construction is shown in *fig*. 3, which exhibits a horizontal section of the plates as they are coiled together: the thick line Z, representing the zinc, and the thinner line C, the copper plate. The zinc sheets were nine inches by six; the copper fourteen by six; more of the latter metal being required; as in every coil it was made to commence within the zinc, and completely to surround it on the outside. Each coil was about two inches and a half in diameter; their number amounted to 80; and by means of a lever they could all be let down at the same moment into as many glass jars, two inches and three quarters diameter inside, and eight inches high, placed so as to receive them, and containing the acid liquor intended to act upon the zinc.

Fig. 3.



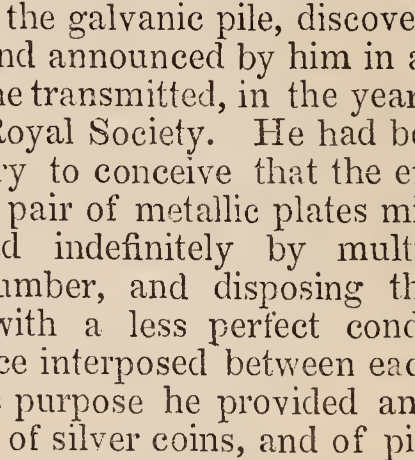
(9.) To the class of simple galvanic circles must also be referred the magnificent battery belonging to the London Institution, and which was constructed under the direction of Mr. Pepys.[*] It consists of two plates only, the one of zinc, and the other of copper, coiled round a cylinder of wood, and prevented from coming into contact by ropes of horse hair, which is a non-conducting substance, interposed in various places between them. The length of each plate is 60 feet, and its breadth two feet; the total surface being 400 square feet. In order to charge this battery, the whole coil is immersed in a tub containing acid of the proper strength.

Chapter III.

Compound Galvanic Circles.

(10.) Many of the effects of galvanism require for their production the combined influence of a number of plates, arranged so as to form what is termed *a compound galvanic circle.* To this class belongs the galvanic pile, discovered by Volta, and announced by him in a paper which he transmitted, in the year 1800, to the Royal Society. He had been led by theory to conceive that the effect of a single pair of metallic plates might be increased indefinitely by multiplying their number, and disposing them in pairs, with a less perfect conducting substance interposed between each pair. For this purpose he provided an equal number of silver coins, and of pieces of zinc, of the same form and dimensions; and also circular discs of card, soaked in salt water, and of somewhat less diameter than the metallic plates, Of these he formed a pile or column, as shown in *fig*. 4: in which the three substances, silver,[†] zinc, and wet card, denoted by the letters S, Z, and W, were

Fig. 4.



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[*] See Thomson's *Annals of Philosophy*, vol. vi. p. 209. While these pages were in the press we have sustained an irreparable loss in the death of Dr. Wollaston, a philosopher whose unrivalled acuteness of observation, soundness of judgment, and integrity of mind, directed to the highest objects of science, place his name among the most eminent of its benefactors.

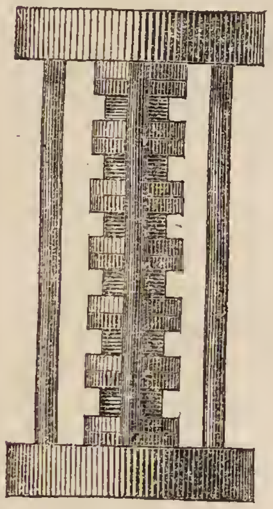
[†] Silliman's *Journal*, iii, 105, and *Annals of Philosophy*, New Series, i. 330.

[*] *Philosophical Transactions* for 1823, p. 187.

[†] Copper might have been used instead of silver, as in the single galvanic circle already described, (§ 4, 5.) with the same effect.

made to succeed one another in the same regular order throughout the series. The efficacy of this combination realized the most sanguine anticipations of the discoverer: it far exceeded in power the single circle already described. If the uppermost disc of metal in the column be touched with the finger of one hand, previously wetted, while a finger of the other hand is applied to the lowermost disc, a distinct shock is felt in the arms, similar to that from a Leyden phial, or still more nearly resembling that from an electrical battery weakly charged. A repetition of shocks is obtained for an indefinite period, whenever the circuit is completed by touching the two ends of the pile with the moistened fingers. The strength of the shock is, as might be expected, greater in proportion to the number of plates of which the pile is composed.

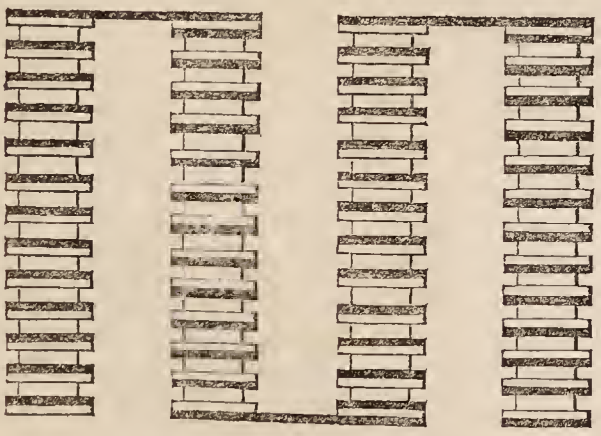
Fig. 5.



If the pile were raised to any considerable height, it would obviously be in danger of oversetting: this may be prevented by placing the discs between three vertical glass rods, properly varnished, and cemented into two thick pieces of wood, one of which serves as a base, and the other as a cover to the pile. See *fig*. 5.

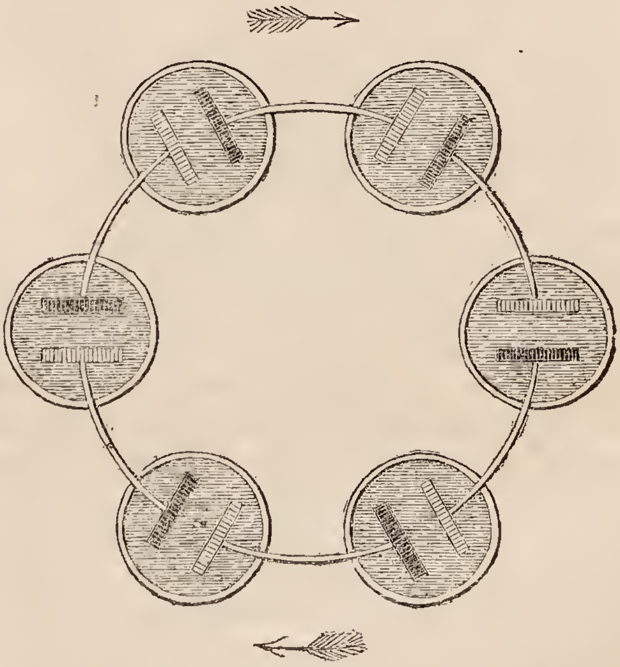
(11.) Any number of these piles may be combined so as to form a battery, by making a metallic communication between the last plate of the one and the first of the next, and so on; taking care that the order of succession of the plates in the circuit be preserved inviolate, as is shown in *fig*. 6, where the dark lines represent the copper, and the light lines the zinc plates.

Fig. 6.



(12.) The component parts of the pile may be arranged in a form somewhat different from the preceding, and corresponding more nearly to the elementary galvanic circle in its simplest state already described (§ 4, 5). In this new arrangement the metallic plates, instead of being piled one above the other, are placed side by side in a vertical position, and combined together in pairs, consisting each of one zinc and one copper (or silver) plate, connected at their upper edges by slips of metal, passing from the one to the other. A sufficient number of glasses being provided, and filled with water, or some acid or saline solution, they are to be placed side by side, so as to form a circle. The two plates belonging to each pair are then to be immersed in the fluids contained in two different, but adjoining, glasses; the zinc plate, for instance, in the first glass, and the copper in the second. The plates of the second pair must be immersed, in a similar way, in the second and third glasses; and so on successively throughout the series, taking care to preserve the same order of alternation in the metals. It is evident that by this arrangement, (of which an horizontal section is shown in *fig*. 7, where the dark lines indicate the copper, and the lighter lines the zinc plates in each pair,) each vessel will contain one plate of zinc and one of copper, which, as they belong to different pairs, are not connected together, except through the medium of the intervening fluid in that particular vessel.

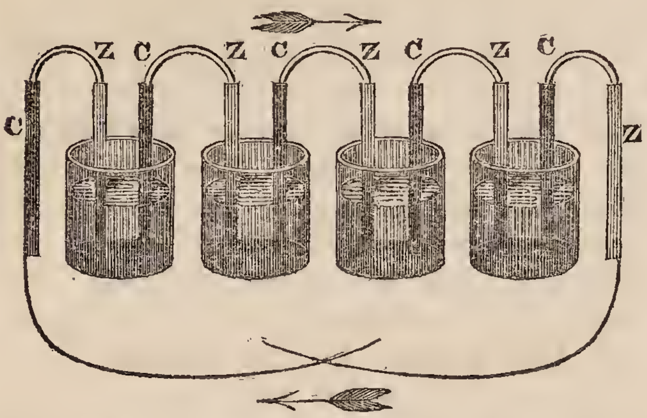
Fig. 7.



(13.) The first apparatus of this kind was constructed by Volta, who employed for that purpose a circular series of cups, and hence gave it the name of

Couronne de tasses. If the circuit be interrupted at any one point, by removing one or more of the vessels, the instrument is rendered similar in its operation to the pile, and the metallic plates at each end of the series which are not immersed in fluid, may be connected together by means of wires in order to complete the circuit. Such an arrangement is shown in *fig.* 8, where the zinc and copper plates are marked respectively with the letters Z and C: and the course of the electric fluid denoted by the arrows.

Fig. 8.



(14.) It is also to be observed, that in every compound galvanic circle, such as is exemplified in this apparatus, the direction of the electric current is precisely the same as in a simple galvanic circle composed of the same elements. In the present case, where zinc and copper are the metals employed, and the fluid acts upon the former so as to oxidate it, a stream of positive electricity is continually circulating from the zinc to the copper plate contained in the same vessel, through the oxidating fluid which separates them; and is transferred from the copper to the zinc plate contained in the next vessel, along the slip of metal which connects them. Following its course in this manner to the end of the series, we find the electric current passing on from the last copper plate, contained in the last vessel, to the zinc plate connected with it, and thence conveyed along the wires of communication, round to the copper plate at the other end of the series. The direction of this current is shown in the figure by the arrows above and below. It is evident, therefore, that that end of the battery which is terminated by a zinc plate is that from which electricity is given out to the wire, and is, consequently, the positive end, or *pole*, as it is called, of the battery. For the same reason, the opposite end, or that terminated by the copper plate, and which receives the electricity from the wire, is the negative pole. The same observations apply to the galvanic pile; the zinc end being the positive, and the copper (or silver) end the negative pole.

(15.) It will be perceived that the denominations of the zinc and copper ends of the pile or compound battery, as being positive and negative, are exactly the reverse of what obtains in the single galvanic circle, where, as we have seen, it is the copper plate which is positive, and the zinc negative, with relation to the communicating wires. But as the direction of the electrical currents is the same in the compound as in the simple circle, this contrariety in the qualities of the poles appears, at first sight, paradoxical. But the difficulty vanishes when we advert to the circumstance, that in the simple galvanic circle the conducting wire communicates directly with that plate which is in contact with the fluid part of the apparatus; while in the compound circle it proceeds, not from the plate immersed in the fluid, but from that which is associated with it, and, therefore, of a different kind. The compound circle reduced to its condition of greatest simplicity would be represented by the following series, consisting of five parts, namely,

copper—zinc—fluid—copper—zinc.

In this arrangement the copper end is negative, and the zinc end positive. By merely removing the two terminal plates,* which, in fact, are no ways concerned in the effect, we bring it to the state of the single circle, consisting simply of

zinc—fluid—copper:

here we find the zinc end negative, and the copper end positive. It is highly necessary to possess clear ideas of this difference, since much ambiguity has arisen from inattention to it in describing experiments, and reasoning upon their results, more especially in the study of electro-magnetism, hereafter to be considered.

(16.) A much more compendious form may be given to a battery constructed on the principle of the *Couronne de tasses*, by employing a trough divided into numerous compartments by partitions, the whole being made of non-conducting materials. This will admit of the plates being brought nearer to each other, and of a much greater number being contained in a given space. The zinc and copper plates are united in pairs, as before, by a slip of metal passing from the one and soldered to the other: each pair being placed so as to enclose a partition between them, and each cell containing a plate of zinc connected with the copper plate of the succeeding cell, and a copper plate joined with the zinc plate in the preceding cell. Such an apparatus is called a *trough battery*, and is represented in *fig*. 9.

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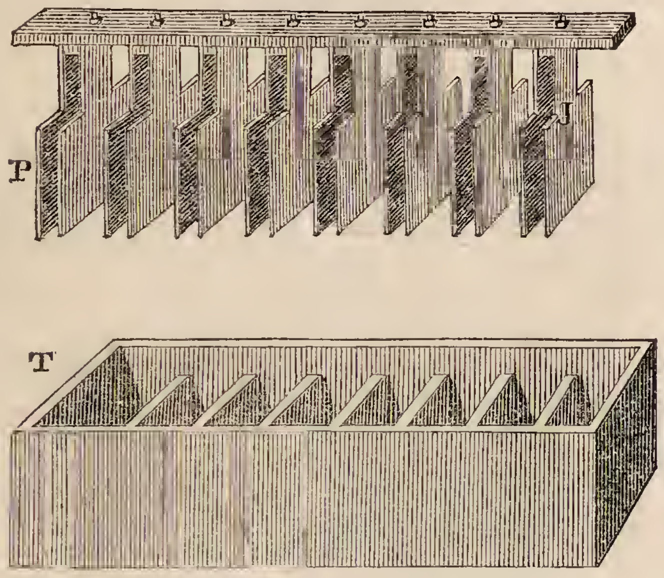
* Volta, in conformity with the theory he had adopted, considered these terminal plates as adding to the galvanic power. But we shall afterwards point out the incorrectness of that theory.

Fig. 9.



The trough, T, may be made of baked mahogany, with partitions of glass: but it is found more convenient to construct the whole of one material, and Wedgwood ware answers best for this purpose. Each trough is usually fitted up with ten or twelve cells. The plates, P, adapted to them, are connected together by a slip of baked wood, so as to allow of their being let down into the cells, or lifted out, together. A further advantage arises from this construction, that the plates and the fluid being independent of each other, the former may be readily cleaned or replaced, when worn or injured, without disturbing the fluid: and the latter may, in like manner, be removed and changed with the utmost facility. A number of these troughs may be combined with great ease, by connecting together the terminal plates of the adjoining troughs, by slips of copper; taking care, as in the case of the pile, (§ 11.) to preserve throughout the whole series the same order of alternation in the plates, by connecting the zinc end of one battery with the copper end of the next.

The voltaic battery belonging to the Royal Institution, which is of immense power, is constructed on the plan above described, and consists of 200 separate parts, each part composed of ten double plates, and each plate containing thirty-two square inches. The whole number of double plates is 2000, and the whole surface 128,000 square inches.

(17.) A trough battery on another construction was invented by Mr. Cruickshanks, and is represented in *fig*. 10. Plates of zinc and of copper, united by their flat surfaces by soldering, are employed to form the partitions themselves, and are fixed into grooves in the sides of a trough of baked wood, which is a bad conductor of electricity, so as to leave sufficient intervals to hold small quantities of fluid. They must, of course, be arranged so that all the zinc surfaces shall be on one side, and all the copper surfaces on the other. The battery is charged by filling the cells with a saline solution, or with dilute acid, and the galvanic circuit completed by bringing the two wires proceeding from the ends of the battery in contact with one another. The section, *fig*. 11, will tend to elucidate the principles of its action. Troughs of this construction, however, are exceedingly liable to get out of order, from the action of the liquid on the wood, which it tends to warp. The plates require to be fixed into the grooves by cement, in order to render them water tight; but this cement is apt to crack from the warping of the wood, and other causes, and the liquid insinuating itself into the fissures, impairs the power of the instrument by destroying the insulation of the cells.

Fig. 10.

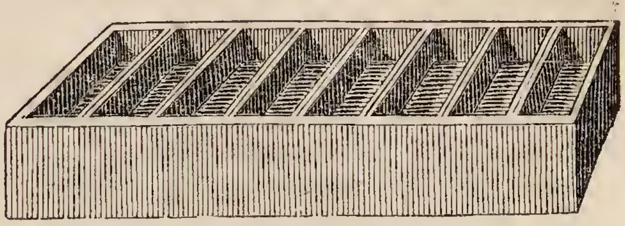
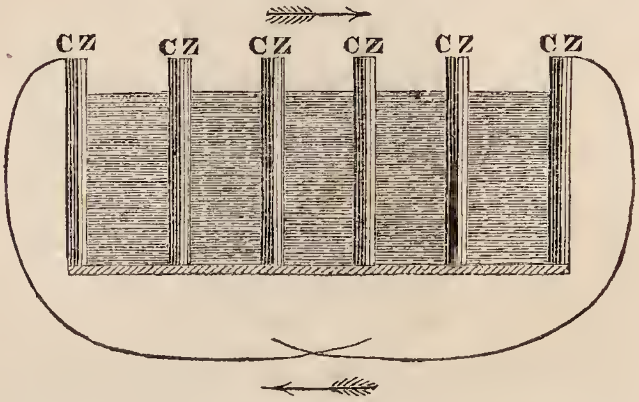


Fig. 11.



(18.) The power of a battery is considerably increased when both surfaces of each plate of zinc, in contact with the oxidating fluid, are opposed to a surface of copper. In order to accomplish this, it will be necessary to add a second copper plate to each pair, so that every cell may contain one zinc and two copper plates, the former being placed between the latter. This plan, which was suggested by Dr. Wollaston, was adopted by Mr. Children in the construction of a very large battery, in which each plate was six feet long, by two feet eight inches broad, so that it presented thirty-two square feet of surface.*

(19.) An ingenious application of this principle was made by Mr. Hart of Glasgow, in the construction of a galvanic battery, requiring no other material for confining the fluid, than the metals themselves which form the circles. This he accomplished by converting the double copper plates into cells, by adding sides and bottoms, so as to enable them to hold the acidulous fluid into which the zinc plates are immersed. The cells are formed by cutting a sheet of copper into the form shown in *fig*. 12.† They are then folded up as seen in *fig*. 13, and the seams grooved. A drop of tin is run into each lower corner to render the cells perfectly tight. *Fig*. 14 represents the zinc plate, having a piece of screwed brass-wire cast into the top of it for the purpose of suspension. *Fig*. 15 is a section of the battery, showing how the copper tail of the first cell is connected with the zinc plate of the second, and so on. The connexion is rendered perfect by joining them with a drop of solder. Each zinc plate is kept firmly in its place by three small pieces of wood. The whole series is then fixed, by means of screw-nuts fitted on to the brass wires, to a bar of baked wood, previously well varnished. When the battery is to be used, it must be lifted off the frame, and dipped into a wooden trough, lined with lead, containing the acid. It is then placed on the frame and is ready for action. Such a battery, with an equal number of zinc plates, is found to possess considerably greater power than the best batteries of the ordinary construction.*

Fig 12.

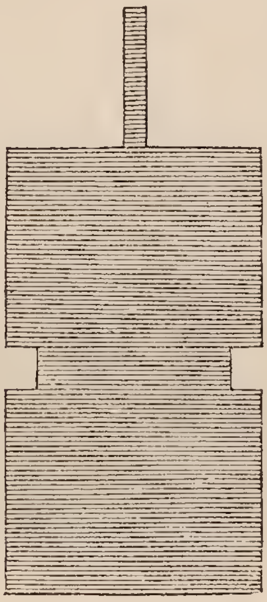


Fig. 13.

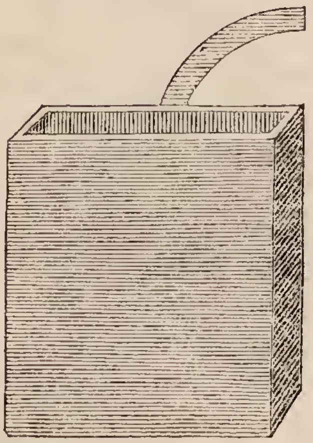


Fig. 14.

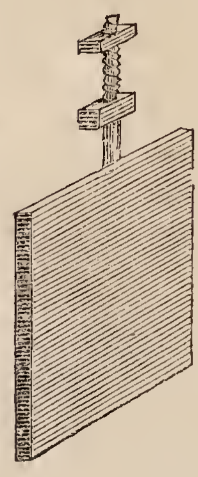
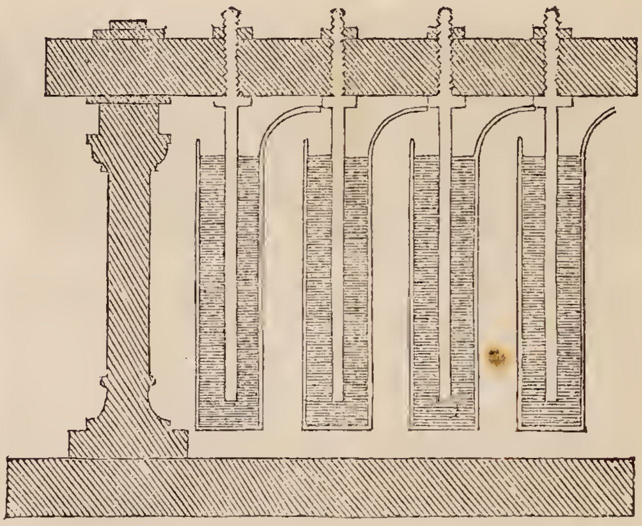


Fig. 15.



(20.) Various contrivances have been employed for converting a compound voltaic battery, consisting of a certain number of alternations of plates, into a battery having a smaller number, or even into one corresponding in principle to the simple battery with a single pair of plates, such as the calorimotor. These changes may be effected by altering the connexions of the plates, and uniting several plates of the same metal together, so that they may act as only one plate; or if the effect of a calorimotor be desired, connecting all the zinc plates together, and also all the copper plates, so that the whole may act only as a single pair.

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* *Philosophical Transactions* for 1815, p. 363.

† The engraving fig. 12, is here reduced in its dimensions from the original drawing. It should have been of the size required to form, when folded, the cell represented in fig. 13.

* *Edinburgh Journal of Science*, iv. 19.

CHAPTER IV.

Effects of Galvanism.

(21.) There are three principal circumstances in which the electricity produced by the voltaic battery differs from that obtained from the ordinary electrical machine; first, the very low degree of intensity in which it exists in the former, when compared with the latter; secondly, the very large quantity of electricity which is set in motion by the voltaic battery; and thirdly, the continuity of the current of voltaic electricity, and its perpetual reproduction, even while this current is tending to restore the equilibrium. The effects of the voltaic pile have been compared by the inventor of that instrument to those of an electric battery of large dimensions, but charged only to a low degree; in which case, as appears from what has already been said on this subject in the Treatise on Electricity, a large quantity of electricity may be contained, with a very small tendency to escape, or, in other words, with a very feeble intensity. The comparison is, in many respects, just; but it fails in regard to the third property we have noticed as belonging to the voltaic apparatus; namely, the continuity of the current arising from its perpetual reproduction and circulation.

However considerable may be the power collected in a highly charged electric battery, the whole of that power is at once expended as soon as the circuit is completed. Its action may, while it lasts, be sufficiently energetic; but it is exerted only for an instant; and, like the destructive operation of lightning, can effect, during its momentary passage, only sudden and violent changes, which it is beyond the power of the experimentalist to regulate or control. On the contrary, the voltaic battery continues, for an indefinite time, to develop and supply vast quantities of electricity, which, far from being lost by returning to their source, circulate in a perpetual stream, and with undimished force. The effects of this continued current on the bodies subjected to its action, will, therefore, be more definite, and will be constantly accumulating; and their amount will, in process of time, be incomparably greater than even those of the ordinary electrical explosion. We shall accordingly find that changes in the composition of bodies are effected by galvanism which can be accomplished by no other means. Hence may be conceived the advantages which have accrued to science from the acquisition of an instrument of such vast power, and admitting of such extensive application in the wide field of chemical research.

It will be convenient to study the effects of galvanism in their relation to the three circumstances which have been noticed as characterizing its operation when contrasted with those of ordinary electricity.

§ 1. *Ordinary Electrical effects resulting from Galvanism.*

(22.) The degree of intensity in which the electricity developed by a single galvanic circle exists, is so extremely low, that its action produces none of the usual phenomena exhibited by the common electrical machine. Even from the largest calorimotor that has yet been constructed, it is not possible to obtain indications of electrical attraction and repulsion, such as are given by the feeblest degree of excitation to a piece of sealing-wax. With a few alternations of plates and interposed fluid, as in the pile or trough battery, electrical indications may be obtained, by means of an ordinary condenser. It is necessary in these experiments to advert to the distinction already pointed out (§ 15.) between single and compound circles as to the denomination of the extremities or poles of the battery. In the compound circles the zinc side is found to be positive and the copper negative. When fifty pairs of plates are employed, a delicate gold-leaf electrometer will be affected, without the aid of the condenser, and with a series of one thousand groups, even pith balls are made to diverge. In order to exhibit these effects, the wire proceeding from one extremity of the battery should be connected with the foot of the electrometer; while the wire proceeding from the opposite extremity is made to touch the cap. It was by means of the revolving doubler (see Electricity, § 195.) that the electrical states of the two ends of the voltaic pile were first ascertained by Messrs. Nicholson and Carlisle.*

(23.) Since the ends of the two wires, which proceed from the two poles of the voltaic battery, are in opposite states of electricity, we might naturally expect that they would attract one another. Such an attraction actually does take place, as Biot found by experiment;† but it does not become sensible, unless a battery composed of a great number of plates is employed.

(24.) The general conclusion deducible from the facts that have now been stated, is that the intensity of the electricity developed by galvanic combinations is increased, according as the number of alternations in the elements which compose them is greater, and that it bears no proportion to the magnitude of their surfaces.

(25.) If the voltaic battery be of sufficient size, its electricity may be transferred to a common electrical battery, which will then become charged to the same degree of intensity. Nothing more is necessary for this purpose than to connect the outer and inner coatings of the electrical battery, respectively, with the two poles of the voltaic battery; when the charge will be instantly communicated to the former. If on removing it from the voltaic battery this electricity be discharged, and the same communications be renewed, a similar charge will again be received; and the same process may be repeated an indefinite number of times. If, instead of removing the electrical battery, we allow it to remain connected with the voltaic battery, a rapid succession of sparks may be obtained from it by connecting a wire with the outer coating, and repeatedly striking the knob of the phial with the other end of the wire. If the series of plates in the voltaic battery consist of three or four hundred alternations, these rapid explosions are so powerful as to ignite the end of the wire, if it be of iron, and to cause it to throw off an abundance of sparks, consisting of small particles of iron in a state of intense combustion. With a series of one thousand, each discharge is attended with a sharp sound, and will burn thin metallic leaves. This is the more remarkable as the same voltaic battery may not have sufficient power to produce these effects by itself, or unconnected with an electrical battery. The shortest possible contact with the voltaic battery is sufficient for giving the whole of the charge which it is capable of communicating. This was apparent in some experiments made by Van Marum and Pfaff with a battery having 137½ square feet of coated surface, and which was charged to the same degree of intensity as the pile with which it was made to communicate, by a contact which did not last for the twentieth part of a second.*

§ 2. *Luminous effects of Galvanism.*

(26.) It is only when the electricity of a voltaic battery possesses a sufficient intensity, that it becomes capable of passing through air. With the calorimotor the intensity is too feeble to enable it to traverse the smallest perceptible interval between metallic conductors, so that they must be brought into actual, or at least apparent, contact, before any sensible effect is produced. In a pile or trough battery, on the other hand, composed of a considerable number of alternations of plates, on bringing together the wires from the opposite poles, the transfer of electricity begins while they are yet at a sensible distance from one another: and as in the case of ordinary electricity, this transit through the air is accompanied by vivid light. The sparks occur every time the contact between the wires is broken, as well as when it is renewed. This phenomenon, which does not take place with the electricity furnished by the ordinary means, is characteristic of voltaic electricity, and is a consequence of its continuous supply. The stream continues to flow, notwithstanding the interruption to the line of circuit, and as long as the conductors remain within the striking distance; and although this happens only for an instant, there is still sufficient time for the appearance of a spark.

(27.) The most splendid exhibition of electric light is that obtained by placing pieces of charcoal, shaped like a pencil, at the ends of the two wires in the interrupted circuit, and bringing their points into contact. When the experiment was tried with the powerful battery of the Royal Institution, already noticed (§ 16.), a bright spark passed between the two points of charcoal, when they came within the distance of the thirtieth or fortieth of an inch; and immediately afterwards more than half of each pencil of charcoal, the length of which was one inch, and the diameter one-sixth of an inch, became ignited to whiteness. By withdrawing the points from each other, a constant discharge took place through the heated air, in a space at least equal to four inches, forming an arch of light in the form of a double cone, of considerable breadth, and of the most dazzling brilliancy. This phenomenon is represented in *fig.* 16; in which W, X, are the conducting wires communicating with the ends of the battery; C, C, the pieces of charcoal, and A the luminous arch of electrical light, making the passage of electricity through the air. When any substance was introduced into this arch, it instantly became ignited: platina melted in it, as wax in the flame of a candle: some of the more refractory substances, as quartz, the sapphire, magnesia, and lime, all entered into fusion: fragments of diamond, and points of charcoal and of plumbago quickly disappeared, and seemed converted into vapour, even when the connection was made in highly rarefied air, and apparently without having undergone previous fusion. When the pieces of charcoal were placed in the receiver of an air-pump, in proportion as the air was abstracted, the distance at which the discharge took place increased: and when the height of the mercury in the barometrical gage was only one quarter of an inch, the sparks were nearly half an inch in length; and by then withdrawing the points from each other, the discharge passed through a space of six or seven inches, producing a most brilliant coruscation of purple light. The whole of the charcoal became intensely ignited, and some platina wire attached to it melted with bright scintillations, and fell down in large globules.* A battery of a hundred pair of plates of six inches square will suffice to exhibit these phenomena on a smaller scale. Charcoal, carefully prepared from some of the harder woods, such as beech, lignum vitæ, or box wood, answers best for these experiments. The arched form of the stream of light passing between the two charcoal points is perceptible even when the points are within half an inch of each other.

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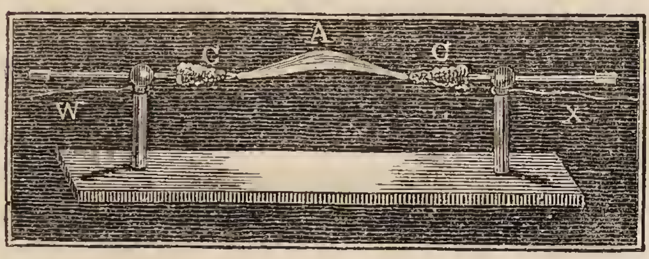
* Nicholson's *Journal*, 4to. iv. 174.

† Biot, *Traité de Physique*, ii. 511.

* *Annales de Chimie*, xl. 289.

Fig. 16.



The light obtained by voltaic electricity in the manner now described exceeds in intensity any other that art can produce. It often exhibits in succession a variety of the prismatic colours; and supplies some of the rays which are deficient in the solar beams. It is so dazzling as to fatigue the eye even by a momentary impression; and it effaces, by its superior lustre, the light of lamps in an apartment otherwise brilliantly illuminated, and which, on the sudden cessation of the galvanic light, appears for a short time as if left in darkness. It is a light which so nearly emulates the brightness of the sun's rays, as to be applicable for the purpose of illuminating objects in a solar microscope; and even with the magic lantern it has been found capable of exhibiting on a large scale, as was done by Mr. W. Allen in his lectures, all the pleasing and endless variations of the kaleidoscope.

(28.) The employment of charcoal in these experiments might lead to a suspicion that the light might, in part at least, arise from combustion; but many circumstances concur to prove that it is quite independent of this cause. During the continuance of the light, although the charcoal be in a state of ignition, yet it suffers but little loss of weight. The light is evolved with equal splendour when the experiment is made in gases that contain no oxygen, such as azote or chlorine, and in which therefore combustion could not be maintained: and it is moreover found that during the ignition, neither the gas nor the charcoal has undergone any chemical change.† Light from voltaic electricity may also be obtained, though with diminished intensity, under water, alcohol, ether, oils, and other fluids of inferior conducting power.

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* Davy's *Elements of Chemical Philosophy*, p. 152.

† Children, *Philosophical Transactions* for 1815, p. 369.

§ 3. *Evolution of Heat by Galvanism.*

(29.) The evolution of heat is one of the effects which accompany the action of the voltaic, as well as of the electric battery; but there is a remarkable difference in the circumstances which favour its production in the two cases. In the common electrical apparatus, heat is not sensibly evolved where the electricity moves with perfect freedom, but only when some resistance is opposed to its passage, and when there is a sudden restoration of its equilibrium, accompanied with light and sound. But in the voltaic battery, an elevation of temperature is observed to take place when the circuit remains complete, when no light is evolved, and when the stream of electricity is conducted in the most silent manner. That the mere passage of voltaic electricity through bodies raises their temperature, is proved by making a wire, forming part of the circuit, pass through a known quantity of water, contained in a vessel, with a thermometer immersed in the fluid. The heat acquired by the water soon becomes sensible by the rise in the thermometer, which even attains the boiling point; and the water continues in ebullition as long as the experiment is continued.

(30.) The circulation of voltaic electricity produces an elevation of temperature, not only in that part of the circuit which connects together the poles of the battery, but also in the battery itself, every part of which, both the plates of metal and the fluid in the cells, become heated when the apparatus is in an active state. But the elevation of temperature is found not to be equal throughout the series; and the difference is dependant on causes which have not yet been accurately determined. Mr. John Murray found a gradual increase of temperature in the successive cells from the negative to the positive pole; and when a number of different troughs were joined together, the cells at the extremities of each were less heated than those towards the middle; the maximum of heat was at a part situated nearer to the positive pole; and the temperature gradually diminished in the direction of the negative pole.*

(31.) Ignition, in various degrees, is produced by the passage of voltaic electricity through metallic wires, when their size and length are properly proportioned to the kind of apparatus, and to the quantity of electric fluid they have to convey. Iron wire is in general easy to ignite, and is often fused into globules; and steel wire is made to burn with a rapid and brilliant combustion. A wire of platina, a metal not susceptible of being acted on by the air, may be kept at a red, or even white heat, for an indefinite length of time, by voltaic electricity. As long, indeed, as the battery retains its power, there appears to be no limit to the continual evolution of heat.

(32.) The order in which the different metals are raised to a red heat by the action of galvanism, was ascertained by Mr. Children, with the aid of a very powerful apparatus of his own construction, to be as follows, namely, platina, iron, copper, gold, zinc, silver. Between copper and gold the difference is inconsiderable; and with regard to platina and iron, their relative places in the scale seem to depend upon the temperature acquired. The relations of tin and lead to the other metals could not be ascertained in these experiments, on account of their melting before they could be raised to a red heat. A beautiful illustration of the difference existing in metals as to their capacity of ignition, is obtained by placing in the circuit a wire or chain composed of alternate portions, or links of platina and silver soldered together; it will then be found that the silver links are not sensibly heated, while all those of platina become equally and intensely ignited.

(33.) It would appear that the heat produced by the voltaic battery is more intense than can be excited by any other process. In the experiments detailed by Mr. Children,* the action of his powerful apparatus raised to a red heat, visible in full daylight, the whole of a wire of platina, one tenth of an inch in diameter, and five feet and a half in length. It also effected the fusion of a variety of substances on which the heat of the best wind-furnaces makes no impression.

(34.) When very thin metallic leaves are placed in the electric current of a powerful voltaic battery they take fire, and by continuing the action, may be made to burn with great brilliancy. In order to exhibit these effects, the metallic leaves should be suspended to a bent wire proceeding from one extremity of the battery, and then a broad metal plate connected with the opposite extremity should be gradually brought near to them till contact is produced. The brilliancy of the effect is heightened by covering the plate with gilt foil. Gold leaf, thus treated, burns with a vivid white light tinged with blue, and produces a dark purple or brown oxide. Silver leaf gives out a brillant emerald green light, and leaves an oxide of a dark grey colour. Copper produces a bluish white light, accompanied with red sparks; its oxide is dark brown. Tin exhibits nearly the same phenomena, excepting that its oxide is of a lighter hue. Lead burns with a beautiful purple light; and zinc with a vivid white light, inclining to blue, and fringed with red. For the distinct appearance of these colours, it is necessary that the contacts should be made with a metal, and not with charcoal; for the intense white light emitted by the latter, would overpower the peculiar colours arising from the combustion of the metal.*

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* *Edinburgh Philosophical Journal*, xiv. 57.

* In the *Philosophical Transactions* for 1815, p. 368—370.

(35.) A beautiful effect, noticed by Van Marum, is produced by connecting a slender iron wire with one of the poles of a powerful voltaic battery, and bringing its end in contact with the surface of some mercury connected with the other pole. Vivid combustion takes place both in the mercury and in the wire; giving rise to an abundant emission of sparks, and appearing like a star or sun dispersing thousands of rays on every side. This splendid spectacle may be prolonged at pleasure, by taking care to continue the depression of the iron wire, in proportion as the metallic particles are dispersed by the combustion.

(36.) Inflammable bodies, such as oils, alcohol, ether, and naphtha, are easily inflamed by means of galvanism, when charcoal points in the circuit of the battery are brought near each other on the surface of these fluids; and gunpowder may readily be made to explode under the same circumstances.

(37.) The difference in the operation of voltaic and ordinary electricity is very manifest in their mechanical effects. The forcible separation of the particles of bodies, and destruction of their cohesion, characterize more especially the electrical explosion, in which the fluid appears to force for itself a passage through every obstacle; while the heat which occasionally manifests itself during this sudden effect, seems as if it were merely the effect of the compression and collision of the particles which are thus forcibly impelled. But the elevation of temperature which accompanies the passage of voltaic electricity, on the contrary, appears to be its immediate and direct effect; for the mechanical texture of the substance which conveys the electricity remains unaltered. If electricity in its common form possess any power of igniting bodies, its operation is too transient and momentary to produce any extensive effect; and its tendency is rather to separate and disperse the body into minute fragments, than to unite the particles into globules by fusion. We have seen that charcoal is very readily ignited by galvanism, but it will sustain a strong discharge from an electric battery without any perceptible rise in its temperature; nor is it possible to ignite it by this means. Whether reduced to fine powder, or cut into thin plates, or made to taper to a point, it resists all attempts to raise it to a red heat, or even to impart to it any sensible warmth, though subjected to the action of the most powerful battery that has yet been tried. Even when an apparently continuous stream of electricity, obtained from a large electrical machine, was made to pass through pointed wires coated with spermaceti, no part of the spermaceti was melted.

§ 4. *Electro-Magnetic Effects of Galvanism.*

(38.) We must rank among the more remarkable of the physical effects produced by the transit of voltaic electricity through conducting bodies, the induction of magnetism in iron, and the influence exerted on bodies which possess magnetic properties. But as the study of the connections which subsist between these phenomena implies a previous knowledge of magnetism, and constitutes, indeed, a distinct branch of science, it will be proper to reserve their consideration for a future treatise. It may be as well, however, to remark in this place, that the discovery of the electro-magnetic effects of galvanism have furnished us with the most delicate tests for detecting very minute portions of voltaic electricity, so that many of the results of simple galvanic arrangements, to be hereafter mentioned, have been obtained by magnetic galvanometers.

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* Singer's *Elements of Electricity*, p. 408.

§ 5. *Chemical changes effected by Galvanism.*

(39.) In the Treatise on Electricity, some of the chemical changes which result from the operation of this agent in its ordinary form were noticed; and experiments were described in which water, and a few saline bodies were decomposed by a succession of electric discharges from a powerful machine. But the power of galvanism to effect changes in the composition of bodies subjected to its action is incomparably greater; and its application has led to a series of discoveries which constitute a new era in chemistry, and rank among the most brilliant in the annals of physical science.

(40.) The chemical agency of galvanism, unlike its power of eliciting heat, is manifested, not while it is traversing substances of great conducting powers, but, on the contrary, when it meets with impediments to its passage; and it is exerted chiefly on substances, generally fluids, which convey electricity only partially and imperfectly. That we may acquire clear ideas of the connection of the chemical phenomena relating to galvanism, it will be necessary to trace them from their origin, and attend to what takes place in the simplest galvanic circle composed of two dissimilar metals and an interposed fluid.

(41.) If a plate of zinc, and another of copper, be immersed in very dilute sulphuric acid, without touching or communicating with each other, the zinc will be acted upon by the acid; part of the water will be decomposed, its oxygen combining with the zinc and forming oxide of zinc; and its hydrogen will be disengaged in the form of gas from the surface of the zinc plate. The oxide of zinc, in proportion as it is produced, will be dissolved by the acid, thereby forming sulphate of zinc. The plate of copper, which has been immersed in the same fluid, will, during all this time, have undergone no change; the acid, in its diluted state, being incapable of acting upon it. But if, while the above process is going on, the metals be brought into contact, either directly, or by the intervention of some metallic intermedium, the following changes will ensue. In the first place, the oxidation and solution of the zinc will proceed with much greater rapidity and energy than it did before; and in the second place, it will not be accompanied by the evolution of the same quantity of hydrogen gas from the oxidating surface. There will, indeed, be a disengagement of hydrogen from the whole fluid, in quantity exactly corresponding to that of the oxygen derived from the water; but the greater part of this hydrogen will now make its appearance on the surface of the copper plate, whence it will arise in a copious stream of bubbles. But still the copper will itself remain apparently unaffected by this change in the circumstances of the experiment. In process of time, indeed, when a considerable proportion of sulphate of zinc has been dissolved in the fluid, the quantity of disengaged hydrogen is found gradually to diminish, and a thin film, composed partly of metallic zinc and partly of filaments of oxide of zinc, is deposited on the surface of the copper; as soon as this happens the galvanic action ceases.

(42.) If an acid, such as the nitric acid, capable of acting upon the copper, as well as upon the zinc, be employed instead of the sulphuric acid, similar phenomena will take place, with this additional circumstance, that the action of the acid upon the copper will cease the instant the galvanic circuit is completed; and instead of nitrous gas being formed on the surface of the copper, which happens before the circuit is formed, only bubbles of pure hydrogen will make their appearance; and the copper is protected from all further action, the zinc being, as in the former case, oxidated and dissolved with additional energy. It is on this principle that Sir H. Davy has effected the protection of the copper sheeting of ships from the corrosion of sea water, by placing in contact with it pieces of zinc or iron, on which sea water exerts a greater chemical action than on copper. See Phil. Trans. for 1824, p. 151, and 242; and for 1825, p. 328.

(43.) In compound voltaic batteries, the same chemical changes which have been just described as occurring in the simple galvanic circle, take place in each of the portions of fluid intervening in the compartments between the plates.

(44.) The chemical agency of galvanism is exerted in a no less remarkable manner on fluid conductors placed in the circuit between the poles of the battery. Among the simplest of its effects is the resolution of water into its two gaseous elements, oxygen and hydrogen. The discovery of this fact is due to the united researches of the late Mr. Nicholson and of Mr. (now Sir Anthony) Carlisle, and was one of the immediate consequences of the invention of the pile by Volta.

(45.) The most convenient mode of exhibiting the decomposition of water by galvanism is to fill with water a glass tube, (see *fig*. 17.) to each end of which a cork has been fitted so as to confine the water, and to introduce into the tube two metallic wires, by passing one at each end through the cork which closes it, allowing the extremities of the wires, that are in the water, to come so near each other as to be separated by an interval of only a quarter of an inch. The wires being then respectively made to communicate with each of the two poles of a voltaic battery, the following phenomena will ensue. If the wire connected with the positive pole of the battery consists of an oxidable metal, it is rapidly oxidated by the water surrounding it—while at the same time a stream of minute bubbles of hydrogen gas arises from the surface of the other wire, which is in connection with the negative pole. But if we employ wires made of a metal which is not susceptible of oxidation by water, such as gold or platina, gas will be extricated from both the wires, and by means of a proper apparatus may be collected separately. This may be accomplished by taking two glass tubes, or receivers, closed at one end, and filled with water; this fluid is retained by inverting them over a sufficient quantity of water contained in a glass vessel, as is shown in *fig*. 18. Each tube is to be furnished with a platina wire, P and N, passed through the closed extremity, and descending within it through its whole length. The open ends are then to be placed as near to each other as their position in the water will allow; and the wires are to be connected respectively with the opposite poles of a voltaic battery. Gas will immediately be seen to rise from each of the wires, but in different quantities. The tube containing the negative wire, N, will be soon filled with hydrogen gas, while the other, which is traversed by the positive wire, P, will, in an equal time, be only half filled with oxygen gas. This arises from the circumstance that the volumes of the two gases, which form water when combined, or which are the products of the decomposition of water, are in the above proportion; that is, the volume of the hydrogen is to that of the oxygen gas as two to one. That the water is in this experiment perfectly resolved into its two elements is satisfactorily proved by mixing together the gases thus obtained, and firing the mixture by the electric spark; when the whole instantly loses its gaseous form, and is reconverted into water.

Fig. 17.

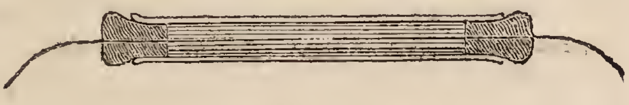
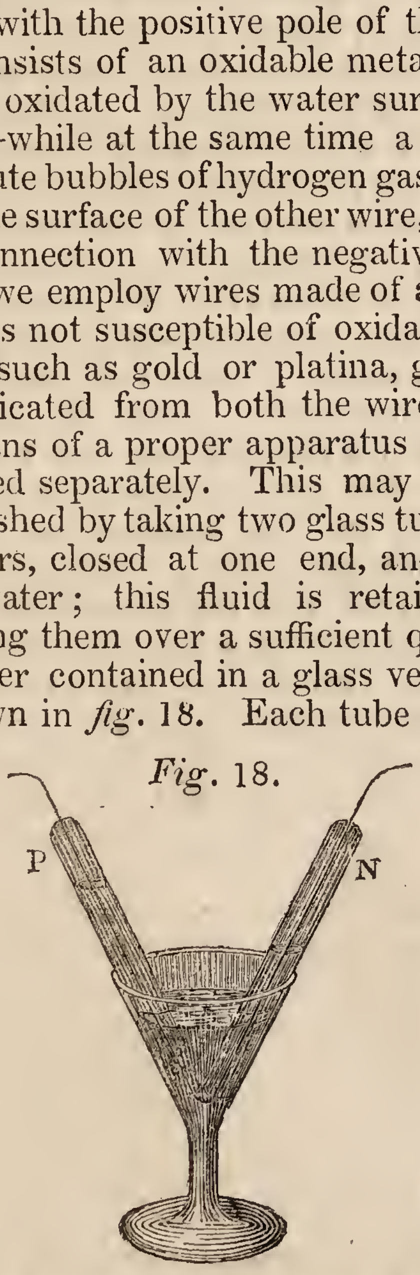


Fig. 18.

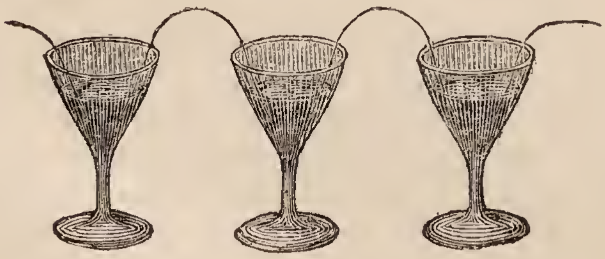


(46.) If the water employed in the preceding experiment be not perfectly pure, other substances besides oxygen and hydrogen will also make their appearance at the two wires, and the apparent formation of such substances from water was the occasion of great perplexity to the earlier experimentalists. But Sir H. Davy succeeded in proving, by a most masterly train of investigation, that when every precaution is taken to ensure the purity of the water subjected to the operation of galvanism, the only products obtained are the two gaseous elements of water, oxygen, and hydrogen.

(47.) In these experiments it became manifest, that under the influence of voltaic electricity neutral salts, existing in any solution, were decomposed, the acid portion being accumulated around the positive wire, on the same points where the extrication of oxygen took place; while the bases, whether earthy, alkaline, or metallic, were, at the same moment, transferred along with the hydrogen to the negative wire. The best mode of exhibiting these decompositions, is to employ two cups, made either of glass, or, where great precision is requisite, of agate, or of gold; the liquids contained in these cups being connected together by a few fibres of moistened asbestos, and subjected to the action of the voltaic battery. If the liquid contain any soluble saline compound, such as sulphate of soda, or common Glauber's salt, and the operation be continued a sufficient time, the whole of the acid contained in the salt will be found collected in the positive cup, and the whole of the alkali in the negative cup. Nor is any considerable solubility in the body placed in the circuit necessary for its decomposition by galvanism. Two cups made of compact sulphate of lime, containing pure water, were connected together by fibrous sulphate of lime, moistened by pure water, and the voltaic current transmitted through them. After an hour the fluids were accurately examined, when it was found that the negative cup contained a pure and saturated solution of lime, partially covered with a calcareous crust; while the positive cup was filled with a moderately strong solution of sulphuric acid. Sulphate of strontites, and fluate of lime, subjected to the same process, yielded similar results: sulphate of barytes, from its greater insolubility, proved more difficult of decomposition; but the difficulty was at length overcome. The analysis of many mineralogical specimens, 'of which the composition was much more complicated, was greatly elucidated by the application of voltaic electricity, which effected the extraction of all the acid and alkaline matters they contained.

(48.) For the production of these effects it is immaterial in what part of the fluid line of circuit the decomposable body happens to be situated. This will appear by placing three cups, side by side, in a line (*fig.* 19.), and connecting them together by moistened asbestos. Let a solution of sulphate of potash, or any other neutral salt, be put into the middle cup, and blue infusion of cabbage into the other cups. When these fluids are placed in the circuit of the voltaic battery, by immersing the wires into the fluid in the outer cups, the sulphuric, or other acid, will collect in the positive cup, and render its blue infusion red, while the alkali will pass into the opposite cup, and tinge its blue contents green.

Fig. 19.



(49.) When metallic solutions are subjected to the decomposing action of galvanism, a deposition of the metal, generally in the form of minute crystals, takes place on the negative wire, and oxide is also deposited around it; while the acid passes over, as before, into the positive cup. This effect takes place with solutions of iron, zinc, and tin, as well as with the more oxidable metals.

(50.) When a solution of nitrate of silver has been placed on the positive side, and distilled water on the negative, the whole of the connecting asbestos becomes covered with a thin metallic film of silver. We have been the more particular in noticing these effects, because, as was before observed (§ 41.), they occur to a greater or less extent in the fluids which occupy the cells of the battery, and have a considerable influence in modifying, and ultimately destroying the power of the instrument.

(51.) Phenomena of a still more extraordinary nature, presented themselves to Sir H. Davy in the further prosecution of these inquiries. It was discovered that the elements of compound bodies were actually conveyed by the influence of the electric current through solutions of substances, on which, under other circumstances, they would have exerted an immediate and powerful chemical action, without any such effect being produced. Acids, for example, may be transmitted from one cup, connected with the negative pole, to another cup on the opposite or positive side, through a portion of fluid in an intermediate cup tinged with any of the vegetable coloured infusions, which are instantly reddened by the presence of an acid, without occasioning the slightest change of colour. The same happens also with alkalies. If three cups be arranged as before, (see *fig.* 19.) and connected with each other in a series by moistened cotton, the middle cup, and also the one next to the positive side of the battery, being filled with blue infusion of cabbage, or of litmus; and the cup next to the negative side containing a solution of sulphate of soda; on the series being placed in the voltaic circuit, a red tinge will soon be perceived in the water of the positive cup, which will become strongly acid. It is evident that the sulphuric acid so transferred must have passed through the fluid in the middle vessel, but without affecting the coloured solution in its passage. By reversing the connections with the poles of the battery, a similar transfer of the alkali will be made; it will be collected in the tinged water of the negative cup, which it will render green; but the intermediate portion of fluid will not, either in this or in the former case, exhibit any trace of the substance which is carried through it by the influence of electricity.

(52.) No union, under similar circumstances, is found to take place, between acids and alkalies, when either of these active chemical principles is transmitted by voltaic electricity through the other, provided the compound which they would form by their union remains soluble; for should the compound be insoluble, the union takes place, and the product, on falling to the bottom of the fluid by its superior gravity, is removed from the line of the electric action. When, for example, sulphuric acid is attempted to be passed through a solution of barytes, or *vice versâ*, barytes through a solution of sulphuric acid, sulphate of barytes is formed, which being insoluble in the fluid, falls down as a precipitate, and being removed from the action of the electric current, proceeds no further in its course. If some basis of mechanical support be provided, whereby this removal from the voltaic influence can be prevented, the transfer may sometimes be continued, notwithstanding the body has assumed a solid form; thus magnesia or lime will pass along moist asbestos, from the positive to the negative sides; but if a vessel of pure water be interposed, they do not reach the negative vessel, but sink to the bottom. In like manner when nitrate of silver was on the positive side, and distilled water on the negative, the silver, as we have already seen, passed along the transmitting fibres of the asbestos, so as to cover it with a thin metallic film.

(53.) When the fluids placed in the same voltaic circuit are connected, not by fluids, but by pieces of metal, such as wires, the changes above described take place in each separate portion of fluid, each alternate metallic surface performing the functions of a positive and negative polarity, according to its place in the circuit of the electric current. Those parts into which the electricity is entering possess properties corresponding to those of the negative wires or poles of the battery; and those which are giving exit to the electricity, act as positive wires. The former will collect around them the several bases of neutral and metallic salts, and the hydrogen of the decomposed water; the latter will collect oxygen, and the compounds in which oxygen predominates, such as the acids.

(54.) The decomposition of the alkalies and of the earths, which crowned this brilliant career of discovery, is, in point of theory, only a particular instance of the general fact above stated, namely, that combustible substances are carried to the negative wire, and oxygen evolved at the positive wire. Various other applications have been made of the voltaic battery to the purposes of chemical decomposition. Sulphuric acid is resolved by its means into oxygen gas and sulphur. Phosphoric acid, in like manner, yields oxygen gas and phosphorus. Ammonia separates into hydrogen and azote, with a small proportion of oxygen. Oils, alcohol, and ether, when acted on by a powerful battery, deposit charcoal, and give off hydrogen, or carburetted hydrogen. But it would be encroaching too far on the province of chemistry to extend our illustrations of this subject to any greater length.

§ 6. *Physiological effects of Galvanism.*

(55.) The action of voltaic, as well as of common electricity, on a living animal is chiefly exerted on the functions of the nervous system. It is shown in the production of sensation, in the excitation of muscular contraction, and in altering the products of secretion.

(56.) If any considerable part of the human body form part of the circuit of a voltaic pile or battery, a separate shock is experienced every time a connection is made with the poles of the apparatus; provided the skin through which the electric current is to pass be sufficiently moist to allow of its being transmitted: for in its usual dry state the cuticle, or outer skin, is scarcely pervious to electricity of such low intensity as that afforded by galvanism. The most effectual method of receiving the whole force of the battery is to wet both hands with water, or what answers still better, with a solution of common salt, and to grasp a silver spoon in each; the circuit is then to be completed by touching one pole of the battery with one spoon, and the opposite pole with the other spoon. Another mode is to plunge a finger of each hand into two separate vessels filled with water, into which the extremities of the two wires from the battery have been immersed. The shock received from the voltaic pile is similar to that resulting from a large electrical battery very weakly charged: and its intensity is greater in proportion to the number of series of elements composing the pile. Twenty pair of plates are generally sufficient to give a shock, which is sometimes felt in the arms: with a hundred pair it extends to the shoulders.

(57.) Independently of the shock felt on the first impression of voltaic electricity communicated from the battery, the continued flow of the current through the body, as long as it forms part of the circuit, is generally accompanied by a continued aching pain. If it pass through any external part deprived of cuticle, it produces a severe smarting or burning sensation, which, if the exposed surface be large, continues to increase till it is scarcely supportable. This painful feeling is experienced if the slightest cut, burn, or excoriation of any kind, happen to be in the path of the electrical current: and it will be excited in these parts, even by a single pair of plates, forming a galvanic combination. It has been remarked by Volta that the pain is of a sharper kind on those sensible parts of the body, included in the circuit, which are on the negative side of the pile; that is, where the electricity flows out from the body, than where it enters: a fact which has also been noticed with regard to the pungency of the common electrical spark.*

(58.) The impression made by voltaic electricity on some of the nerves of the face, when they form part of the circuit, is accompanied by the sensation of a vivid flash of light. The simple application of a piece of zinc and one of silver to the tongue or lips, frequently gives rise, at the moment of the contact of the metals, to this perception of a luminous flash: but the most certain way of obtaining this result is to press a piece of silver as high as possible between the upper lip and the gums, or to insert a silver probe into the nostrils; while, at the same time, a piece of zinc is laid upon the tongue; and then to bring the two metals into contact. Another mode is to introduce some tinfoil within the eyelid, so as to cover part of the globe of the eye, and place a silver spoon in the mouth, which must then be made to communicate with the tinfoil by a wire of sufficient length; or conversely, the tinfoil may be placed upon the tongue, and the rounded end of a silver probe applied to the inner corner of the eye; and the contact established as before. The flash which results from the action of a pile, applied in this way, is very powerful; and if the plates were numerous, the experiment might occasion permanent injury to the sight. This phenomenon is evidently produced by an impression communicated to the retina, or optic nerve, and is analogous to the effect of a blow on the eye, which is well known to occasion the sensation of a bright luminous coruscation, totally independent of the actual presence of light. In like manner the flash from galvanism is felt whether the eyes be open or closed, or whether the experiment be made in day-light or in the dark. If the pupil of the eye be watched by another person when this effect is produced, it will be seen to contract at the moment when the metals are brought into contact. A flash is also perceived at the moment the metals are separated from each other.

(59.) The peculiar taste which is perceived when different metals are applied to different parts of the tongue, and made to touch each other, has already been noticed. It is essential to the success of the experiment, that the surface of the tongue should be moist; for when the tongue is previously wiped very dry, the effect is considerably diminished, and it is not at all perceptible, if the surface is absolutely dry. The quality of the metal laid upon the tongue influences the kind of taste which is communicated; the more oxidable metal giving rise to an acid, and the less oxidable metal to an austere or alkaline taste. Similar differences have been observed by Berzelius, with regard to the sensations excited in the tongue by common electricity, directed in a stream upon that organ, from a pointed conductor; the taste of positive electricity being acid, and that of negative electricity caustic and alkaline. This circumstance would tend to prove that the taste perceived in the galvanic experiment is owing to the actual presence of acids and alkalies, derived from the chemical decomposition of the salts contained in the saliva, by the galvanic action; and that it is not merely the effect of a direct impression of the electric current on the nerves of the tongue.

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* Nicholson's *Journal*, 4to. iv. 180.

(60.) When the current of voltaic electricity is made to pass along a nerve distributed to any of the muscles of voluntary motion, these muscles are thrown into violent contractions of a convulsive kind. It was an observation of this kind that led the way, as we have already seen, to the discovery of the galvanic influence. The muscles of a frog are, indeed, peculiarly sensible to this influence, and are therefore the fittest for the exhibition of this phenomenon, with very weak galvanic powers. The susceptibility of some of the animals belonging to the class of *vermes*, is also very great. If a crown piece be laid upon a plate of zinc of larger size, and a living leech be placed upon the silver coin, it will suffer no inconvenience as long as it remains in contact with the silver only; but the moment it has stretched out its head so as to touch the zinc, it suddenly recoils, as if it had experienced a painful shock. An earthworm will also exhibit the same kind of sensitiveness; and the same effect is still more strikingly exhibited by the *nais*, which is an aquatic worm. Humboldt found that the *lernæa*, or water-serpent, and even the *tænia*, *ascaris*, and other species of intestinal worms, had their movements accelerated by the influence of galvanism, which also speedily destroyed their life. Powerful shocks from a voltaic battery are no less immediately fatal to animals, than discharges from an ordinary electric battery.* Small animals are easily killed by discharges which would only produce a temporary stunning effect on larger animals.

(61.) Convulsive movements may be excited by galvanism in the muscles of an animal, after its death, as long as they retain their contractility. These effects become exceedingly striking, when large animals are made the subjects of experiment, and when powerful batteries are employed. Thus if two wires, connected with the poles of a battery of a hundred plates, be inserted into the ears of an ox, or sheep, when the head is removed from the body of the animal recently killed, very strong actions will be excited in the muscles of the face every time the circuit is completed. The convulsions are so general, as often to impress the spectator with a belief that the animal has been restored to the power of sensation, and that he is enduring the most cruel sufferings. The eyes are seen to open and shut spontaneously, they roll in the sockets as if again endued with vision; the pupils are at the same time widely dilated. The nostrils vibrate as in the act of smelling; and the movements of mastication are imitated by the jaws. The struggles of the limbs of a horse galvanised, soon after it has been killed, are so powerful as to require the strength of several persons to restrain them.

(62.) It is needless to enter into the details of experiments of a similar kind performed in hospitals on limbs removed by amputation; or on the bodies of criminals soon after their execution. A great number of these are stated to have been made at Turin, on the victims of the guillotine; and in this country, Aldini, by operating with a considerable number of plates on the body of a criminal executed at Newgate, produced effects very similar to those already described in the sheep and ox; but which were necessarily of a more impressive character, from their conveying the more terrific expressions of human passion and of human agony.

(63.) Muscles whose actions, like those of the heart, are not under the guidance of the will, are less easily affected by galvanism than the muscles of voluntary motion. But Fowler, Vassali, Humboldt, Nysten, and others, have sufficiently proved that even these muscles may, by the proper application of this power, be made to contract.

(64.) The most curious and hitherto unexplained of the physiological effects of galvanism, are those on the functions of secretion, especially on that of the gastric juice, a fluid which is essentially subservient to the process of digestion. But these topics appertain more to physiology than to the subject of the present treatise.

Chapter V.

Theory of Galvanism.

(65.) The various attempts which have at different times been made to explain the phenomena of galvanism, by the application of the laws which are known to govern those of ordinary electricity, have on the whole been attended with very indifferent success; and the theory

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* See Treatise on Electricity. § 182.

of this science remains, even at the present day, involved in considerable uncertainty and obscurity. No very distinct or satisfactory account has yet been given of the nature of that force, which originally disturbs the electrical condition of the different parts of the voltaic apparatus, and constitutes the primary source of galvanic power. It was long the prevailing hypothesis, that this force was the same with that which gives rise to the developement of electricity during the contact of dissimilar metals; a fact, the principal circumstances attending which have been stated in the treatise on Electricity. (§ 203.) But in proportion as a more extensive acquaintance with the phenomena afforded the means of a more accurate analysis, the insufficiency of this, which was termed the *Electrical Theory*, became more apparent; and it is now fully established, that the primary agent in the evolution of electricity, is the force of chemical attraction. This latter view of the subject, has led to what may be called the *Chemical Theory of Galvanism.*

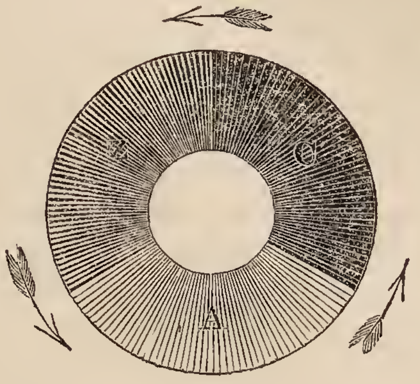
(66.) Every scientific theory must have for its basis some general fact, comprehending a multitude of subordinate phenomena, which are its more or less direct consequences. The chemical theory of galvanism assumes the following as the most general fact in that science: namely, that chemical action, occurring between a fluid and a solid body, is always accompanied by the disturbance of electric equilibrium; in consequence of which a certain quantity of electricity is developed, or, in other words, converted from a latent into an active state. So intimate, indeed, is the connection between the electrical and the chemical changes, that the chemical action can proceed only to a certain extent, unless the electrical equilibrium which has been disturbed be again restored. The oxidation of metallic bodies (that is, their combination with oxygen) is more especially accompanied by the developement of large quantities of electricity. Thus it has been ascertained, that when a plate of zinc is chemically acted upon by dilute sulphuric acid, which produces first oxide, and then sulphate of zinc, the metal becomes negatively electrified, while the liquid is in the same degree positively electrified. This fact, when stated conformably to the hypothesis of Franklin, implies the abstraction of the electric fluid from the zinc, and its transference to the liquid product of the combination: but, when translated into the language of the hypothesis of a double fluid, must be understood as the separation of the two electricities by the chemical action, and the determination of the resinous or negative electricity in the direction of the zinc, and of the vitreous or positive electricity in the direction of the oxidating liquid. In order to avoid perplexity, however, we shall continue to adhere to the simpler of these hypotheses; and advert only to the conditions and movements of positive electricity. (§ 4.)

(67.) That two conducting bodies, such as zinc and acid, thus remain, the one in a negative, and the other in a positive electrical state, notwithstanding their being in contact, is known to us as a matter of fact; but it is a fact which is not explicable by any of the laws of ordinary electrical phenomena, or, in other words, it is not reducible to any other more general fact. We must for the present, therefore, be content to leave it as a subject of future inquiry, to determine to what peculiarity in the circumstances attending the changes of chemical composition it is owing, that the electric equilibrium is permanently disturbed, and what is the unknown obstacle that prevents its restoration. A similar difficulty occurring in the case of the electricity produced by contact, has been noticed in the treatise on Electricity. (§ 204.)

(68.) As long as the chemical action proceeds, the transfer of electricity from the metal to the fluid continues; but the rapidity of the process is checked by the circumstance, that as soon as the quantity transferred has accumulated so as to reach a certain degree of intensity, which is generally exceedingly low, all action ceases, the chemical affinities being balanced by an opposing electrical force. But in consequence of the gradual absorption of electricity by the metal from surrounding bodies, and the gradual dissipation of the superabundant electricity of the fluid, this state is never reached; or, if attained, does not long subsist: and the chemical affinities continue to produce their effects, though more slowly than if their operation were uncontrouled by the electrical force. But if, on the other hand, by the interposition of good conductors, a ready passage be afforded for the electricity from the fluid, where it is accumulated, to the metal where it is deficient, then the obstacle to the further exertion of the chemical affinities between these two bodies will be removed, and the action will now proceed with much greater energy. This is precisely what is accomplished by galvanic combinations. Some metal, such as copper, silver, gold, or platina, not susceptible of oxidation by the fluid employed, is applied to this fluid, collects from it the redundant electricity, and then being brought into contact with the zinc, or metal acted upon by the fluid, communicates to it this electricity, and thus continually restores the electric equilibrium, the very instant after it has been disturbed. We find, accordingly, that under these circumstances, that is, whenever the galvanic circuit is completed, the oxidation of the zinc proceeds with renewed activity; but ceases, or at least takes place more slowly, whenever this circuit is interrupted.

(69.) In order to take a more comprehensive view of the subject, we may state the following as the conditions that are essential to galvanic action. First, the presence of three elements is required, which we shall designate by the letters A, Z, and C. Between the two first of these, A and Z, some chemical affinity must exist, adequate to produce combination and developement of electricity; while the same action, or at least the same degree of that action, is not exerted between the third element C, and either of the former. Secondly, it is necessary that one of the two first bodies, which we shall suppose to be Z, be a solid,* and that it possess a high degree of conducting power with regard to electricity. As it is a general law in chemistry that no chemical action can take place between two bodies, unless one of these bodies be in a fluid state, it follows that as Z is a solid, so A must be a fluid body; on the other hand, the body C may be either solid or fluid. Thirdly, it is requisite that all the three bodies be in mutual contact, so as to compose a kind of circular arrangement, as is represented in *fig* 20. If all these conditions be fulfilled, it is found that a continued stream or current of electricity will circulate in a determinate direction through the bodies thus placed, as long as the chemical action continues. If the bodies Z, A, and C, be respectively zinc, acid, and copper, the surface of contact between Z and A will be that at which the chemical action and consequent developement of electricity takes place; for C may be considered as acting merely the part of a conductor of that electricity between A and Z; and the current will circulate in the direction denoted in the figure by the arrows, that is, from A to C, and thence to Z.

Fig. 20.



(70.) The absolute quantity of electricity which is thus developed, and made to circulate, will depend upon a variety of circumstances, such as the extent of the surfaces in chemical action, the facilities afforded to its transmission, &c.,—causes the operation of which we shall afterwards have occasion to examine. But its degree of intensity, or *tension* as it is often termed, will be regulated by other causes, and more especially by the energy of the chemical action. In a single galvanic circle, however, it is necessarily very low, being limited by the nature of the process to which it owes its origin, and to which it is in some respects opposed. It may be much increased, however, by combining together the power of a number of circles, as is done in the pile and voltaic battery. Taking the common trough battery as an example, and tracing the several steps of the process, we shall find that the electricity which the liquid in the first cell has acquired from the first plate of zinc exposed to its action, is taken up by the copper plate belonging to the second pair, and transferred to the second zinc plate, with which it is connected. This second plate of zinc, having thus acquired a larger portion of electricity than its natural share, is capable of supporting a more intense chemical action than it would otherwise have done; and hence it communicates a larger quantity of electricity to the fluid in the second cell. This increased quantity is again transmitted to the next pair of plates, and renders the third zinc plate capable of maintaining a still more powerful chemical action than the preceding plate; and thus every succeeding alternation is productive of a further increase, both in the quantity and intensity of the electricity developed.

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* Sir H. Davy has shown that chemical action taking place between two fluids, although intense, is not attended with the disturbance of the electric equilibrium. *Philosophical Transactions* for 1826, p. 399, 400.

(71.) The simplest cases are those in which no chemical action whatever is exerted either between the fluid A and the body C, or between C and Z; and the force of the electric current will then be proportional simply to the energy of the chemical action taking place between A and Z. But either A and C, or C and Z, may also have some chemical action on one another; and it will depend on the nature of that action whether the electric force to which it gives rise opposes or concurs with the force resulting from the action between A and Z. If the two actions be of the same kind, as, for example, if they should both be oxidating actions, the electric forces resulting from them will be in opposition to each other; for while the one is impelling the current from Z to A, the others will tend to impel it from C to A, or from Z to C, that is, in a contrary direction. The effective electromotive force will, in all these cases, be equal to the difference between the two that are thus opposed to each other. On the other hand, if the chemical actions between A and C, or between Z and C, should happen to be of an opposite kind, with regard to their electrical tendencies, to that between Z and A, they will communicate to the developed electricity an impulse in the same direction, and the resulting electromotive force will be equal to the sum of the conspiring forces.

(72.) We have seen that the third element C may be either a solid or a fluid body, and we may therefore distinguish galvanic circles into two kinds, according as C has the one or the other of these two forms. In the first, the circle is composed of two solids and one fluid; in the second, of one solid and two fluids. Of the solid elements capable of forming galvanic combinations, the most efficacious are the metals, and charcoal. Of fluid elements, those which exert a powerful chemical action upon the former, such as the mineral acids, alkaline solutions, sulphurets, solutions of neutral salts, and water containing oxygen gas, or atmospheric air. The energy of the galvanic power will depend altogether upon that of the chemical action, and can never be excited when the latter condition is wanting. Thus silver and gold evolve no galvanic influence when in contact with pure water, which is incapable of acting chemically upon either of these metals; but the addition of nitric acid, or any other fluid decomposable by silver, to the water, immediately renders this combination of elements an active galvanic circle.

(73.) With regard to the direction given to the electrical current by the chemical action of two bodies, we may lay it down as a general rule, to which there are but few exceptions, that the electricity is determined from the solid to the fluid which acts upon it chemically. This we have already seen exemplified in the instance so frequently referred to of the ternary arrangement of zinc, acid, and copper. Another, and very common mode of expressing the same fact is, to say that the zinc is rendered positive with regard to the copper, and, *vice versâ*, the copper negative with reference to the zinc. In this sense, that is with relation to the action of acids and other oxidating fluids, every oxidable metal is positive with regard to a metal which is oxidable in a less degree.

In order to determine beforehand the effect of any combination of two metals in a galvanic circle with any of the acids, it will be convenient, therefore, to arrange the metals in the order of their oxidability. With this view the following catalogue has been given by Sir Humphry Davy:* viz.

Potassium and its amalgams.
Barium and its amalgams.
Amalgam of zinc.
Zinc.
Cadmium.
Tin.
Iron.
Bismuth.
Antimony (?).
Lead.
Copper.
Silver.
Palladium.
Tellurium.
Gold.
Charcoal.
Platina.
Iridium.
Rhodium.

(74.) In a ternary galvanic arrangement with acids, then, each metal in the above list is positive to all those which follow it; and the more so in proportion as the two metals are more distant from each other in the scale. Thus zinc and iron will compose a weaker circle than zinc and silver; and zinc and platina will form one of still greater power. It may be observed, however, that the precise order in which the metals stand in such a scale as the above, must be understood as only strictly true with relation to the particular acid employed, and even to the particular degree of dilution that has been given to the acid. For we find slight variations in the order of relation of the metals with different acids, or even with the same acid in different states of concentration.

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* *Philosophical Transactions* for 1826, p. 408.

(75.) When alkaline solutions are employed as the fluid agent, instead of acids, the same general order is observed in the metals, with regard to their mutual electrical relations. The principal exception is with regard to iron, which is here found to occupy a place intermediate between copper and silver. Thus a combination of iron and copper will, by immersion in an acid, form a circle in which the electricity will be determined from the iron to the acid, thence to the copper, and thence to the iron; that is to say, the iron will be positive with regard to the copper. But if the same combination of iron and copper be acted upon by an alkaline solution,* and more especially by ammonia, the iron is negative with regard to the copper; for here the chemical action of the fluid upon the copper is stronger than upon the iron, and the electricity is therefore determined to the fluid from the copper, and not from the iron as in the former case. The same results are obtained when tin is employed in conjunction with copper, and with ammonia.†

(76.) With solutions of hydro-sulphurets, the several metals stand also nearly in the same order, as to their electrical relations, as with acids—with a few exceptions, however, as will appear from the following catalogue, given by Sir H. Davy:—

Zinc.
Tin.
Copper.
Iron.
Bismuth.
Silver.
Platina.
Palladium.
Gold.
Charcoal.

We may observe, that here also copper is positive with regard to iron; so that when these two metals form a circle with a solution of hydro-sulphuret, the electrical current is in an opposite direction to what it is when the same combination of metals is plunged in acids.

(77.) It need hardly be observed, that every thing that has been stated with regard to single galvanic circles applies also to compound circles, whether in the form of the pile, or the trough battery, composed of the same elementary parts.

(78.) We have next to consider the second class of galvanic circles; those, namely, which are composed of a single solid and two fluid elements.

The arrangement assumed in this case by the three elements of the circle, may be represented by the same diagram as before, *fig.* 20. Z will then denote the solid; A the acting fluid, and C the conducting fluid. As there is a necessity for separating the two fluids, they may be contained in separate vessels, and be made to communicate by means of a bent tube, inverted like a syphon, full of some conducting liquid, and passing over from the one to the other of the two fluids. Sir H. Davy uses, in many of his experiments, fibres of moistened asbestos in place of the tube, for establishing a communication between the fluids. Two plates of the same metal are then to be immersed in the fluids, and made to communicate by wires, or slips of the same metal.

(79.) Sir H. Davy has distinguished three different kinds of circles of the second class.‡

The first and most feeble is composed of single metallic plates, arranged in such a manner, that two of their surfaces are in contact with different fluids, one capable, and the other incapable, of oxidating the metal. Zinc, acid, and water, occupying the situations of Z, A, and C, in the diagram, may be taken as an example; and it will be seen that the only difference between this arrangement and those of the former class, consists in the substitution in the circle of water for copper; but the function of each of these parts is essentially the same, namely, that of simply conducting electricity between the other two elements. As the conducting power of fluids, however, is much inferior to that of metals, the electrical indications will be more feeble than in circles of the first class; and, indeed, will scarcely be sensible unless we employ the more easily oxidable metals, such as tin and zinc. But powerful effects may be obtained by combining a number of such circles in a pile or battery. For constructing an instrument of the former kind, Sir H. Davy directs pieces of polished tin, about an inch square and one-twentieth of an inch thick, to be piled up with woollen cloths of the same size, moistened some in water, and some in dilute nitric acid, in the following order,—tin, acid, water, and so on. It is proper to observe the precaution of placing the cloth moistened with acid underneath the one which is moistened with water; for, as the acid is specifically heavier than the water, little or no mixture of fluid will then take place. Twenty such alternations will produce a battery capable of acting weakly on the organs of sense, and of slowly decomposing water. When zinc is the metal used, it is necessary, on account of its rapid oxidation in water containing atmospheric air, to use three cloths; the first moistened with a weak solution of hydro-sulphuret of potash, which has no power of acting upon zinc, and which prevents it from being acted upon by the water; the second moistened with a solution of sulphate of potash, of greater specific gravity than the solution of hydro-sulphuret; and the third wetted with an oxidating fluid, such as an acid, specifically heavier than either of the solutions. In this case, if, proceeding upwards, the order be as follows — zinc, — oxidating solution,—solution of sulphate of potash,—solution of hydro-sulphuret of potash, very little mixture of the fluids, or chemical action between them will take place; and an alternation of twelve series of this kind, forms a battery capable of producing sensible galvanic effects. The direction of the electrical current is, as usual, from the zinc to the oxidating fluid.

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* Davy, *Elements of Chemical Philosophy*, p. 148.

† De la Rive, *Annales de Chimie*, xxxvii, 232.

‡ *Philosophical Transactions* for 1801, p. 398.

(80.) It has often been remarked that porter drank out of a pewter pot has a brisker taste than when taken out of a glass. Professor Robison ascribed this to the influence of galvanism, arising from the circle formed by the metal and two different fluids. He considered that in the act of drinking, one side of the pewter pot is exposed to the action of the saliva which moistens the lip, while the other side of the metal is touched by the porter; the circle being completed when the latter fluid comes in contact with the tongue.

(81.) The second kind of galvanic combinations with a single metal, consists of a series of plates composed of a metal capable of being acted upon by sulphuretted hydrogen, in contact with solutions of hydrosulphurets on the one side, and water on the other, placed in a regular order of alternation. Under these circumstances, a current of electricity is produced, the direction of which is the reverse of what it is in the former case, the surface of the metallic plate in contact with the solution of sulphur being positive, while that in contact with acid is negative. Eight series will produce sensible effects. Copper, silver, and lead are each capable of forming this combination; their comparative activity being in the order in which they are here enumerated, that is, copper the most, and lead the least.*

(82.) A familiar instance of the operation of galvanism in promoting the combination of sulphur with silver, occurs in the employment of a silver spoon in eating the yolk of an egg; a galvanic circle of the second kind being formed by the yolk, which contains sulphur, the silver spoon, and the saliva of the tongue.

(83.) The third kind of combinations unite the power of the two former, and consist of a single metal, acted upon on one side by an acid, and on the other side by the hydro-sulphurets. Copper, silver, or lead may here be employed, and the order of their powers is the same as in the preceding instance. The pile may be constructed in the same manner as the pile with zinc in the first kind of combination; the cloths moistened with acid being separated from those moistened with solution of hydrosulphuret by an intermediate cloth soaked in solution of sulphate of potash. Three plates of copper, or silver, arranged in this manner, in proper order, produce sensible effects; and a pile composed of twelve or thirteen series is capable of giving weak shocks and of rapidly decomposing water. The current of electricity is determined as in the two former cases.

(84.) Greater permanency may be

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* *Philosophical Transactions* for 1801, p. 400.

given to the effects of these combinations of a single metal with two fluids, by a disposition of the plates similar to the trough of Cruickshanks, with partitions alternately of metal and of horn or glass; and with the cells filled alternately with the different solutions, according to the kind of combination employed; these fluids being connected in pairs with each other, by slips of moistened cloth, carried over the non-conducting plates.

(85.) Efficient galvanic circles may also be formed with a single metal and with the same fluid solvent, (an acid, for example,) provided the action of the latter is different on the two sides of the metal, by being of different degrees of strength. Thus, if one of the branches of a tube, bent in the form of a V, contain concentrated sulphuric acid, while the same acid in a diluted state occupies the other branch, in which case the two fluids will, on account of the difference in their specific gravities, remain without mixing with each other; and two portions of the same metal, zinc for instance, be then immersed in these fluids, and made to communicate with each other, galvanic electricity will be evolved, and determined from the metal to the diluted acid, in consequence of the action of this portion of acid upon the zinc being greater than that of the concentrated acid. But with those metals, which are more acted upon by the latter than by the former, the influence of the concentrated acid will preponderate, and the current will be determined in an opposite direction. In like manner it has been observed, that two solutions of common salt, the one concentrated, the other diluted, form a galvanic circle with copper; that metal being more acted upon by the latter than by the former, became negative to the one and positive to the other.*

(86.) The application of these principles will explain a variety of apparently anomalous facts, which are continually presenting themselves in the course of experimental researches. Sir H. Davy observed, for instance, that when two pieces of the same polished copper were introduced at the same moment into the same solution of hydro-sulphuret of potash, there was, as might be expected, no action; but if they were introduced in succession, there was a distinct, and often, if the interval of time was considerable, a violent electrical effect; the piece of metal first plunged in being negative with relation to the other. This is owing to the rapid formation at the surface of contact of sulphuret of copper, which, by its presence, prevents, or at least diminishes, the further action of the fluid; the clean surface of the plate last introduced is therefore attacked comparatively with greater force, and determines a galvanic effect.* Many singular and apparently capricious changes of electric states occur in these and other experiments of the same kind, whenever new substances are produced by the chemical action, which at first adhere to the metal, but are liable to be detached in smaller or larger portions, and thus occasion sudden alterations in the conditions of the galvanic elements.

(87.) Having thus seen how, under certain circumstances, it is possible to form various galvanic combinations with a single metal and a single fluid, it remains for us to notice the attempts that have been made to produce the same effect without the aid of any metallic substance, or even of charcoal. Lagrave announced that by placing upon each other alternate layers of muscle and of brain, from a human body, with pieces of moistened cloth or leather interposed, he formed a pile which produced galvanic effects.† Dr. Baconio, of Milan, composed a galvanic pile entirely of vegetable substances: namely, discs of red beet-root, two inches in diameter; and discs of walnut-tree, of the same size, divested of their resin by digestion in a solution of cream of tartar in vinegar. With a pile so constructed, and with a leaf of scurvy-grass as a conductor, he is said to have excited galvanic convulsions in a frog.‡ Aldini also succeeded in producing the same effect without the intervention of any metallic substance; sometimes by bringing into contact the nerve of one animal with the muscle of another, and at other times by employing the nerves and muscles of the same animal. In some of his experiments the most powerful contractions were excited, by bringing the parts of a warm-blooded animal into contact with those of a cold-blooded animal. On introducing, for example, into one of the ears of an ox recently killed a finger of one hand, moistened with a solution of salt, and holding in the other hand a prepared frog, when the spine of the frog was made to touch the tongue of the ox, convulsions took place in the limb of the frog. In like manner, when he held a prepared frog by one hand, moistened with solution of salt, and applied the crural nerves of the animal to the tip of his own tongue, convulsions were produced.* Many of these experiments were made in presence of the members of a commission of inquiry appointed by the French Institute: and they have since been repeated with success in London, at the Anatomical Theatre in Great Windmill Street.

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* Becquerel, *Annales de Chimie et de Physique*, xxxv. 120.

* *Philosophical Transactions* for 1826, p. 393.

† *Journal de Physique*, lvi. 235; and Nicholson's *Journal*, v. 62.

‡ Nicholson's *Journal*, xviii. 159.

(88.) It is well known that several fishes, such as the *torpedo*, which is a species of ray; the *gymnotus electricus*, or the electric eel; the *silurus electricus*, a species peculiar to some of the rivers in Africa; and also the *trichiurus indicus*, and *tetraodon electricus*, which are fishes found in the Indian ocean, possess the power of giving electrical shocks to animals that touch them, or communicate with them by electrical conductors. Anatomical investigation has shown that this power resides in organs of a very peculiar construction. In the torpedo they are composed of a great multitude of vertical and parallel membranous plates, arranged in longitudinal columns of quadrangular, pentagonal, or hexagonal forms, with a loose net-work of tendinous fibres passing transversely and obliquely between the columns, and uniting them firmly together. Each column is, moreover, divided by a great number of thin horizontal partitions, placed over each other at very small distances, and forming numerous interstices, which appear to contain a fluid. All these parts are supplied by a great abundance of blood-vessels, and by a still more extraordinary proportion of nerves.

(89.) In the regular arrangement of their plates these organs have a marked resemblance to a voltaic battery; we know nothing, however, of the immediate source from which they derive electrical properties. Mr. Cavendish compared the action of the torpedo to that of a large electrical jar very weakly charged: and Volta considered it as still more analogous to that of the galvanic pile. Sir Humphry Davy, with a view to ascertain the justness of Volta's comparison, passed the shocks given by living torpedos through the interrupted circuit made by silver wire through water, but could not perceive that it produced the slighest decomposition of that fluid. The same shocks made to pass through a fine silver wire less than one thousandth of an inch in diameter did not produce ignition. Volta, to whom Sir H. Davy communicated the results of these experiments, considered the conditions of the organs of the torpedo to be best represented by a pile, of which the fluid substance is a very imperfect conductor, such as honey; and which, though it communicated weak shocks, yet did not decompose water. Sir H. Davy also ascertained that the electrical shocks given by the torpedo, even when powerful, produced no sensible effect on an extremely delicate magnetic electrometer. In a paper recently read at the Royal Society, he explains these negative results by supposing that the motion of the electricity in the organ of the torpedo is in no measurable time, and wants that continuity of current requisite for the production of magnetic effects.

(90.) Mr. Geoffroy St. Hilaire has found an organic structure very similar to that of the torpedo in other animals of the ray genus, which, nevertheless, do not possess any electrical powers.

(91.) Electrical effects are obtained from a pile composed of thin plates of different metals in the usual order, with discs of writing paper interposed between them. This species of pile was the invention of Mr. De Luc, who gave it the name of the *electrical column*. It may be constructed of pieces of paper, silvered on one side, by means of silver leaf, and alternated with thin leaves of zinc; taking care that the silvered surfaces of the paper discs are always in the same direction. A very large number of these may be contained in a glass tube of moderate length, previously well dried, having its ends covered with sealing-wax, and capped with brass. The most extensive instrument of this kind was made by Mr. Singer, and consisted of twenty thousand series. Each of the two ends or poles of the column affect the electrometer, and exhibit electrical attractions and repulsions; the apparatus will even give sparks, and communicate shocks of considerable force: but it possesses no sensible power of chemical decomposition when applied to fluids in the interrupted circuit. If two upright electrical columns be placed side by side, with their poles in opposite directions, and connected at their upper ends, while a small bell is attached to the lower end of each; the whole will act as one column, and each bell will, in consequence of the electrical actions, be alternately struck by a brass ball suspended between them; and thus a continual ringing will be produced as long as the machine remains in action, which is generally for a considerable time. This action is, however, kept up solely by the presence of moisture in the paper, for it does not take place at all when the paper is perfectly dry; and although the process of oxidation is very slow, the more oxidable metal is in process of time found to be tarnished.

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* Nicholson's *Journal*, iii. 298.

(92.) An apparatus somewhat analogous to that of De Luc was constructed by Hachette and Desormes with pairs of metallic plates, separated by layers of farinaceous paste, mixed with common salt. To this instrument, although it evidently owed its efficacy to the moisture of the paste, they gave the very inappropriate name of *dry pile.* It has the same properties as the electric column, except that it is unable to give a shock. A pile having nearly similar powers was also constructed by Professor Zamboni, of Verona, with discs of paper, gilt or silvered on one of their sides, while the other side was covered with a layer of pulverized black oxide of manganese, mixed with honey. Both this and the former instrument retained their power for a great length of time.

(93.) Piles formed simply of discs of copper and moistened card, placed alternately, were found by Ritter to have no power of developing electricity by their own action, but to be capable of receiving a charge by being placed in the circuit of a powerful voltaic battery, and of thus acquiring, though in an inferior degree, all the properties of the battery itself from which it derived its activity. The properties of these *secondary piles*, as they have been called, are obviously the effect of a series of electrical inductions, extending from end to end; and the apparatus is found to retain its charge for a very considerable time, provided it be kept insulated, and the communications between the two poles are not renewed too frequently.

(94.) Having thus traced the various ways in which galvanic power may be excited, we have next to examine the influence of different circumstances, by which its quantity, intensity, and mode of action are regulated. We have already seen that the intensity of the electricity developed by a single galvanic circle, bears no relation to the extent of surface of the elements which compose that circle. It follows, therefore, that however much we may increase the quantity of electricity by employing very large plates, as in the calorimotor, we cannot obtain from such an instrument any of those effects which require for their production a certain intensity, as well as quantity of electricity. In order to obtain these latter effects, we must employ the compound battery, consisting of a considerable number of alternations of the same elements. The former of these instruments, accordingly, will be capable of producing such effects as depend upon mere quantity, without regard to intensity; such as the evolution of heat, the ignition and deflagration of the metals, and electro-magnetic phenomena. The compound apparatus, on the other hand, will afford the more ordinary electric appearances, (such as the spark, and the phenomena of attraction and repulsion,) will affect the electrometer, or condenser, and will communicate a charge to a Leyden jar; for in all such operations, intensity of electricity is the most essential requisite, and the power of the battery to produce them is found to be augmented by every increase in the number of the alternations. But there is also a third class of effects, more peculiarly appertaining to galvanism, which take place by the transmission of the electric current through bodies of inferior conducting power; such as liquids of various kinds, and living organized structures, both animal and vegetable: producing in the former chemical decomposition, and in the latter various physiological effects, such as nervous excitation, muscular contraction, and affections of secretion. For the production of these effects it is necessary, not only that the electricity be sufficiently powerful, both in respect to intensity and to quantity, but also that it should flow in a continuous current. It is from the difficulty of supplying this latter requisite, that the electricity derived from the common electrical machine, is, under ordinary circumstances, incapable of decomposing water in the way that is so readily accomplished by voltaic electricity. It is from deficiency of intensity, on the other hand, that we are unable to obtain the same effects from the calorimotor, which amply fulfils the conditions of quantity and continuity. The electricity which it furnishes, however abundant in quantity, does not possess sufficient intensity to overcome the obstacle presented by the smallest thickness of water, or other liquid of low conducting power; and is, for the same reason, incapable of penetrating through the skin, or traversing through any other part of an animal body. Hence we can obtain from it neither chemical nor physiological effects. The electricity furnished by the electric column of De Luc, again, though of sufficient intensity to produce the shock and other effects of a sudden influx, is too deficient in quantity to produce chemical action; and the same general observations apply to the electricity of the torpedo.

(95.) Every circumstance that facilitates the passage of the electric current in all parts of the circuit, will tend to increase the quantity that circulates. The degree of conducting power possessed by the fluid parts of the circle, will, therefore, have an important influence on the power of the apparatus. Hence the addition of various saline bodies to the fluid is found to increase the efficacy of the voltaic battery, probably, in part at least, by increasing the conducting power of the fluid; but as such substances generally also promote chemical action, it is always in some degree doubtful what part of the effect is to be ascribed to the one or the other of these causes.

(96.) As the fluid element of the circle is the part having the smallest conducting power, the electric current will be retarded by having to pass through any considerable extent of fluid. With a view to augment the activity of the battery, it is an object to bring the two metallic surfaces of Z and C very near each other, so that the distance the electricity has to pass from the one to the other, through the fluid, shall be as small as possible; and for the same reason the surface of C, which collects the electricity from the fluid, should be sufficiently extensive to effect this purpose completely. We hence perceive the reason of the advantage derived from employing in the common trough battery, according to the suggestion of Dr. Wollaston, a double plate of copper to each plate of zinc, so that each surface of the latter metal acted upon by the fluid, may have a surface of copper opposite to it, (§ 18.); and also of enveloping each coil of zinc plate, in the calorimotor, by a coil of sheet copper, (§ 8.) Mr. Marianini has extended this principle still further, and has found that the maximum of effect takes place when the surface of the copper is no less than eight times greater than that of the zinc.

(97.) There is yet another cause of impediment to the motion of the electric current of a singular kind, and which produces very considerable effect. It appears from the experiments of Mr. Augustus De la Rive, that voltaic electricity, in passing out of one conducting body into another of a different kind, always sustains some loss of its intensity.* The amount of this loss varies much in different cases, according to the nature of the two conductors; and it is different with different degrees of intensity. In the case of the passage of the electricity from a fluid to a metal, or *vice versâ*, it is very great, and it is sensible even when it has to pass from one liquid to another, or along a mixed conductor composed of two different kinds of solids. The impediment arising from the mere change of conductor is quite independent of the peculiar conducting powers of the one or the other of the substances through which the electricity passes. Mr. De la Rive found, for example, that a much greater obstacle existed to the transmission of the electricity between sulphuric acid, especially when concentrated, and platina, than between nitric acid and the same metal; and accordingly, on sending the electric current from a voltaic battery through a number of portions of sulphuric acid, contained in separate glasses, and connected by arcs of platina wire, it proved to be a much worse conductor than when nitric acid was employed in a similar arrangement. But the conducting powers of each system of compound conductors were immediately rendered equal by dipping the ends of the platina wires in nitric acid, before immersing them in the sulphuric acid.†

(98.) In general the more readily a metal is acted upon by liquid conductors, the less is the diminution of intensity which is sustained by the passage of electric currents through them. Mr. De la Rive states it to be a general law, that, independently of the effects of chemical action, the influence of the obstacle opposed to the passage of electricity from a fluid to a solid conductor, is such, that when two metallic surfaces, either of the same or of different metals, are immersed in a fluid, so as to form a galvanic circle, that metal which transmits the electricity with the least loss of intensity is positive with respect to the other metal.*

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* *Annales de Chimie et de Physique*, xxxvii. 267.

† Ibid. p. 273.

(99.) The influence of this retarding cause varies also with the intensity of the current itself. The loss of electricity, from its passage through a number of metallic plates, is scarcely sensible when the current is very energetic, as, for instance, when it proceeds from a battery composed of a great nnmber of plates; but it becomes more and more perceptible, according as the original intensity of the current is less considerable. It is also remarkable that the current is disposed to pass more readily through imperfect conductors, which present a great degree of resistance, when it has previously been made to traverse a great number of metallic plates. This was illustrated in two comparative experiments, in the first of which a current, originally of high intensity, was reduced, by passing through a considerable number of plates, till it was equal in intensity to one originally weaker, that had, in the second experiment, passed through a smaller number; of the two currents, thus apparently rendered equal in every respect, it was nevertheless found that the one which had previously passed through the greater number of plates, was thereby rendered capable of passing through any succeeding plate with less loss of intensity than the other current. The phenomena, he states, correspond to those which would take place, if we could imagine that there were two distinct kinds of electric current—the one capable of passing indiscriminately through all sorts of conductors, good or bad; the other capable of passing through good conductors alone. The passage of the currents through successive plates gradually effect the separation of these two portions, the plates arresting the one which cannot pass so readily through bad conductors, and giving free passage to the other portion.

(100.) M. De la Rive has applied this theory to the explanation of the different effects resulting from the increase of the number of the plates. If the pile, he says, consist only of a small number of plates, the electricity produced by it, not having undergone the above process of *filtration*, as it may be called, only one part of it will be capable of passing through an imperfect conductor, which is presented to it, and the other part will be arrested; but if a good conductor be presented, the whole of the electricity finds a ready passage, and will produce corresponding effects. Electricity of the former kind only will be capable of producing chemical decompositions, and of passing through organized bodies; but, in the latter case, it will be adequate to the production of all the calorific and magnetic effects. These modifications of electricity would, if this theory were established, have a remarkable analogy with those of light and of heat, under circumstances somewhat parallel.

(101.) It must be observed, however, that one source of the diminution of effect consequent on the multiplication of surfaces, exists in the transfer of elements which takes place in the fluid from galvanic action. This transfer, as is remarked by Sir H. Davy, in as far as it has actually occasioned the deposit of a positive element on the negative surface, and *vice versâ*, has an immediate influence in checking the further progress of the galvanic action; and arrests it completely when it has proceeded to a certain extent. Hence the powers of batteries are found to diminish by the continuance of their action, and ultimately to cease. This change we have already noticed in treating of the chemical actions of the simple galvanic circle. (§ 41.)

(102.) It is obvious that the several causes of retardation now stated render it exceedingly difficult to determine, previous to actual experiment, the relative powers of different batteries, composed of different materials, and consisting of different numbers of alternations of its parts.

(103.) It is not easy to understand the manner in which the chemical elements of a body decomposed by galvanism, are carried to their respective stations in the voltaic circuit. Thus if the influence of a powerful battery be transmitted through water, it will operate in decomposing that fluid, although the wires which form the communication with the poles be at a considerable distance from each other. They may even be placed in separate vessels, provided the portions of water in which they terminate are made to communicate with one another by means of a syphon full of water, or even by moistened threads. We find, under these circumstances, the whole of the oxygen of the decomposed water transferred to the positive, while the hydrogen is collected at the negative wire. Two questions may here be asked: first, in what part of the circuit does the decomposition take place? secondly, in what mode are the elements of the decomposed particles transferred to such distant points, without any indication being afforded of their movements, which must be exceedingly rapid, in order to traverse through so long a space? The velocity of this transfer would appear to be very considerable from the following experiment made by Dr. Roget, in the year 1807. The ends of two platina wires, communicating with the poles of a powerful battery, were introduced into two separate vessels of water, communicating by means of a long tube, bent into the form of a syphon, and filled with a solution of common salt. The whole length of the fluid part of the circuit between the two wires was 46 inches. Microscopes were applied to the ends of the wires, for the purpose of enabling the observer and an assistant, (who was the late Mr. Sylvester,) to ascertain the precise moment when the gases made their appearance at the respective wires. No sensible interval of time could be perceived between the appearance of the oxygen gas at the positive, and of the hydrogen gas at the negative wire, when the communications with the battery were made.

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* *Annales de Chimie et de Physique*, xxxvii. 284.

(104.) The transfer of material and ponderable substances, such as those which constitute the elements of water, might be expected, even with a moderate velocity, to occasion visible currents in the fluid through which they pass; for their motion, by whatever force produced, must be accompanied by a certain momentum, sufficient to displace the particles of the fluid through which they pass. Dr. Roget could, however, detect no appearance of current or displacement of fluid; such as would be indicated by movements among the minute globules of dust, or other extraneous matters suspended in the water, even with the assistance of the microscope. Mr. Wilkinson and Professor de la Rive, have also arrived at the same conclusion, by employing microscopes of high magnifying power.

(105.) These phenomena of transfer have appeared to some so inexplicable, upon the commonly received doctrine of the composition of water, that they have had recourse to a new hypothesis in order to solve the difficulty. Professor Ritter was led to consider water as a simple substance, forming oxygen by its combination with positive electricity, and hydrogen by its union with negative electricity; and this theory was adopted by several other philosophers. Monge endeavoured to account for the phenomena, by supposing that water formed compounds with excess of oxygen on the one hand, or excess of hydrogen on the other; which compounds passed in opposite directions between the two wires, each depositing on their arrival the superabundant ingredient. Dr. Bostock conceived that the water was decomposed at the positive wire only, where its oxygen was disengaged; and its hydrogen, uniting with electricity, was carried invisibly along with it to the negative wire, where this union being dissolved, the electricity passed on through the wire, and the hydrogen appeared in its gaseous form.

(106.) The following mode of explaining these phenomena was suggested by Dr. Roget, in a paper which was read to the Philosophical Society of Manchester, in 1807.

"We may conceive the agency of electricity to extend throughout the whole of the fluid line connecting the two wires. The hydrogen existing in every particle of water in this line, will, if it possess a positive electrical polarity, according to the hypothesis of Mr. Davy, be repelled by the positive, and attracted by the negative wire. We may consider the row of particles of hydrogen abstractedly from those of oxygen. While the former are moving together, by the agency of the electricity, in a direction towards the negative wire, all those particles which have not yet reached that wire, will merely have to pass over in succession from one particle of oxygen to the next, among those of the other row. They will not appear in the form of gas, because the instant each has quitted the particle of oxygen with which it was associated, it meets with another to combine with; and this process will be continually repeated, until it has arrived at the end of the line, when, finding no oxygen to unite itself with, it will make its appearance in the form of gas. In like manner, the first particle of hydrogen, in the series, by its abandoning the first particle of oxygen, which finds no other particle of hydrogen to replace it, causes the oxygen to appear at that point in the form of gas. We have thus the two gases formed at each end, not from the same individual particle of water, but from the two which happen at that moment to be in contact with the wires. The production of the two gases will take place at the same instant in both places, each particle having only to move one step, that is, from one particle to the adjoining one, instead of having to traverse the whole extent of the line, and no current will be perceptible in the fluid. If this theory be correct, the operation of gravity in favouring the descending current of the heavier element, namely oxygen, might be rendered sensible; and that this is actually the case appears by an observation of Mr. Sylvester, that when the wire giving out oxygen is placed at a much lower level than that which gives out hydrogen, the effect is sensibly greater than when the positions are reversed."

(107.) Similar explanations of the mode of transfer have been given by Dr. Henry, and by Grotthus; and from the following passage in Sir H. Davy's last paper on the subject,* it would seem that he entertained views somewhat similar. "If it be supposed that the fluid is divided into two zones, directly opposite in their powers to the poles of the battery, the virtual change may be regarded as taking place in the two extremities of these zones nearest the neutral point; so that by a series of decompositions and recompositions, the alkaline matters and hydrogen separate at one side, and oxygen, pure, or in union, at the other. In this way the electricity may be regarded as the transporter of the ponderable matters, which assume their own peculiar characters at the moment when they arrive at the point of rest." That visible motions are sometimes produced in fluid conductors when transmitting the electric current, has been shown by Sir H. Davy, who noticed the very singular convulsive agitations into which mercury is thrown, when placed within the circuit of a powerful voltaic battery discharged through water.† These motions, which are frequently of a violent and capricious kind, have also attracted the attention of Mr. Herschel, and he has made them the subject of an interesting research, of which an account is contained in the Philosophical Transactions.*

(108.) The following singular fact has been noticed by Mr. Porrett:—If a vessel be divided by a membranous partition into two compartments, of which the one is filled with water, and the other contains but a very small quantity, and if the positive wire from a voltaic battery be inserted into the former, and the negative wire into the latter, the water will be impelled from the first compartment into the second, through the partition, and will at length rise to a higher level in the latter than in the former.† Mr. A. De la Rive, upon repeating these experiments, arrived at the same result, when he employed distilled or river water, which has but a small conducting power; when, however, a saline solution of sufficient strength was used, no such effect of impulsion was perceptible. But the reality of such an effect under the above circumstances, is sufficient to establish the existence of a mechanical force derived from the current of voltaic electricity.

(109.) We have already had occasion to observe that a theory, founded upon totally different views of the sources of galvanic power from those which have now been stated, has been applied to the explanation of the phenomena. As this, which has been termed the *electric theory of galvanism*, has been adopted by several eminent philosophers, it ought not to be considered as undeserving of notice in this place.

(110.) It was conceived by Volta, the original author of this theory, that the primary source of the electricity liberated during the action of a galvanic apparatus, might be traced to the contact of the dissimilar metals. He assumed as a fundamental fact, that during the whole time that these metals are in contact, a certain force is in constant operation, tending to effect a transfer of electricity from the one metal to the other. To this force he gave the name of *electromotive force*. When, for example, zinc and copper are in contact, the alleged operation of this force is to impel the electricity from the copper to the zinc, so as to maintain in the latter a positive state, when compared with the former, which will, consequently, itself be in a negative state with relation to the zinc. If either of these states be reduced to a more neutral condition by communication with other bodies; that is, if the redundant electricity of the zinc be carried off, and the deficiency of electricity in the copper be supplied from other sources, the electromotive force will, he conceived, immediately renew this difference of condition, and thus maintain a continual and rapid current of electric fluid, flowing always in the same direction.

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* *Philosophical Transactions* for 1826, p. 416, 417.
† *Elements of Chemical Philosophy*, p. 172.

* For 1824, p. 163.
† Thomson's *Annals of Philosophy*, viii. 74.

(111.) It was further assumed in this theory that liquids have no electromotive power when in contact with metals: and that this negative property enabled them to transmit the electricity evolved by the contact of the zinc and copper, and which is accumulated in the zinc, back again to the copper; whence it is again transferred to the zinc; and so on in a perpetual circle. In compound galvanic circles, the electromotive force residing in the surfaces of contact between the two metals in each pair of plates, are all tending in the same direction; and the several impulses they give to the electricity conspire together to increase the effect, which will therefore be the sum of all the forces taken separately. Thus will a continued and powerful stream of electricity be determined from the negative to the positive pole of the battery, ready to circulate through any conducting line of communication extending between the two poles. The office of the fluid is considered, in this theory, as simply that of conducting the electricity from the one metal to the other: its chemical action on either of these being regarded as a mere accidental circumstance, not in any way concerned in the production of galvanic or electrical effects. The effective quantity of electricity which actually circulates in the voltaic battery is supposed to be determined altogether by the degree of conducting power possessed by the liquid: for it is assumed that the quantity which the electromotive force existing at even the smallest surface of contact between dissimilar metals could set in motion, if the movements of that electricity were not impeded by the difficulty of its transmission through fluids, would be incomparably greater than that which any conducting fluid can discharge.

(112.) Such is the general outline of the electric theory, which it is scarcely necessary to pursue in its various applications, because there are several facts which appear so totally at variance with the immediate consequences of its fundamental hypothesis, as to warrant us in rejecting it. Chemical action between some of the elements of a galvanic combination is so invariably connected with the production of electrical effects, that it would be a violation of all just rules of philosophy not to consider these two classes of phenomena as standing to each other in the relation of cause and effect. The quantity of galvanic effect is always in proportion to the energy of the chemical action. The extent of contact between the two metals, on the other hand, appears to have no relation to the quantity of electricity which is developed. Combinations producing galvanic effects may be formed, as we have seen, with a single metal only, when two fluids are present; and indeed, on other occasions, without the presence of any metallic substance whatever. We have also seen that the same metals do not in all cases stand in the same invariable electrical relation to each other; but that this relation is determined by the chemical properties of the fluid with which they are placed in contact. (§ 75.) All these facts are irreconcilable with the electric theory.

(113.) Were any further reasoning necessary to overthrow it, a forcible argument might be drawn from the following consideration. If there could exist a power having the property ascribed to it by the hypothesis, namely, that of giving continual impulse to a fluid in one constant direction, without being exhausted by its own action, it would differ essentially from all the other known powers in nature. All the powers and sources of motion, with the operation of which we are acquainted, when producing their peculiar effects, are expended in the same proportion as those effects are produced; and hence arises the impossibility of obtaining by their agency a perpetual effect; or, in other words, a perpetual motion. But the electro-motive force ascribed by Volta to the metals when in contact, is a force which as long as a free course is allowed to the electricity it sets in motion, is never expended, and continues to be exerted with undiminished power, in the production of a never-ceasing effect. Against the truth of such a supposition, the probabilities are all but infinite.

ROGET, Peter Mark (6)

ELECTRICITY.

Chapter I.

General Facts and Principles.

(1.) The science of Electricity, which now ranks as one of the most important branches of Natural Philosophy, and which embraces so many subjects of inquiry, exceedingly curious in themselves, and highly interesting from their relations with every department of nature, is wholly of modern creation. The ancients were, indeed, acquainted with a few detached facts, depending on the agency of electricity; such as the attractive power which amber acquires by being rubbed, the benumbing shocks which are experienced on touching the *torpedo* (or electrical eel), and the appearance of those sparks or streams of light which, on some occasions, are seen to issue from the human body. But no suspicion was entertained that these phenomena had any connexion with one another; and far less was it imagined that they were the effects of a power pervading all material bodies, and extensively concerned in all the operations of nature.

(2.) It was only by slow degrees that this knowledge was acquired. The first step towards a generalization of the phenomena was made by Dr. Gilbert, an English physician, who, in the year 1600, published a very original and valuable treatise on the magnet. He remarked that several other bodies besides amber can, by friction, be made to attract light bodies; and he was thus led to the discovery of a property common to all of them. The Greek name for amber being ηλεϰτϱον (*Electron*), the bodies possessed of this property were denominated *Electrics;* and the power they manifested was termed Electricity. The observations of Boyle, Otto Guericke, Newton, and a few other philosophers of the same period, contributed somewhat to the extension of our knowledge on this curious subject; but even the information collected during the whole of that century amounted to nothing that could be entitled to the name of science. The real science of Electricity can, properly speaking, be considered as taking its rise only in a later age; and it was the first fruit of that active spirit of investigation, which at the commencement of the eighteenth century was rapidly diffusing itself over Europe. The establishment of the Royal Society of London appears to have had considerable influence in promoting the cultivation of electricity: for we find that almost every discovery of importance in this science was made by the members, and is recorded in the Transactions of that Society. But it was not until the present century that the extensive relations which connect electricity with so many other branches of physical science, were discovered, and their importance appreciated. Already have we seen, in this short era, the rise of a new science, founded on that peculiar modification of Electricity, which is known by the name of Galvanism. Hence, have we derived new instruments of analysis, new paths of research, and new powers of extending the dominions of science; hence, have we been able to trace alliances between several of the great agents concerned in the phenomena of the material universe. Electro-Chemistry has thus arisen as one of the connecting branches between remote divisions of the Philosophy of Nature. Still more recently there has been opened to us, in the subject of Electro-Magnetism, another new province of science, which establishes a natural connexion between two powers hitherto regarded as distinct.

So rapid has been the march of scientific improvement, that it is difficult for those whose attention has not been steadily and exclusively devoted to these particular objects, to keep pace with the progress of discovery. The materials collected by the numerous labourers in these wide fields of inquiry have poured in upon us so fast, that there has scarcely yet been time for marshalling them in their proper places, and for disposing them in the order best fitted for instruction. It is to be lamented that there exists as yet no general and comprehensive treatise embracing the whole of these extensive and complicated subjects of modern research; and that the student has still to gather the information he seeks from a multitude of journals and other miscellaneous sources, where they lie irregularly scattered, and are not to be arranged, or even found, without a great expenditure of time and labour. It is the aim of these treatises to supply, in some degree, this deficiency, in as far, at least, as relates to the instruction of those who have no previous acquaintance with the subject, and are desirous of being initiated in the principles of the science.

(3.) In order to convey the clearest and most philosophical views of the subject we are about to treat, we shall begin by stating, independently of all theory, the most general facts relating to Electricity; presenting them at first in their simplest form. We shall, in the second place, review the theories which have been framed for the purpose of connecting these facts in the mind. We shall thus be enabled, lastly, to study their combinations, to unravel their complicated results, and to follow them in their practical applications.

(4.) The general facts relating to Electricity may be reduced to the six following heads:—

1. Excitation.
2. Attraction.
3. Repulsion.
4. Distribution.
5. Induction.
6. Transference.

§ 1. *Of Excitation, Attraction, and Repulsion.*

(5.) If a piece of amber, or sealing-wax, or a smooth surface of glass, perfectly clean and dry, be briskly rubbed with a dry woollen cloth, and immediately afterwards held over small and light bodies, such as pieces of paper, thread, cork, straw, feathers, or fragments of gold leaf, strewed upon a table, these bodies will be seen to fly towards the surface that has been rubbed, and adhere to it for a certain time. The surfaces which have acquired by friction this attractive power are said to be *excited;* and the substances thus susceptible of excitation are termed *electrics*, in contradistinction to such as are not excitable by a similar process and which are, therefore, termed *non-electrics*.

(6.) The principal electric substances in nature are the following: viz. amber, gum-lac, resin, sulphur, glass, talc, the precious stones, silk, the fur of most quadrupeds, and almost all vegetable substances (excepting charcoal), which have been thoroughly deprived of moisture, as, for example, baked wood, and very dry paper.

(7.) After the bodies which had been attracted by the excited electric have remained in contact with it a certain time, the force which held them together ceases to operate: the bodies then recede from the electric, and if the latter be again presented to them, they will, provided they have touched no other body, be repelled, or driven off, instead of attracted. This change from attraction to repulsion takes place more slowly with some substances than with others: some bodies will adhere to the electric a considerable time before they recede; while others, and especially metallic bodies, are repelled the instant after contact:—the reason of this will afterwards be seen.

(8.) It is also to be noticed that two bodies which have both of them been in contact with the same electric, mutually repel each other.

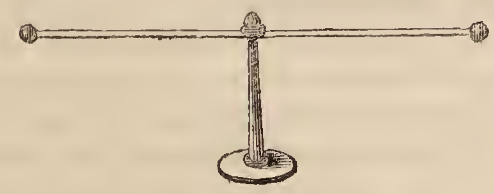
(9.) The phenomena of electrical attraction and repulsion are best observed when electrics of considerable size are employed. For the experiments we are about to describe, it is convenient to have them of a cylindrical shape, which admits of their being more easily carried in the hand, and more readily transferred to wherever we may wish to place them. We may employ as our electric a thick cylinder of sealing wax, or one of sulphur. If glass be chosen, it should be in the form of a tube of considerable diameter, and should, previously to the experiment, be gently warmed before the fire, in order to expel all moisture from its surface. As a rubber we may use a silk handkerchief, a piece of clean flannel, or the fur of a quadruped; but the material which produces the greatest effect when rubbed with glass is an amalgam (or mixture) of mercury with tin or zinc. Whatever be the substance employed, it should be perfectly dry; to ensure which condition it should, previously to being used, be held for some time before the fire.

(10.) When, by attending to these precautions, a sufficiently powerful excitement of the cylinder has been obtained, we may observe several other remarkable phenomena, besides those of attraction. If the experiment be performed in a dark room, flashes of light, of a bluish colour, will be perceived during the friction, extending over every part of the surface rubbed; and sparks, attended with a sharp snapping sound, will be seen to dart around it in various directions. If a round body, as a metallic ball, be presented to it, and moved from one end to the other, a succession of sparks will be obtained as the ball passes along the surface; and if the knuckle be presented instead of the metallic ball, each spark will be accompanied by a pricking sensation. When the excited cylinder is brought near to the face, an unpleasant sensation of tickling is felt in the skin, as if it had been covered with a cobweb.

(11.) If a globe of metal be suspended in the air by silk threads, and if, while in this situation, it be rubbed by an electric, such as silk, fur, or the outside of the skin of a cat, it will also become electrical, and exhibit the same properties of attraction and of repulsion as if it had been itself an electric. The circumstance of its being thus insulated or cut off from the contact of any substance, except the air and the electric which sustains it, is essential to the success of this experiment.

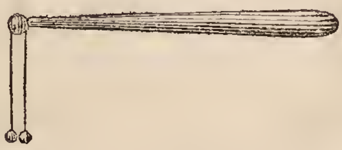
(12.) Various modes have been devised for exhibiting distinctly the attractive and repulsive agencies of electricity; and for obtaining indications of its presence, when it exists only in a feeble degree. Instruments for this purpose are termed *Electroscopes*. One of the simplest of these is the Electroscope of Haüy, which is very similar to that formerly proposed by Dr. Gilbert. It consists of a light metallic needle, terminated at each end by a light pith ball, which is covered with gold leaf, and supported horizontally by a cap at its centre, on a fine point. The attractive or repulsive power of any electrified body presented to one of the balls, will be indicated by the movements of the needle.

Fig. 1.



(13.) In some cases it is more convenient to employ a pair of similar balls, suspended from a brass ball fixed to the end of a glass handle, by very fine silver wires, or by hempen threads, previously steeped in a solution of salt, and afterwards dried. See *fig.* 2.

Fig 2.



(14.) Cavallo has contrived an electroscope of the same kind, which has the advantage of being more portable than that of Haüy, while it is, perhaps, equally sensible. It is formed by two fine silver wires, each carrying at one of their ends a little ball made of cork, or of the pith of the elder tree; the other ends of the wires being suspended from a cork, which is rather long, and tapering at both ends, so as to fit either way into the mouth of a varnished glass tube, serving both as a handle to the instrument when in use, and as a case for it when carried in the pocket. When it is to be employed as an electroscope, the wires with pith balls are placed so as to hang out from the end of the tube, and will indicate by their divergence any electricity which may be communicated to them. (*Fig.* 4.) When the instrument is not in use, the wires are put into the tube, by inverting it, and closing it with the other end of the cork. (*Fig.* 3.)

Fig. 3.

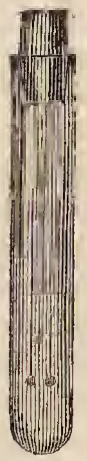
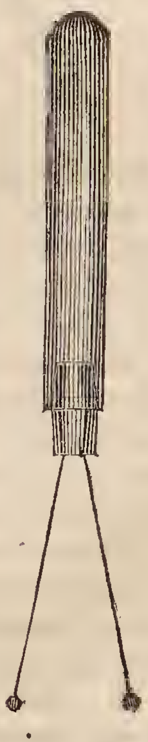


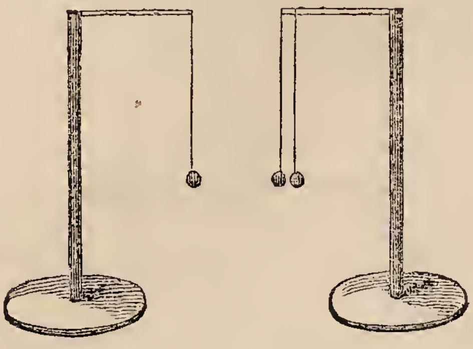
Fig. 4.



(15.) For studying the circumstances

attending electrical attraction, we should be provided with stands, from the ends of which are suspended by their respective threads one or two pith balls, about the size of a small pea, as shown in *fig.* 5.

Fig 5.



§ 2. *Distribution and Transference.*

(16.) If an excited electric be brought near a pith ball suspended by silk, the ball will, in the first place, approach the electric (*fig.* 6.), indicating an attraction towards it, and if the position of the electric will allow, the ball will come into contact with it and adhere to it for a short time; but it will presently afterwards recede from the electric, showing that it is now repelled. (*fig.* 7.) If we now remove the electric, and present to the ball which has thus touched it, a second ball which has had no previous communication with any electric, we find that these two balls attract one another, and come into contact. The same actions are repeated between this second ball and a third, which may be presented to it; and so on in succession, but with a continued diminution of intensity. This diminution plainly indicates a diminished power, in consequence, as it would seem, of its being distributed among a number of bodies.

Fig. 6.

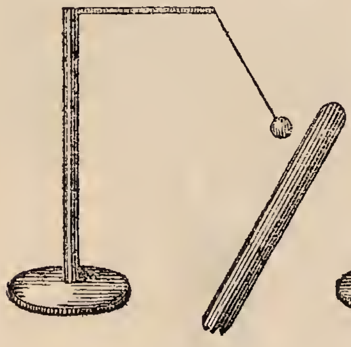
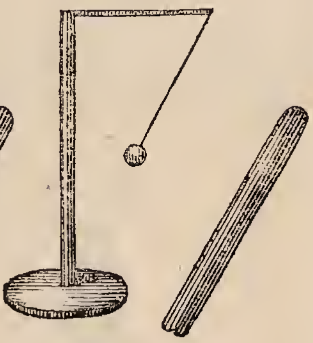


Fig. 7.



(17.) In the prosecution of these experiments, therefore, the effects will be more distinct, if, instead of small pith balls, we employ a globe of metal of larger size, which will allow of the reception of a considerable quantity of this electric influence by contact with the excited electric. A globe, suspended by silk threads, as the pith balls are, and which has extensively touched the electric, will act upon these balls precisely in the same way as the original electric would have acted upon them, and may accordingly be substituted for it in all these experiments. It is, indeed, exactly in the same condition as the globe rubbed by an electric, already mentioned. (§ 11.)

(18.) From the whole of these facts we necessarily infer that the electric has imparted to the ball or globe which came in contact with it, properties exactly similar to those which had been excited in itself by friction. By repeated contact with a number of bodies, an excited electric is found to lose its electrical powers in the same degree as these powers have been acquired by the bodies themselves; and fresh excitation alone can renew them. It is evident, then, that the unknown agent, which we have termed *Electricity*, is capable of transference, in the same sense in which we speak of heat being communicated or transferred from one body to another, and that, like heat, it is weakened by diffusion among a number of bodies.

(19.) If the electrified ball be touched with the finger, it will be deprived of the whole of its electricity, which will pass into the body of the person who touches it. It is now reduced to its original or natural state, and is again susceptible of being attracted, either by an excited electric, or by another body to which electricity has previously been communicated.

If the electrified body, instead of being touched with the finger, had been touched by a rod of metal held in the hand, the effect would have been the same in both cases: hence we may infer that the metallic rod is capable of conveying away from the body the whole of its electricity. But if a glass rod be substituted, the result is very different; the body touched is found to retain the whole of its electricity, notwithstanding the contact of the glass rod. We are thus led to the conclusion that some substances, such as glass, are incapable of conducting electricity; while others, such as the metals and the human body, readily convey that influence.

(20.) It is invariably found that all *electrics* are, at the same time, *non-con-*

ductors. Conductors, on the other hand, are *non-electrics.* The two qualities of a capability of excitation, and a power of conducting electricity, appear to be incompatible with each other—for the one is always found to diminish in proportion as the other increases. The permanence of electricity in metallic bodies, which are suspended in the air by silk threads, shows that the air as well as the silk is a non-conductor. Bodies which are in this way surrounded on all sides by non-conductors are said to be *insulated.* When this condition is not observed, that is, when the body is in contact with conducting bodies which communicate with the earth, its electricity will escape by the channel which is thus opened for it, and will be lost by diffusion in the mass of the earth, which is formed of conducting materials, and which may be regarded as the great reservoir both for the absorption and supply of electricity. Hence we see why it is not possible to accumulate electricity in a conducting body while it is held in the hand, and why electrics alone are capable of permanent excitation.

(21.) The insulating power of atmospheric air depends principally upon two circumstances, its density and its dryness. Air with the density which it has under the ordinary pressure of the atmosphere, if perfectly dry, is a remarkably good insulator, even although it be rapidly renewed on the surface of the electrified body. This is shown by an experiment of Franklin's, in which he whirled an electrified ball round his head, by means of a silk line, with great rapidity, so as to make it perform many hundred revolutions, without being able to perceive that it had thereby lost any sensible portion of its electricity. Neither an increase, nor a diminution of temperature, appears to lessen its insulating power. But in proportion as the air is rarefied by the removal of the superincumbent pressure, its power of confining electricity diminishes, till, at last, when the rarefaction is very great, it opposes scarcely any resistance to the passage even of very feeble electricity; and it may be then classed among conductors. This is the case with the imperfect vacuum produced by the air-pump, from which it is almost impossible to exclude minute quantities of air. Even in the space left in the upper part of the tube of a barometer by the descent of the mercury, or the Torricellian vacuum, as it is called, there is in general present a minute portion of air, as well as of mercurial vapour, which are sufficient to conduct electricity. Small globules of air usually adhere to the mercury, and to the sides of the tube; and these, upon the removal of pressure, expand into an atmosphere. It has been asserted by Mr. Morgan that if great care be taken to remove every source of error, by the employment of very pure mercury, and by boiling it for a long time in the tube, a perfect vacuum may be obtained, which does not conduct electricity. His experiment, however, was made long ago, and requires careful repetition, before the result can be confided in.

(22.) The circumstance which chiefly determines the conducting power of common air is that of its containing a greater or less quantity of moisture. Water is a very good conductor of electricity; and that portion which is suspended in the air tends powerfully to carry off electricity from the bodies which are charged with it, and which are surrounded by air. Moisture also easily attaches itself to glass and other electrics, and deprives them of the power of insulation. Hence, the same experiments which succeed in a clear dry day, will often fail when tried in damp weather: and hence we see the utility of previously drying every part of the apparatus, in order to exclude as much as possible the interference of moisture.

(23.) The conducting powers of most bodies are influenced by changes of temperature, and also of form. Thus, although water in its liquid state is a good conductor, yet, when congealed, in the form of ice, its conducting powers are much impaired; and at a very low temperature, namely, at —13° of Fahrenheit's scale, ceases altogether. Mr. Achard, who observed this fact, formed ice of this temperature into a spheroid, and mounting it upon an axis, was able to excite it by friction as any other electric. On the other hand, by raising the temperature of water, its conducting powers are increased. Charcoal is also found to transmit electricity with more facility when hot, than when cold. Glass, which is a non-conductor when cold, becomes a tolerably good conductor when heated to redness; and a similar change takes place in sulphur and in resinous bodies when melted; and also in baked wood when heated. Reducing substances to powder has often an effect upon their powers of conducting electricity. Snow conducts less readily than ice of the same temperature. The same is the case with powdered charcoal, when compared with the same substance in its entire state. But glass, on the contrary, acquires some conducting power by being pulverized, as was ascertained by Van Swinden, who extended the same observation to sulphur.

Many bodies, which, in their usual state, are good conductors of electricity, lose this power when they are made very dry. This is the case with recent vegetable and animal substances, their conducting power appearing to be derived solely from the fluids they contain.

(24.) Strictly speaking, there is no substance hitherto known that is perfectly impervious to electricity; for the intensity of that agent may be so increased as to force it, for a certain small distance, through all bodies: neither is there any body in which the conducting power is infinitely great; that is, which opposes no resistance to the transmission of electricity. If the degree of conducting power which bodies possess could be ascertained with sufficient precision, they might be arranged in progressive order; but the present state of our knowledge affords only an approximation to such a series. As a table of this kind, however, with all its imperfections, may be of great use, we subjoin the following, in which the different bodies are arranged in one series, beginning with those which have the greatest conducting power, and terminating with those that have the least. The order in which they possess the power of insulating is, of course, the reverse of this.

Catalogue of Bodies in the Order of their conducting Power.

The perfect, or least oxidable metals.
The more oxidable metals.
Charcoal prepared from the harder woods, and well burned.
Plumbago.
The concentrated mineral acids.
Powdered charcoal.
Dilute acids.
Solutions of metallic and neutral salts.
Metallic ores.
Animal fluids.
Pure water.
Ice above —13° Fahrenheit.
Snow.
Living vegetables.
Living animals.
Flame.
Smoke.
Steam.
Metallic salts.
Salts with alkaline or earthy bases.
Rarefied air.
Vapour of alcohol.
Vapour of ether.
Earths and stones in their ordinary state.
Pulverized glass.
Flowers of sulphur.

Dry metallic oxides.
Oils.
Vegetable ashes.
Animal ashes.
Dry transparent crystals.
Ice below —13° Fahrenheit.
Phosphorus.
Lime.
Dry chalk.
Native carbonate of barytes.
Lycopodium.
Caoutchouc, or Indian rubber.
Camphor.
Siliceous and argillaceous stones in proportion to their hardness.
Dry marble.
Porcelain.
Baked wood.
Dry atmospheric air, and other gases.
White sugar, and sugar crystallized.
Leather.
Dry parchment.
Dry paper.
Cotton.
Feathers.
Hair, especially that of a living cat.
Wool.
Dyed silk.
Bleached silk.
Raw silk.
Transparent gems.
Diamond.
Talc.
Metallic vitrifications.
Glass, and other vitrifications.
Fat.
Wax.
Sulphur.
Resins, and bituminous substances.
Amber.
Gum-lac.

Although the precise point in the scale which forms the separation between conducting and insulating bodies must, of course, be somewhat indefinite, we have endeavoured to mark it by the division in the above table.

(25.) It appears, from the experiments

of Mr. Coulomb, that a thread of gum-lac is the most perfect of all insulators, and is ten times more effectual than a silk thread as dry as it can be made; for the former, when only one inch and a half in length, insulated as well as a fine silk thread of fifteen inches. When the thread of silk was dipped in fine sealing-wax, it was equal in power to a thread of pure lac of six inches, that is, of four times its length. Professor Robison found that the conducting power of silk thread depends greatly on its colour; or, in other words, on the nature of the drug with which it is dyed. When of a brilliant white, or a black, its conducting power is the greatest; and a high golden yellow, or a nut brown, renders it the best insulator. Glass, even in its dryest state, and in situations where it was impossible that moisture could have access to it, is stated by the same author to insulate considerably better than silk; and when drawn into a slender thread, and coated with gum-lac, it acted as well as a thread of lac of one-third of the length. It was found, however, at the same time, that extreme fineness was requisite; for it dissipated in proportion to the square of its diameter. The insulating power of glass is remarkably injured by having a bore, however fine, unless that bore admits of being also coated with lac. Human hair, when completely freed from every thing that water could wash out of it, and then dried by lime, and coated with lac, was equal to silk. Fir, cedar, larch, and the rose-tree, when split into filaments, and first dried by lime, and afterwards baked in an oven, which just made paper become faintly brown, seemed scarcely inferior to gum-lac. The white woods, as they are called, and mahogany, were much inferior. Fir, baked and coated with melted lac, seems, therefore, the best support when strength is required. The lac may be rendered less brittle by a minute portion of pure turpentine, which has been cleared of water by a little boiling, without sensibly increasing its conducting power. Lac, or sealing-wax, dissolved in spirits, is far inferior, for these purposes, to what it is when melted by heat.

(26.) The laws which regulate the gradual dissipation of electricity from bodies in a state of imperfect insulation have been investigated with great ability by Coulomb. Three causes chiefly operate in depriving a body under these circumstances of its electricity:—First, the imperfection of the insulating property in the solids by which it is supported. Secondly, the contact of successive portions of air, every particle of which carries off a certain quantity of electricity. Thirdly, the deposition of moisture upon the surface of the insulating body, which establishes communications with its remote ends, and may be considered as virtually increasing its conducting power.

(27.) With regard to the first cause, Mr. Coulomb has completely ascertained that for all fine cylindrical fibres, such as hair, silk, filaments of gum-lac, &c. if the nature of the substance, the diameter of the fibre, and the dispersive state of the air are supposed constant, the length of the fibre requisite for the complete insulation of a given intensity of electricity, varies as the square of that intensity. Theory, therefore, leads to the conclusion that, however great may be the intensity, there is always a certain length beyond which a filament of any of these bodies becomes a perfect insulator; and we find, in practice, that by diminishing the intensity of the electricity, or increasing the length of the substance it has to traverse, a sufficiently accurate degree of insulation may be obtained. With respect to the second source of dissipation, it was found that in a given state of the atmosphere, as far as it could be determined by the indications of the barometer, thermometer, and hygrometer, the dissipation at each instant of time, varied directly as the intensity of the electricity.

(28.) There is one very material circumstance relating to the dissipation of electricity that should here be mentioned, although its explanation must be deferred till the principles on which it depends have been developed; and it is, that the power of retaining electricity in any body is much influenced by its shape. The form most favourable to its retention is that of a sphere; next to which is a spheroid, and a cylinder terminated at both ends by a hemisphere. On the other hand, electricity escapes most readily from bodies of a pointed figure, especially if the point projects to a distance from the surface. In such bodies it is scarcely possible, indeed, to accumulate any sensible degree of electricity, on account of its rapid dissipation from the point. In like manner pointed bodies receive electricity more readily than those of any other form.

§ 3. *Of the two species of Electricity.*

(29.) We have hitherto viewed electrical phenomena as arising from the operation of a single agent, which could be called into action, and transferred from one body to another. We have seen that bodies, which have received their electricity from excited glass, repel one another, and are likewise repelled by the excited glass. The same thing happens with respect to those bodies which have received their electricity from excited sealing-wax. But upon examining the action of any of the bodies belonging to the one set, upon any of those belonging to the other, we find, that instead of repelling, they attract each other. Thus, the ball which has received its electricity from the glass, attracts that which has been electrified by the sealing-wax, and is attracted by it; but, what is still more remarkable, the moment these balls have come into contact, provided they have both been electrified in the same degree, they cease at once to exhibit any signs of electricity, as if the electricities of both were suddenly annihilated by their mutual communication. Thus there appears to be two different, and, in some respects, opposite kinds of electricities; the one obtained from glass, the other from sealing-wax. Du Fay, by whom this distinction was first noticed, denominated the former the *vitreous*, and the latter the *resinous* electricity.

(30.) The mode of action which these two electricities exert on matter, may be expressed by the following law: namely, that *bodies charged with either species of electricity, repel bodies charged with the same species, but attract bodies charged with the other species; and that, at equal distances, the attractive power in the one case is exactly equal to the repulsive power in the other.*

Accordingly, if we wish to ascertain what is the species of electricity with which a given body is charged, we have only to approach it to a small insulated pith ball, which has previously been touched either with excited glass or with excited sealing-wax. If the body in question repel it in the former case, or attract it in the latter, its electricity is vitreous; if the contrary happens, it is resinous.

(31.) Although each of these two electricities, when taken separately, acts in a manner precisely similar to the other, they nevertheless exhibit in all their relations to each other a marked contrariety of nature. Hence they are naturally viewed as agents having opposite qualities, which completely neutralize one another by combination.

(32.) Another remarkable circumstance which characterizes these agents, is, that the excitation of one species of electricity is always accompanied by the excitation of the other; and both are produced in equal degrees. Thus, when glass is rubbed by silk, or flannel, just as much resinous electricity is produced in the silk, or flannel, as there is vitreous electricity produced in the glass; and whatever electrified bodies are repelled by the one are attracted in the same degree by the other. If one of the substances happen to be a conductor, and be held in the hand, the whole of the electricity which the friction excites in it will disappear as soon as it is produced, from its escaping through the body of the person holding it, and being lost in the earth. But if the precaution be taken of insulating the rubber, its electricity will become manifest, and is always found to be of the opposite species to that which is excited in the body which is rubbed.

(33.) Since the two surfaces rubbed acquire opposite electricities, it follows as a consequence of the law above stated, that they must attract one another; and this is found invariably to be the case. If a white and a black ribbon of two or three feet long, and perfectly dry, be applied to each other by their flat surfaces, and are then drawn repeatedly between the finger and thumb, so as to rub against each other, they will be found to adhere together, and if pulled asunder at one end, will rush together with great quickness. While united they exhibit no sign of electricity, because the operation of the one is just the reverse of that of the other, and their power is neutralized and inoperative. If completely separated, however, each will manifest a strong electrical power, the one attracting those bodies which the other repels.

(34.) The very act of separation is accompanied by appearances which indicate that considerable portions of the electricities excited on each of the surfaces fly back to the opposite surface, and by their union become as it were extinguished or inoperative; and it is only the remaining quantities which have adhered more tenaciously to the surfaces, that retain their activity. When the experiment is made in the dark, flashes of light attend these sudden exchanges of electricity, passing between the two surfaces, and accompanied with a rustling noise.

(35.) Numberless experiments have been made with a view of ascertaining the conditions that determine the species of electricity excited in the respective bodies of which the surfaces are made to rub against each other, but they have led to no satisfactory conclusion. The mechanical configuration of the surface appears to have a greater influence in the result than the peculiar nature of the substance itself. If a plate of glass with a polished surface be rubbed against one which is roughened, the former always acquires the vitreous, and the latter the resinous electricity. No approach to an explanation of this peculiarity has ever been made. Smooth glass acquires vitreous electricity by friction with almost every substance, except the back of a cat, which gives it the resinous electricity; but roughened glass, if rubbed with the same substances, becomes charged with resinous electricity, while the rubbing bodies acquire the vitreous. Sealing-wax, rubbed with an iron chain, acquires, if polished, the resinous electricity; but if its surface is previously rough with scratches, the vitreous. Silk, rubbed by resin, takes the resinous, but with polished glass, the vitreous electricity. The following is a list of several substances which acquire vitreous electricity when rubbed with any of those which follow it in the order in which they are set down; and resinous electricity if rubbed with any of those which precede:—

The back of a cat.
Polished glass.
Woollen cloth.
Feathers.
Wood.
Paper.
Silk.
Gum-lac.
Roughened glass.

In the experiment just mentioned, in which a black and a white ribbon are rubbed together, the former is found to be resinously and the latter vitreously electrified. But if two pieces of the same ribbon of the same length be rubbed, the one being drawn lengthwise and at right angles over a part of the other, the one which has suffered friction in its whole length acquires vitreous, and the other resinous electricity. In like manner, when the whole length of the bow of a violin is drawn over a limited part of the string, the hairs of the bow exhibit a vitreous, and the string a resinous electricity, the body whose excited portion is of the least extent being generally found to be resinously electrified. But in truth, the slightest difference in the conditions of these and similar experiments on the species of electricity arising from friction, will be often sufficient to produce opposite results.

(36.) Electrical excitation may also be produced by the friction of liquids or of gases against solid bodies. This is the case when mercury is made to fall, in a fine shower, under the exhausted receiver of an air-pump, against the glass. If a current of atmospheric air be directed against a pane of glass, by means of a pair of bellows, the glass becomes vitreously electrified.

§ 4. *Induction.*

(37.) Another class of electrical phenomena must here be noticed. Whenever a body is charged with electricity, although it be perfectly insulated, and of course all escape of that electricity prevented, it tends to produce an electrical state of the opposite kind in all the bodies in its vicinity. Thus the vitreous electricity tends to induce the resinous electricity in a body that is situated near it; and this with greater energy, as the distance is smaller. This effect is termed the *induction* of electricity, and may be ranked among the general facts, or laws of the science. The further development of the consequences it leads to, must, for the present, be postponed, as we shall hereafter be better prepared to understand them. But there is one of its results which we shall now point out, as it refers immediately to the phenomena that have already occupied our attention.

(38.) If an electrified body, charged with either species of electricity, be presented to an unelectrified or neutral body, its tendency, in consequence of the law of induction, is to disturb the electrical condition of the different parts of the neutral body. The electrified body induces a state of electricity contrary to its own in that part of the neutral body which is nearest to it; and consequently a state of electricity similar to its own in the remote part. Hence the neutrality of the second body is destroyed by the action of the first; and the adjacent parts of the two bodies, having now opposite electricities, will attract each other. It thus appears, that the attraction which is observed to take place between electrified bodies and those that are unelectrified, is merely a consequence of the altered state of those bodies, resulting directly from the law of induction; and that it is by no means itself an original law, or primary fact in the science.

(39.) The effects of induction will be in proportion to the facility with which changes in the distribution of electricity among the different parts of a body can be effected, a facility which corresponds with the conducting power of the body. Hence the attraction exerted by an electrified body upon another body previously neutral, will be much more energetic if the latter be a conductor, than if it be an electric, in which these changes can take place only to a very small extent. This is confirmed by the following experiment: suspend by fine silk threads of equal length, two small balls of equal dimensions, both made of gum-lac, but one having its surface covered with gold leaf. Place these two pendulums, as they may be called, at a little distance from one another, so as to admit of a comparison of their motions; and then present to them an excited electric, which may be either a tube of glass, or a cylinder of sealing-wax. It will at once be seen that the ball, with a metallic covering, which readily admits of the transfer of electricity from one side to the other, will be much more readily and powerfully attracted, than the other ball which allows of no motion in its electricity. The latter ball will, by slow degrees, however, assume electrical states of the same kind as the gilt ball, and will be feebly attracted. As this change is very slowly effected, so it is more permanent when once produced; and the plain ball adheres for a considerable time to the electric which has attracted it. The gilt ball, on the contrary, is sooner repelled, by its readily receiving the charge of electricity imparted to it by the electric. A degree of permanent electricity, however, is also induced on this ball, in consequence of its gradual penetration into the substance of the gum-lac.

Chapter II.

Theories of Electricity.

(40.) It is impossible to arrive at the full comprehension of the multifarious facts relating to any of the physical sciences without the aid of some leading principles, or modes of viewing them, by which their connexions can be represented to the mind, so as to combine them into an intelligible system. We begin by classing the different agents in nature, designating them by specific names; we next endeavour to conceive these agents as possessed of certain powers or qualities adapted to the production of the observed effects. In the case of light, for example, we may conceive the phenomena to result from the action of material particles, emanating in all directions from the luminous body, and obeying certain laws in their course; or we may adopt another hypothesis, namely, that they proceed from the undulations of an elastic medium pervading space. By employing either the one or the other of these hypotheses, we acquire great facility in tracing the connexions of the phenomena of optics, and retaining them in our minds. This advantage is not immediately dependent on the truth of the particular hypothesis we employ for that purpose: for, in the example before us, it is evident they cannot both be true, and yet they both answer this end. But, of course, the utility of an hypothesis will be proportionate to the degree of exactness with which it accords with the phenomena. No inconvenience can arise from its adoption, as long as we bear in mind that our reasonings are founded on a mere hypothesis, and as long as we hold ourselves in readiness to abandon it, the moment we meet with facts with which it is decidedly inconsistent.

(41.) The hypothesis which naturally suggests itself for the explanation of electrical phenomena is that of a very subtile and highly elastic fluid, pervading the earth and all other material bodies, but itself devoid of any sensible gravity. We must suppose this fluid to be capable of moving, with various degrees of facility, through the pores or actual substance of different kinds of matter. In some, as in those we call *conductors*, or *non-electrics*, such as the metals, it moves without any perceivable obstruction: but, in glass, resin, and, in general, in all bodies called *electrics*, or *non-conductors*, it moves with great difficulty. Moreover, as the phenomena appear to point out the existence of two distinct kinds of agencies, we may further assume that there are two distinct species of electric fluid, which we shall, for the present, name the *vitreous* and the *resinous electricities*. They must each have, when separate, the same general properties as have already been enumerated; but, in relation to each other, there must be a complete contrariety in their natures, so that when combined together, their actions on the bodies in their vicinity, or on the particles of electric fluid contained in those bodies, are exactly balanced; and all visible action ceases. It is in this state of union, in which they perfectly neutralize one another, that they exist in bodies which may be said to be in their *natural state* with regard to electricity.

(42). Thus, then, may the problem be solved, in which it is required to conceive an agent, analogous, in many respects, to other known agents, and to assign to it such properties as will, in their results, correspond to all the observed phenomena. In order to apply to it this latter test, we must trace all the consequences which flow from the suppositions we have made, and strictly compare them with the facts both as presented to us by nature, and as resulting from experiment. These facts, it will be recollected, are reducible to those of excitation, attraction, and repulsion, distribution, induction, and transference.

(43.) *Excitation.* From various causes, of which the friction of surfaces is one, the state of union in which the two electricities naturally exist in bodies, is disturbed; their latent powers are called forth by their separation; the vitreous electricity is impelled in one direction, while the resinous is transferred to the opposite side; and each can now manifest its peculiar energies. When accumulated in any body, or part of a body, each fluid acts in proportion to its relative quantity, that is, to the quantity which is in excess above that which is still retained in a state of inactivity by its union with electricity of the opposite kind. Thus when glass is rubbed with a metallic amalgam, a portion only of the electricities at the two surfaces is decomposed; the vitreous electricity resulting from this decomposition attaches itself to the glass; the resinous, to the amalgam. What remains in each surface undecomposed continues to be quite inert, and has no other influence on the phenomena, than being ready, on the continuance of the decomposing action, to furnish a fresh supply of both fluids to the bodies in the vicinity.

(44.) *Distribution.* Each of these fluids, being highly elastic, their particles repel one another with a force which increases in proportion as their distance is less: and this force acts at all distances, and is not impeded by the interposition of bodies of any kind, provided they are not themselves in an active electrical state. From the most careful analysis of the phenomena, it has been deduced that the exact law of this force is the same as that of gravitation, namely, that its intensity is inversely as the square of the distance.

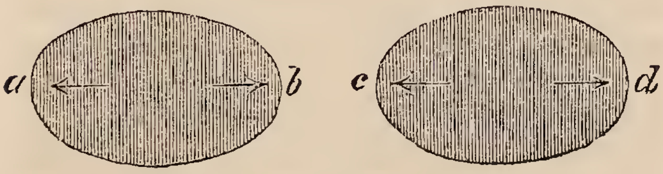
The mode in which the electricity imparted to a conducting body, or to a system of conductors, is distributed among its different parts, is in exact conformity to the results of this law, as deduced by mathematical investigation. But we reserve the examination of this subject for a future chapter.

While the particles of each fluid repel those of the same kind, they exert an equally strong attraction for the particles of the other species of electric fluid. This attraction, in like manner, increases with a diminution of distance, and follows the same law as to its intensity, namely, that of the inverse ratio of the square of the distance. This force, also, is not affected by the presence of any intervening body.

(45.) *Transference.* Since the two electricities have this powerful attraction for each other, they would always flow towards each other and coalesce, were it not for the obstacles that are opposed to their motion by the non-conducting properties of electrics. When these obstacles are overcome, and a free channel is open for the passage of the electricities, they rush into union with great force and velocity, producing, in their transit and confluence, several remarkable effects. After their coalescence, their power seems to be at once annihilated, or, more properly speaking, it remains dormant, until called into play by the renewed separation of the fluids.

(46.) *Attraction and Repulsion.* The repulsion which is observed to take place between bodies that are insulated and charged with any one species of electricity, for other bodies similarly charged, is derived from the repulsive power which the particles of this fluid exert towards those of their own species. Let us suppose a body charged with electricity to be suspended in the air, or otherwise surrounded by a non-conducting medium, which allows it to move freely. As long as this body remains alone, the outward pressure which the electric fluid exerts against the insulating medium that confines it, will, by the laws of hydrostatics, be equal on all sides; and the body, thus balanced by equal and opposite pressures, will have no tendency to move. But if another body, similarly circumstanced, be brought near it, the repulsive action between the similar electricities contained in these bodies, will diminish the outward pressures of each fluid against the sides of the bodies, (*b*, *c*, *fig.* 8.) which are adjacent to each other; and it will, at the same time, increase the outward pressure on the opposite or remoter sides (*a*, *d*.) Both these causes conspire to destroy the equilibrium; each body is impelled in the direction of the preponderating force, that is, in a direction from the other body; and an effect, which may be called repulsion, takes place. The very same explanation, it is evident, applies to both kinds of electricity, their properties being in this respect exactly alike.

Fig. 8.



If, on the other hand, a body charged with vitreous electricity be presented to one that is charged with resinous electricity, the attraction of these two fluids will diminish the outward pressure on the remote sides of the bodies, and increase it on the adjacent sides; hence, the bodies will be urged towards each other, and motions indicative of attraction will result. Thus, in all cases, do the movements of the bodies represent the forces themselves which actuate the particles of the developed electricities they contain.

(47.) *Induction.* The law of induction is a direct consequence of the hypothesis we are considering. Wherever one of the electricities exists in an active state, it must repel the particles of the same electricity in all surrounding bodies, and attract those of the opposite species: or, in other words, it tends to decompose their united electricities, accumulating the electricity of the opposite species towards the nearest side, and impelling that of the same species towards the remote side. The body thus acted upon is no longer neutral, although it contains, on the whole, its natural quantities of both electricities; but, in consequence of their partial distribution, electrical appearances will be exhibited in its different parts. The further prosecution of this branch of the subject must also be postponed to a subsequent chapter, our present object being merely to point out, in a general way, the coincidence of the fundamental facts with the proposed theory.

(48.) Thus far we have proceeded upon the hypothesis of there being two distinct electric fluids, having certain properties in common, but each being characterized by a certain modification of these properties. It is, however, equally possible to account for all the phenomena with the same exactness, on the supposition of their resulting from the agency of a single electric fluid. This simplification of the theory may be considered as the discovery of the immortal Franklin, although it had occurred at the same period to Dr. Watson; for it was Franklin who first pointed out the mode in which it might be successfully applied to explain some of the most remarkable phenomena of the science. Several particular points in his theory, as he originally proposed it, were defective, and were found on strict examination to be at variance with ascertained facts. It is to Æpinus and to Cavendish that we owe the rectification of these errors: and the theory of Franklin, as thus amended, whatever alterations the future progress of discovery may oblige us to make in it, will ever remain one of the most beautiful specimens of this kind of reasoning which philosophy has produced. Of this hypothesis we shall now present a brief outline; and point out the mode in which it explains the phenomena.

(49.) We set out, then, with supposing that there exists in all bodies a subtile fluid, which we shall call the *electric fluid*;—that its particles repel one another with a force varying inversely as the square of the distance;—that they attract the particles of all other matter, or some specific ingredient in that matter, with a force following the same law of the inverse square of the distance;—that this fluid is dispersed through the pores of bodies, and from some unknown peculiarity, can move through them with various degrees of facility, according as they are conductors or non-conductors. Bodies are said to be in their natural state with regard to electricity, when the repulsion of the fluid they contain for a particle of fluid at a distance is exactly balanced by the attraction of the matter in the body for the same particle. In this state they may be considered as *saturated* with the electric fluid. Whenever they contain a quantity of fluid greater than this, they are said to be *positively electrified*, or to have *positive electricity*. When, on the other hand, there is a quantity less than that required for saturation, the body is said to be *negatively electrified*, or to have *negative electricity*. In the former case, it is the fluid that is redundant, or in excess; in the latter, it is the matter which is left unsaturated that should be considered as the redundant principle. The state of positive electricity, then, consists in a redundance of fluid, or in matter that is *over-saturated*, as it has been termed; that of negative electricity, in a deficiency of fluid, or in matter *under-saturated*, or, what is an equivalent expression, in *redundant matter*. In mathematical language, the former condition may be expressed by the sign *plus;* the latter by that of *minus*. In considering the mutual electrical actions of bodies, the portions in which the matter and the fluid mutually saturate each other, need not be taken into account, since their actions, as we have seen, are perfectly neutralized: and we need only attend to those of the redundant fluid and the redundant matter.

(50.) When a body contains more than its natural proportion of electric fluid, the surplus will, by the repulsive tendency of its particles, overflow and escape, if such escape be allowed, until the body is reduced to its neutral state. When under-saturated, the redundant matter will attract fluid from all quarters from which it can receive it, until it is again brought to its neutral state. This efflux, or influx, is prevented either when the body is surrounded on all sides by substances, through the pores of which the fluid cannot pass, or when the body itself is of that nature.

(51.) The mutual recession of two positively electrified bodies is a direct consequence of the repulsion of the redundant fluids contained in each, which, being attached to the matter by their attraction for it, impel it in the direction of their own repulsion. In the same way the mutual approximation of two bodies in opposite electrical states is the immediate effect of the attraction of the redundant fluid in the one, for the redundant matter in the other; and *vice versâ*, for this attraction is mutual.

(52.) A difficulty does, indeed, occur when we attempt to apply the theory to the case of two bodies which are both in a state of negative electricity, that is, in which there exists in both certain quantities of matter unsaturated with electric fluid. What action does the theory, as hitherto stated, point out as the result in this particular case? Plainly none. All those portions of the matter of each body which are still saturated, together with the fluid which saturates them, can have, as we have already seen, no effect either of attraction or repulsion. The only active element is the unsaturated matter; but the hypothesis does not assign any action of this matter upon other matter at a distance. Yet we learn from experience that the bodies, under these circumstances, actually repel one another. In order, therefore, to render the hypothesis conformable to fact, we are obliged to annex to it another condition; namely, that the particles of simple matter, that is, of matter uncombined with the electric fluid, exert a repulsive action on one another. It is singular that so acute a mind as that of Franklin should not have discerned this defect in his own theory, or perceived that this further condition was absolutely requisite for the explanation of the phenomena. Without it, indeed, we should be unable to explain the want of action between two neutral bodies; for the repulsion of the fluids in both bodies being balanced by the attraction of the fluid in the one for the matter in the other, the remaining attraction of the fluid in the second body for the matter in the first would be uncompensated by any repulsion, and the forces would not be held in equilibrium, as we find they really are.

(53.) The law of electrical induction is an immediate consequence of the Franklinian theory. When a body charged with electricity is presented to a neutral body, the redundant fluid of the former exerts a repulsive action on the fluid in the latter body; and if this happens to be a conductor, it impels a certain portion of that fluid to the remote end of this body, which becomes at that part positively electrified; while its nearer end, which the same fluid has quitted, is consequently in the state of negative electricity. If the first body had been negatively electrified, its unsaturated matter would have exerted an attractive force on the fluid in the second body, and would have drawn it nearer to itself, producing an accumulation or redundance of fluid at the adjacent end, and a corresponding deficiency at the remote end: that is, the former would have been rendered positive, and the latter negative. All this is exactly conformable to observation.

(54.) The phenomena of transference are easily explicable on this hypothesis; and they arise from the destruction of the equilibrium of forces, which confined the fluid to a particular situation or mode of distribution.

(55.) There is, indeed, no fact explicable by the hypothesis of a double fluid, which is not explained with equal facility by that of a single fluid, with the condition already stated. The explanation by the first is easily converted into an explanation by the second, by substituting the expressions of *positive* and *negative* for those of *vitreous* and *resinous electricities;* and considering the action of the latter as arising from the influence of redundant or unsaturated matter, to which is ascribed in the Franklinian hypothesis a similar operation to that of the resinous electricity in the hypothesis of Du Fay. The hypothesis of a single fluid has, it must be allowed, the advantage of greater simplicity: but, on the other hand, it lies open to the objection of its involving a condition which appears, at first view, to be at variance with our preconceived notions of the primary laws of matter, and more especially with that of gravitation; namely, that which implies the mutual repulsion of its particles when void of electricity.

When viewed as a mere hypothesis calculated to facilitate our comprehension of the phenomena and of their connexions, it is a matter of indifference which we employ, for they will either of them answer the purpose. In our future explanations we shall, in general, adhere to the language of the Franklinian theory, as being the simplest, and generally the most convenient; and because a conversion of terms the reverse of that just now pointed out will in all cases enable us to supply the explanation of the same phenomenon according to the theory of Du Fay. As to the question which of these two hypotheses approaches the nearest to the real state of things, we are not yet prepared to discuss the arguments that could enable us to decide it; and we must, therefore, wait till we can resume the subject in the sequel.

The further development of these theories, and of the law of induction, in particular, must, for the present, be postponed, since they require us to be acquainted with many practical details relating to the accumulation of electricity, and its management when applied to various objects of experimental research.

Chapter III.

Electrical Machines.

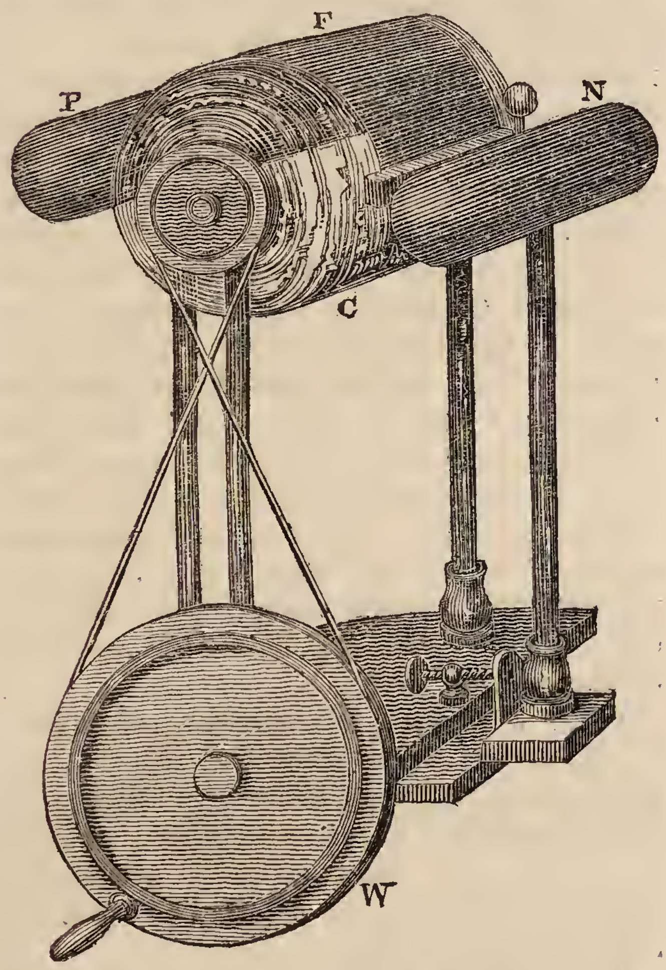
(56.) The essential parts of an instrument for procuring large supplies of electricity for the purposes of experiment, or an *electrical machine*, as it is called, are the electric, the rubber, the prime conductor, the insulator, and the machinery for setting the electric in motion.

(57.) The electric, by the excitation of which the electricity is to be developed, may be made of various materials. Globes of sulphur were employed by the earlier electricians for that purpose; but polished glass is found, on the whole, to be the most convenient substance. The original form given to it by Hauksbee, who was the inventor of the electrical machine, was that of a globe, which he caused to revolve upon a vertical axis. The most convenient forms, however, are those of a hollow cylinder, or of a flat circular plate, revolving upon a horizontal axis. When used in the form of a globe or cylinder, it has sometimes been found advantageous to line the inside of it with a thin layer of a resinous composition, consisting of four parts of Venice turpentine, one of resin, and one of bees' wax. This must be introduced in sufficient quantity into the inside of the globe or cylinder, and, when the glass is brought gradually to an equal degree of heat throughout the melted substances, is allowed to spread itself over the interior surface, by turning the globe or cylinder about its axis. The principal use of such a coating is to improve bad machines, for it is not required in good ones.

The earlier electricians contented themselves with using the hand as a rubber, till a cushion was introduced for that purpose by Professor Winkler, of Leipsic. The cushion is usually made of soft leather, generally basil skin, stuffed with hair or wool, so as to be as hard as the bottom of a chair, but yet sufficiently yielding to accommodate itself, without much pressure, to the surface of the glass to which it is applied.

(58.) Of cylindric machines, the simplest and most perfect construction is that invented by Nairne, and which is represented in *fig*. 9. The glass cylinder C is from 8 to 16 inches in diameter, and from one to two feet long, supported, for the purpose of insulation, on two upright pillars of glass, which are fixed to a firm wooden stand. Two hollow metallic conductors P, N, equal in length to the cylinders, are placed parallel to it, one on each side, upon two insulating pillars of glass, which are cemented into two separate pieces of wood that slide across the base so as to allow of their being brought within different distances from the cylinder. To one of these conductors N, the cushion is attached, being fastened to it by the intervention of a bent spring, the purpose of which is to keep it equally pressed against the cylinder in every part of its revolution. The pressure of the cushion is also further regulated by an adjusting screw adapted to the wooden base, on which the glass pillar that supports the conductor is fixed. From the upper edge of the cushion there proceeds a flap F of thin oiled silk, which is sewed on the face of the cushion about a quarter of an inch from its upper edge. It extends over the upper surface of the glass cylinder to within an inch of a row of metallic points, proceeding like the teeth of a rake from a horizontal rod, which is fixed to the adjacent side of the opposite conductor P. The motion of the cylinder must always be given in the direction of the silk flap; and it may be communicated either by a single handle, or by a multiplying wheel W, as in the figure: the latter produces more electricity in the same time, but the labour of turning is increased nearly in the same proportion. On some accounts it is more convenient to place the conductor to which the rubber is not attached, at right angles to the cylinder; and this is the plan adopted in the common electrical machines.

Fig. 9.

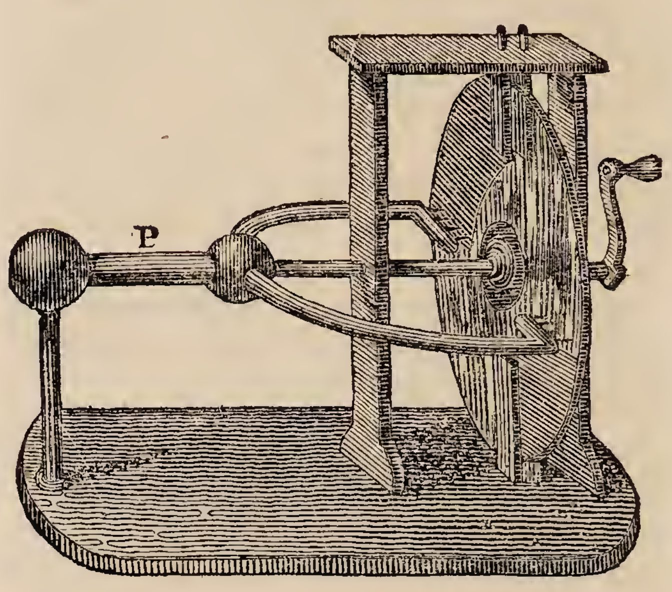


(59.) The conductor P, to which the rubber is not attached, is generally called the *prime conductor*, or the *positive conductor*, as the electricity with which it becomes charged is positive. It is a cylindrical tube, each end terminating in a hemisphere. There is no advantage in its being made of solid materials, for the electricity is contained only at the surfaces. It may be made of thin sheet brass, or copper, or tin, or of pasteboard, covered with gold leaf or tin foil. Care must be taken that its surface be free from all points and asperities; and the perforations which are made in it, and which should be about the size of a quill, for the purpose of attaching wires, and other kinds of apparatus, should have their edges well rounded and smoothed off. For the more perfect insulation of the conductor, it is advisable to apply upon the glass pillar which supports it, a varnish of gum-lac, or of sealing wax.

(60.) The degree of excitation produced in the glass depends much upon the substance employed as a rubber. Mr. Singer observes that dry silk is very efficacious, but that the most powerful effects are obtained by the use of an amalgam of tin, zinc, and mercury, applied by means of hog's lard, to the surface of leather or oiled silk. That part of the cushion which comes in contact with the glass cylinder, should be coated with an amalgam of this kind, spread evenly over its surface, until level with the line formed by the seam which joins the silk flap to the face of the cushion. No amalgam should be placed over this seam, nor on the silk flap; which last should be wiped clean whenever the continued motion of the machine shall have soiled it, by depositing dust or amalgam on its surface. The same attention is requisite to the surface of the glass, which often becomes covered with black spots and lines, more particularly when the amalgam has been recently applied. It is essential to remove these as often as they are formed in any quantity, since they tend to lessen the power of the machine. The surface of the amalgamated cushion is also soon soiled; for the excited glass constantly attracts dust from surrounding bodies, and this dust is collected by the rubber as the glass passes it. If the dust is removed after every course of experiments, by separating the cushion from the negative conductor, and gently rubbing its surface, and the surface of the silk flap, with a dry linen cloth, the machine may be kept in good order without a frequent renewal of the amalgam; such renewal being only necessary when that which has been applied becomes irregularly distributed over the cushion, or impregnated with dust.

(61.) The amalgam recommended by Mr. Singer, is made by melting together one ounce of tin and two ounces of zinc, which are to be mixed, while fluid, with six ounces of mercury, and agitated in an iron, or thick wooden box, till cold. It is then to be reduced to very fine powder in a mortar, and mixed with a sufficient quantity of hog's lard to form it into a paste. When amalgams have a large proportion of mercury, their action is variable and transient. The best cement for attaching the cylinder to its pivots, is made by mixing five pounds of resin, one pound of bees' wax, one pound of red ochre, and two table-spoonfuls of plaster of Paris. The ochre and plaster of Paris should be well dried, and then added to, and alternately mixed with the other ingredients, when they are in a state of fusion.

Fig. 10.



The plate machine, *fig.* 10, was originally proposed by Dr. Ingenhouz, and has been since much improved by Cuthbertson. This machine, in its most perfect form, consists of a circular plate of glass, turning on an axis that passes at right angles through its centre: it is rubbed by two pair of cushions, fixed at opposite parts of the circumference by elastic frames of thin mahogany, which are constructed so as to press the glass plate between them with the requisite force, by means of regulating screws. A brass conductor P, supported by glass, is fixed to the frame of the machine, with its branched extremities opposite to each other, and near the extreme diameter of the plate, in a direction at right angles to the vertical line of the opposite cushions. The branched extremities of the conductor are furnished with pointed wires, that serve to collect the electricity from the surface of the excited plate.

(62.) It is not quite determined which of these two arrangements affords the greatest quantity of electricity from the same surface; but the cylinder is less expensive, and less liable to accidents than the plate, and it appears to possess nearly equal power.

(63.) From what has already been explained of the general laws of electricity, the mode in which these machines act will readily be understood. The friction of the cushion against the glass cylinder produces a transfer of electric fluid from the former to the latter; that is, the cushion becomes negatively, and the glass positively, electrified. The fluid which thus adheres to the glass, is carried round by the revolution of the cylinder; and its escape is at first prevented by the silk flap which covers the cylinder, until it comes to the immediate vicinity of the metallic points, which being placed at a small distance from the cylinder, absorb nearly the whole of the electricity as it passes near them, and transfer it to the prime conductor. Positive electricity is thus accumulated in the prime conductor, while the conductor connected with the cushion, being deprived of this electricity, is negatively electrified.

But if both these conductors are insulated, this action will soon have reached its limit: for when the cushion and its conductor have been exhausted of their fluid to a certain degree, they cannot by the same force of excitation supply any further quantity to the glass. In order to enable it to do so, we must replenish it, as it were, that is, restore to it a quantity equal to what it has lost. This purpose will be answered by placing it in communication with a conducting body of large dimensions; or, what is still more effectual, by making it communicate with the earth, which is an inexhaustible source of electric fluid. In order, therefore, to supply the prime conductor with a constant stream of electricity, we must destroy the insulation of the cushion, by placing on the conductor to which it is fixed, a metallic chain, or wire, extending to the ground. If, on the other hand, we wish to obtain negative electricity, by means of the same machine, we must keep the negative conductor insulated, and connect the prime conductor with the ground, in order to allow the fluid to escape from it as soon as it is collected from the cylinder. The fluid will thus continue to be drawn without interruption from the negative conductor, as it now meets with no impediment to its discharge on the opposite side of the machine.

That the quantity of positive electricity produced in one conductor is exactly equal to that of the negative electricity in the other, is proved by the fact that, if the two conductors are connected by a wire, no signs of electricity are obtained in any of the conductors on turning the machine: but if the wire be not continuous, but interrupted by short intervals, a succession of sparks appear at each interval, indicating the passage of a stream of fluid from the one side to the other of the apparatus.

(64.) A person standing on a stool with glass legs is thereby insulated; and if, in this situation, he touch the prime conductor, either with his hand, or through the intermedium of a metallic rod, or chain, he may be considered as forming part of the same system of conductors. When the machine is worked, therefore, he will partake with the conductor of its charge of electricity, and sparks may be drawn from any part of his body by the knuckle of any other person who is in communication with the ground.

Chapter IV.

Effects of Electrical Attraction and Repulsion.

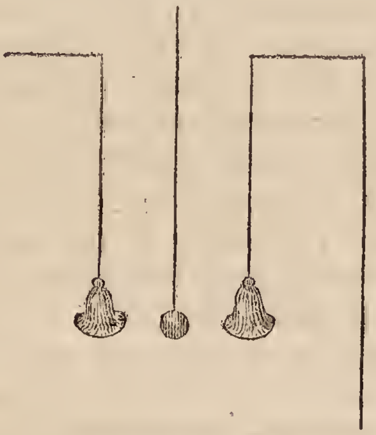
(65.) Having obtained, by the electrical machine, the means of accumulating considerable quantities of electricity, we are enabled to multiply and extend our observations of the phenomena, and to examine with more precision their correspondence with the results of theory. The effects of electrical attractions and repulsions may be exhibited much more distinctly, and on a larger scale than with the simpler instruments we had previously employed. The experiments formerly mentioned on the alternate approach and recession of light bodies, may be repeated with either conductor of the machine, when charged with electricity, and we may note with more accuracy the differences which occur in the rapidity with which the changes from one electrical state to another take place according as the bodies are more or less good conductors of electricity. A pith ball, or a fragment of gold leaf, is very strongly and immediately attracted by the electrified conductor, and the instant after it has come into contact with it, is repelled; but it is now attracted by the other bodies in its neighbourhood, to which it communicates its own electricity, and then is again in a state to be influenced by the conductor, and to be again attracted: and this alternation of effects will continue as long as the conductor remains charged.

(66.) These alternate and rapid movements are best seen by placing these small bodies between two metallic plates, placed as in fig. 11, the one over the other, at a certain distance; the upper one communicating with the prime conductor, the lower one with the ground. If figures of men and women are cut out of paper and placed between the two plates, they will exhibit a rapid dance, while they fetch and carry the electricity from the upper to the lower plate; or contrariwise, if the conductor be in the negative state.

Fig. 11.

(67.) This alternation of attractions and repulsions accompanying the transferring electricity by moveable conductors, is also illustrated by the motions of a ball (*fig.* 12.), suspended by a silk thread, and placed between two bells, of which the one is electrified, and the other communicates with the ground. The alternate motion of the ball between the two bells will produce a continued ringing. As thus described, it is a mere toy, but the same arrangement has been applied to the philosophical purpose of giving notice of changes taking place in the electrical state of the atmosphere.

Fig. 12.



(68.) The mutual repulsion of bodies that are similarly electrified gives rise to many amusing appearances. The filaments of a feather will separate from each other and diverge, when electrified, presenting a singular and unnatural appearance. A small figure in the shape of a human head, covered with hair, when placed upon the conductor and electrified, will exhibit the appearance of terror from the general bristling up and divergence of the hair. A lock of wool highly charged with electricity will, in like manner, swell out to a large size, in consequence of the mutual repulsion of the filaments which compose it. On approaching a needle to it, held in the hand, whereby its electricity is quickly drawn off, the cotton will suddenly shrink into its original dimensions.

(69.) We have already adverted to the effects of fusion in rendering some bodies conductors, which in their solid state had the contrary property. This is the case with sealing-wax; and accordingly, if melted sealing-wax be electrified, its particles will tend to separate by their mutual repulsion, and to draw out into filaments. Let a piece of sealing-wax be fixed on the end of a wire, and be set fire to, but the flame immediately afterwards blown out. While the surface of the wax is still melted, present it, at the distance of some inches, to the electrified conductor, a number of extremely fine filaments will immediately dart out from the sealing-wax to the conductor, on which they will be condensed into a kind of net-work resembling wool. If the wire with the sealing-wax be stuck into one of the holes of the conductor, and a piece of paper be presented at a moderate distance to the wax, just after it has been ignited, on setting the machine in motion, a net-work of wax will be formed on the paper. The same effect, but in a slighter degree, will be produced, if the paper be briskly rubbed with a piece of Indian rubber, and the melting sealing-wax be held pretty near the paper immediately after it has been rubbed. If the paper, thus covered with filaments of sealing-wax, be gently warmed before the fire, the wax will adhere to it, and exhibit permanently the result of the experiment. Still more beautiful are the appearances produced by camphor subjected to a similar process. For the purpose of obtaining them, a spoon, holding a piece of lighted camphor, must be kept electrified by working the machine, while it communicates with the conductor; the camphor will then throw out curious ramifications, which appear to shoot like those of a vegetable.

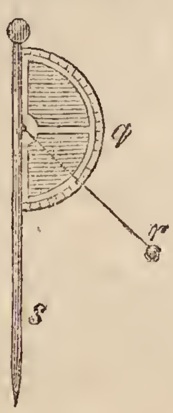
(70.) It is on the same principle that

the escape of a conducting fluid, such as water, through a narrow aperture, is promoted by electrifying it. If a small metallic vessel filled with water be suspended from the prime conductor, and there be placed in the water one end of a glass syphon, with a capillary bore of such a diameter as that the water will scarcely drop from it; upon turning the cylinder of the machine so as to convey electricity to the vessel and its contents, the water immediately flows in a stream, and, if the electrical charge be very powerful, the descending current will be seen to separate into several branches.

(71.) If a sponge, saturated with water, be suspended from the prime conductor, the water will at first only drop gradually from the sponge; but when the conductor has become strongly electrified, the drops will fall plentifully, and, in the dark, will produce the appearance of a luminous shower of rain.

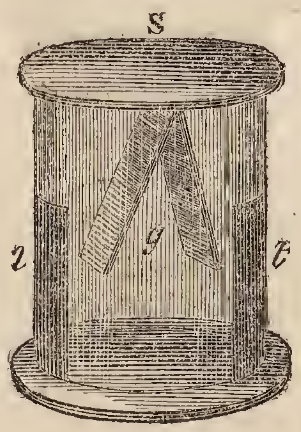
(72.) Advantage is taken of the repulsive property of electrified bodies for the construction of an *Electrometer*, or instrument adapted to measure the intensity of the electricity they may contain. Henley's electrometer (*fig.* 13.) consists of a slender rod of very light wood, *r*, serving as an index, terminated by a small pith ball, and suspended from the upper part of a stem of wood, *s*, which is fitted to a hole in the upper surface of the conductor. An ivory semicircle, or quadrant, *q*, is affixed to the stem, having its centre coinciding with the axis of motion of the rod, for the purpose of measuring the angle of deviation from the perpendicular, which the repulsion of the ball from the stem produces in the moveable rod. The number of degrees which is described by the index, affords some evidence of the quantity of electricity with which the apparatus is charged; though the instrument has obviously no pretensions to being an exact measure of its intensity.

Fig. 13.



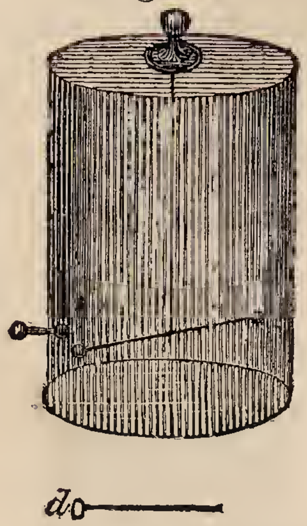
(73.) One of the most delicate instruments for detecting the presence of electricity is that which was invented by Mr. Bennet, and is usually called the *gold-leaf electrometer;* although it is, properly speaking, only an electroscope. It consists (*fig.* 14.) of two narrow slips of gold leaf, *g*, suspended parallel to each other, in a glass cylinder, which secures it from disturbance by accidental currents of air, and attached to the end of a small metallic tube, which terminates above either in a flat surface, S, of metal, or in a metallic ball. Two slips of tin-foil *t t*, are pasted to the inside of the cylinder, on opposite sides, in a vertical position, and so placed as that the gold leaves may come in contact with them, when their mutual repulsion is sufficiently powerful to make them diverge to that extent. These slips of tin-foil terminate in the foot of the instrument, and thus are in communication with the earth. A very minute charge of electricity communicated to the upper end of the tube, is immediately transmitted to the gold leaves, which are thus made to repel each other; but if the repulsion is such as to make them strike against the tin-foil, their insulation ceases, and their electricity is carried off; and being now rendered neutral, they cease to repel one another, and, collapsing, resume their original position.

Fig. 14.



(74.) The most perfect electrometer for measuring very small quantities of electricity, is the apparatus contrived by Coulomb, and to which he has given the name of the *torsion balance.* It is represented in its simplest form in *fig.* 15, and consists of a cylindrical glass jar, covered at the top by a circular glass plate, with a hole in its centre. Through this hole a single fibre of

Fig. 15.



the web of the silken worm descends nearly to the bottom of the jar, and carries at its lower extremity a transverse needle. This needle consists of either a filament of gum-lac, or a silk thread or piece of straw coated with sealing-wax. At one end it is terminated by a small pith-ball, and at the other by a disc of varnished paper, acting merely as a counterpoise to the ball. The upper end of the silk fibre is affixed to a kind of button having a small index, and capable of being turned round upon a circular plate divided into degrees. One side of the jar is perforated to allow of the insertion of a short horizontal bar, having a small metallic sphere at each of its ends, the one being in the inside and the other on the outside of the jar; and the former being so situated as just to allow the ball of the suspended needle to come in contact with it in the course of its revolution. By turning the button, or the index, the needle may be brought into this, or any other required position with regard to the ball. It is found by experiment that the angle of torsion of the silk fibre is, within a certain range of distance, very nearly in the direct ratio of the force which acts in producing the torsion; and therefore, if the two balls be placed in contact by turning the button, and then similarly electrified, the distance to which they are repelled by the angular motion of the suspended ball, affords a measure of the repulsive force exerted. In like manner, the distance which the suspended ball is made to move when it is attracted by the fixed ball, when the two have opposite electricities, gives accurate measures of the attractive forces. It was by the employment of this apparatus, in a very elaborate series of experiments, that Coulomb was enabled to establish very satisfactorily the exact law of variation, both of the attractive and repulsive forces, arising from electricity, with relation to the distance, which we have already stated.

Chapter V.

Distribution of Electricity.

(75.) It had long been observed, that the quantity of electricity which bodies are capable of receiving, does not follow the proportion of their bulk, but depends principally upon the extent of their surface. It was found, for instance, that a metallic conductor in the form of a globe, or cylinder, contains just as much electricity when hollow, as it does when solid. Hence it was evident that the electricity resides altogether at the surface, or at least does not extend equally throughout the whole mass of the body. But it was only by applying to the theory all the refinements of mathematical investigation, that precise notions could be formed of the exact distribution of the electric fluid in bodies of different shapes. The labours of Cavendish, Coulomb, Poisson, and Ivory, have furnished the means of determining this problem in every case, however complicated; and whenever a comparison has been instituted between the results of experiment and of theory, the most perfect agreement has been found between them. Thus all the phenomena of electricity are found to be in exact conformity with the mechanical consequences of the theory: they can be anticipated with rigorous precision, and can even be reduced to numerical calculation in their minutest details, as well as in their most intricate combinations.

(76.) For the purpose of measuring the proportional quantities of electricity with which different parts of the same, or of different bodies are charged, no instrument is so well fitted as the balance of Coulomb, of which an account has just been given. What peculiarly adapts it for these experiments, is its extreme sensibility, by which the slightest variation in the intensity of the attractive or repulsive force produces a very considerable effect in the movement of the horizontal needle. In some of the experiments related by Coulomb, a force only equal to the 279th of a grain was sufficient to make the needle perform an entire revolution round the circle: the 360th part of this force, therefore, or less than the 100,000th of a grain, might be estimated by each degree of its angular motion.

In order to apply to the instrument only such forces as it is capable of measuring, and of collecting at the same time from the different parts of bodies such minute quantities of electricity as are exactly proportional to those with which they are themselves charged, Coulomb employed what he calls a *proof plane*, which is simply a small circular disc of gilt paper, *d*, (*fig.* 15,) fixed to the extremity of a very slender cylinder of gum-lac, and thus completely insulated. If we wish, then, to ascertain the proportions in which electricity is distributed on the surfaces or interior of any particular body, we first insulate that body as completely as possible, and impart to it a small quantity of electricity by a spark from the prime conductor. We next touch any of the points on its surface, the electricity of which we may wish to measure, with the little gilt disc, holding it by the other end of its insulating handle; then carrying the plane to the torsion balance, of which the moveable ball has been previously charged with an electricity of the same kind, we bring it for an instant in contact with the fixed ball. We then withdraw it, and the fixed ball being now electrified in the same manner as the moveable one, repels the latter with a force measured by the angle of torsion, at which the moveable ball stops. While the little plane and the balls of the balance remain the same, the division of the electricity between the little plane and the moveable ball preserves the same uniform proportion; and thus the repulsive force which results, and which drives off the moveable ball, is proportional to the quantity of electricity with which the little plane is charged. It has been proved, by a series of well-contrived experiments, that this quantity is exactly proportional to the quantity of electricity which really exists at the point of the body with which it has been placed in contact. By applying the same test and method of admeasurement to various other points of the body we are studying, we may determine the manner in which the electricity is distributed in all its parts; for the method is applicable even to the interior of the body, if we pierce it with a small hole terminating at the part whose electricity we wish to examine, and pass the proof plane into it till it is applied to the bottom of the aperture. Care must be taken, however, in conducting these last experiments, that the proof plane be not suffered to touch any other part of the body except that of which the electricity is to be determined, and not even the sides of the aperture through which it is introduced, as such contact would entirely falsify the result.

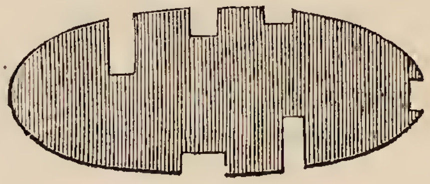
The following are among the principal results of these investigations:—

(77.) In a solid body having the form of a perfect sphere, and charged with positive electricity, the whole of the fluid is, in consequence of the repulsion of its own particles, which is everywhere directed from the centre outwards, accumulated in a thin stratum at the very surface of the sphere. If the body be charged with negative electricity, the deficiency of fluid will take place only in the superficial stratum of matter.

(78.) If, instead of being spherical, the body have any other form, the electricity will still be chiefly confined to the surface; and if it have an elongated form, there will be a greater charge in the remoter parts than in those nearer to the middle.

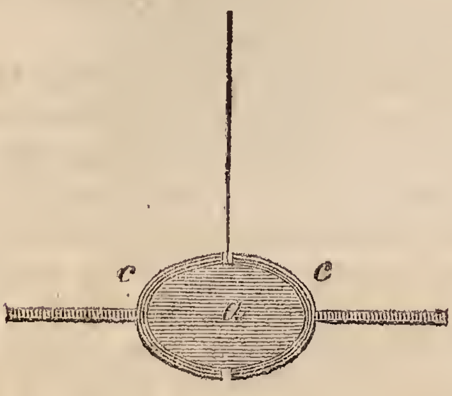
(79.) This result of theory, respecting the limitation of electricity to the mere surface, is confirmed in the most decisive manner by the experiments of Coulomb. A conducting body of the form represented by the section, *fig.* 16, had small pits made in various parts of its surface. They were half an inch in diameter, and some of the most shallow were not depressed more than one-tenth of an inch below the surface. When the body was electrified, and the small proof plane applied in accurate contact to the bottom of these pits and depressions, care being taken that it should not touch their margin, and then applied to the electrometer, no indication of its having received any electricity could be perceived; whereas the contact of the same proof plane with any part of the even surface showed the latter to be strongly electrified.

Fig. 16.



(80.) The following experiment of Biot's contains also a striking practical illustration of the same truth. Let *a* (*fig.* 17.) represent a section of any spheroid of conducting matter, suspended by a thread which perfectly insulates it. Let *c c* be two caps formed of gilt-paper, tin-foil, or any conductor, and such that, when united, they accurately fit the surface of the spheroids; and let them be also furnished with insulating handles of gum-lac. Let there be communicated to the ball, *a*, any degree of electricity; and then let the two caps, held by their insulating handles, be carefully applied to its surface. Upon the removal of these caps, it will be found that the whole of the electricity has been abstracted from the spheroid, so that it will no longer affect the most delicate electrometer; whilst the two caps will be found, upon accurate trial, to have acquired precisely the same quantity of electricity which had at first resided in the body *a*.

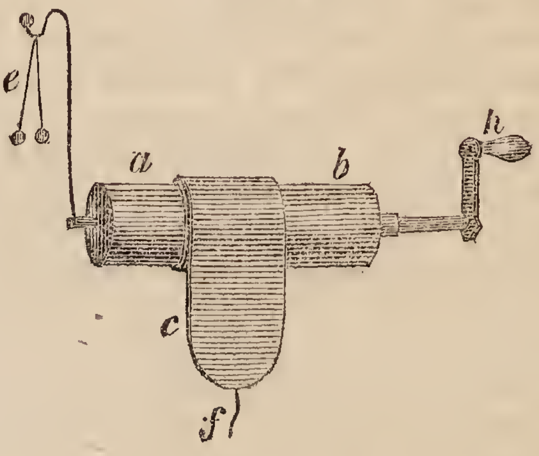
Fig. 17.



We may conclude, both from theory and experiment, therefore, that although, strictly speaking, the electricity must reside within the substance of conducting bodies, it extends, in fact, to a depth so small as to be inappreciable by any known methods of observation.

(81.) The effect of an expansion of surface in lessening the intensity of electricity, while its absolute quantity remains the same, is well illustrated by the following experiment mentioned by Biot. *Fig.* 18 represents an insulated cylinder, *a b*, moveable round a horizontal axis, and capable of being turned by an insulating handle *h*. Around the cylinder is coiled a thin lamina of any metal, *c*, the end of which is semicircular, and has attached to it a silk thread *f*. The whole apparatus communicates with an electroscope *e*, formed of two linen threads, each terminating in a pith ball. On communicating a charge of electricity to the cylinder, the threads and balls of the electroscope diverge. Upon taking hold of the silk thread, and unrolling the metallic lamina from the cylinder, the balls gradually collapse; thus indicating a diminution in the intensity of electrical repulsion. If the lamina be sufficiently long, the electrical charge may be spread over so great an extent of surface, as to allow the balls to hang perpendicularly and come in contact. But on winding up the lamina, the intensity of the electricity is restored, and the balls diverge to the same extent as before, allowance being made for the small dissipation of electricity which may have occurred from the contact of the air during the experiment.

Fig. 18.



(82.) In the case of a long and slender lamina of conducting matter, charged with electricity, Coulomb found that its intensity continued nearly uniform from the middle of the lamina to within a short distance from the ends; at that part it rapidly increased; and at the very extremity it became twice as much as at the middle part. In a circular plate, the electricity is accumulated in much greater quantities at the circumference than about the centre; the intensities being in the proportion of 2.9 to 1: that is, the intensity at the centre is nearly one-third of that at the circumference.

(83.) If the body be an oblong spheroid, arising from the revolution of an ellipse on its smaller axis, the thickness of the strata of electricity, or, in other words, its intensity, at the extremities of the two axes, is exactly in the proportion of the respective axes themselves. It thus appears, that if the ellipsoid be much elongated, the intensity must be very feeble at the equator, but very great at the poles. A still more rapid augmentation of the relative intensity at the extremities takes place in bodies of a cylindric or prismatic form; and the more so as their length bears a greater proportion to their breadth. Coulomb found by experiment that, in a cylinder thirty inches long and two inches in diameter, the intensity of the electricity at the ends was to its intensity at the middle, or at any part more than two inches from the extremity, as 2.3 to 1. Pursuing this train of reasoning, it will lead us to a conclusion of some importance, namely, that if the conducting substance be drawn out into a point, the intensity of the electricity at that point will be exceedingly great; and that the point will accordingly absorb and draw into itself nearly the whole of the electricity that is contained in the body. This vast concentration of electricity is found actually to take place in all points that project beyond the general surface.

Chapter VI.

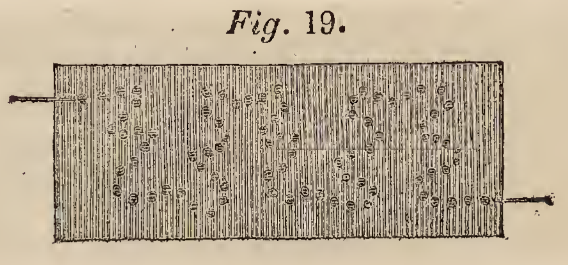
Transference of Electricity.

(84.) We are next to consider the condition of bodies during the prevalence of those forces which tend to overset the electric equilibrium, over those which tend to preserve it. The pressure exerted by the electric fluid against the non-conducting medium, such as the air, which opposes an obstacle to its escape, is in a ratio compounded of the repulsive force of its own particles at the surface of the stratum of fluid, and of the thickness of that stratum; but as one of these elements is always proportional to the other, the total pressure must, in every point, be proportional to the square of the thickness. If this pressure be less than the resistance, or coercive force, as it has been called, of the air, the electricity is retained; but the moment it exceeds that force, in any one point, the electricity suddenly escapes, just as a fluid confined in a vessel would rush out if it were to burst open a hole in the side of the vessel.

(85.) It is only a certain proportion of the whole quantity of electricity in the conducting body that thus suddenly escapes; but the irruption of it is marked by many very striking phenomena, all indicative of the abruptness and violence with which the change is effected. A sharp snap is heard, accompanied by a vivid spark, and there are evidences of an intense heat being evolved in the line which the electricity takes.

(86.) The passage of the electric fluid through a perfect conductor is unattended with light. Light appears only where there are obstacles in its path by the interposition of imperfect conductors; and such is the velocity with which it is transmitted, that the sparks appear to take place at the very same instant along the whole line of its course. Thus, if a row of small fragments of tin-foil be pasted on a piece of glass, *fig.* 19, and electricity be sent through them by connecting one of its ends with the conductor of an electrical machine, while the other end communicates with the ground, it will not be possible to detect any difference of time in the occurrence of the light in the different parts, so that the whole series of luminous points, if sufficiently near, appear, in the dark, like a vivid and continuous line of light. By varying the arrangement of the tin-foil, we may distribute the light in any manner we please, so as to exhibit a brilliant delineation of the figure they represent. Even when conducting bodies appear to be in contact, if the experiment be made in the dark, a spark is generally seen to pass between them, unless the bodies be pressed together with considerable force. Hence, a chain appears luminous at each link, while conveying a charge of electricity.

Fig. 19.



(87.) The longest and most vivid sparks are obtained between two conductors having a rounded form, and the more so in proportion as they are both portions of spheres of large diameter. This may be exemplified in a common electrical machine, by presenting a metallic ball of large size to that side of the prime conductor which is furthest from the cylinder of the machine. In such cases, however, the electricity being of weaker intensity, the distance between the conducting bodies requisite for the transfer of electricity through the air, or what is termed *the striking distance*, is necessarily small. If the ball is of smaller diameter, or the conductor of a more elongated shape, the electricity at its surface is of higher intensity, and will therefore pass through a greater extent of air; the spark is in this case of considerable length, appearing as a long streak of fire extending from the conductor to the ball, and instead of being directed towards one point, being distributed to various points throughout a certain extent of the surface of the ball.

Often, when very long, the spark is seen to have an angular or zig-zag course, (see *fig.* 20.) exactly like that of a flash of lightning.—This irregularity is probably occasioned by the fluid darting obliquely in its course to minute conducting particles that are floating in the air, a little removed from the direct line of passage. Even particles of moisture suspended in the air would be sufficient to occasion these deviations. The presence of such particles will account also for the appearance of lateral scintillations, which frequently seem to diverge from the principal stream of electricity. The greater the number of such intermediate conductors, or stepping-stones, as it were, for the electricity, the more readily will the balance between the forces be overset, and the irruption of electric fluid determined.

Fig. 20.

When the air is either sufficiently moistened, or sufficiently rarefied, the electric fluid passes through it with comparative facility, and its track is indicated by streams of light, probably occasioned by many parallel series of minute sparks passing from particle to particle.

(88.) Electrical light differs in no respect from the light obtained from other sources. Dr. Wollaston found that, when observed through a prism, the ordinary colours arising from the decomposition of light are obtained; but the prevailing tint of colour will vary according to the different substances through which the sparks pass, or to the nature of the surface from which they emanate, or by which they are received. Dr. Brewster found that it is capable of undergoing polarization, either by transmission through a doubly refracting crystal, by reflection at the proper polarizing angle from a polished plane surface, or by oblique refraction through a series of glass plates.

(89.) The brilliancy of the electrical spark is proportional to the conducting power of the bodies between which it passes. When an imperfect conductor, such as wood, is employed, the electric light appears in the form of faint red streams; but metals afford them of great brilliancy. Its colour is subject to variation, from a great number of different circumstances. Sparks passing through balls of wood or ivory, are of a crimson colour; but this depends also upon their position with regard to the surface. If two pointed wires be inserted obliquely and in opposite directions into a piece of soft deal, having their points an inch and a half distant, but penetrating to different depths below the surface, and so that the line joining them is in the direction of the fibres, the sparks passing from the one to the other, will exhibit different colours at different depths; and if one of the points be inserted deeper than the other, all these colours will appear at once, according as the electric light is transmitted at various depths. Electric sparks passing from one polished metallic surface to another are white; but if the finger be presented to an electrified conductor, the sparks obtained are violet. They are green when taken from the surface of silvered leather; yellow when taken from finely powdered charcoal; and of a purple colour when taken from the greater number of imperfect conductors. If one of the bodies between which the spark takes place is a green plant, the light is red; and the same is the case with water or ice. In the vapour of ether green sparks are seen when the eye is placed close to the tube: but they appear reddish when viewed at a considerable distance. Even between the same two metallic conductors the colour may vary from the most brilliant white to the most delicate violet, according to the distance through which the electricity is transmitted, and according to the resistance of the medium which it is compelled to traverse. In exceedingly rarefied air, the colour of the spark is green; in denser air, it acquires a blue tint, and passes to a violet and purple, in proportion as the condensation of the air is increased. Transmitted through other gases, the colour varies according to their density, In carbonic acid gas, the spark is white and vivid; in hydrogen gas, it is faint and red.

(90.) It should be recollected, in making these experiments, that in proportion as the medium is more rare, its conducting power increases, and a smaller intensity of electricity is required for the production of light. In the ordinary vacuum produced by the air-pump, the passage of electricity is rendered sensible by streams or columns of diffused light occasionally varying in their breadth and intensity, and exhibiting movements which give them a marked resemblance to the coruscations of the Aurora Borealis. After rarefying the air contained in a glass jar, about one foot long and eight inches in diameter, to the 500th part, Mr. Smeaton placed the jar upon a lathe, and caused it to revolve rapidly, whilst at the same time he rubbed it with his hand. A considerable quantity of lambent flame appeared under his hand, variegated with all the colours of the rainbow. The light was steady; but every part of it was constantly changing colours. When a very perfect vacuum is made in a glass cylinder covered with a brass plate, the electric stream will pass between it and the plate of the receiver of the air-pump, in a continued stream of the same size throughout its whole length. If a Torricellian vacuum be formed in the upper portion of a long bent glass tube filled with mercury, and inverted, by placing the legs of the bent tube in separate basins of mercury, when electricity is transmitted through the tube, light is seen to pervade the vacuum in a continued arch of lambent flame, without the least divergency.

(91.) It was natural to suppose, before sufficient consideration had been bestowed upon the subject, that the light which appears during the passage of electricity, was actually the electric fluid itself, which, at some certain degree of accumulation, was in itself luminous; and such was the notion entertained by the early electricians. But since we know that common atmospheric air becomes luminous by violent compression, and we must also presume that electricity exerts a very sudden and powerful pressure upon the air by its passage through that resisting medium, we are certainly justified in drawing the inference that the same phenomena proceed in both cases from the same cause. Biot has adopted this opinion, which appears to be more consonant with philosophical views of the subject than any other: for it is certain that the whole of the electrical light that appears is not more than what may proceed from the mechanical compression of the air, the vapours, and other constituents of the medium through which the passage of the electricity is effected.

(92.) The sound which accompanies these various modes of transference is subject to corresponding modifications, dependent likewise, no doubt, upon the degree and the suddenness of the impulses given to the air. The full, short, and undivided spark is attended with a loud explosion; the more lengthened spark, with a sharper snap, which becomes more broken and rattling in proportion to the distance it has to traverse. The luminous streams produced by a succession of minute sparks are scarcely productive of noise, but are accompanied only by a faint rustling sound, like that of a stream of wind through a narrow chink.

(93.) A peculiar odour has sometimes been perceived in the neighbourhood of an electrical machine which has been briskly worked, so as to emit for some time a great number of sparks; and it has been thought to resemble that of phosphorus. This is also probably owing to some unknown chemical decomposition effected by the electricity during its passage through the air.

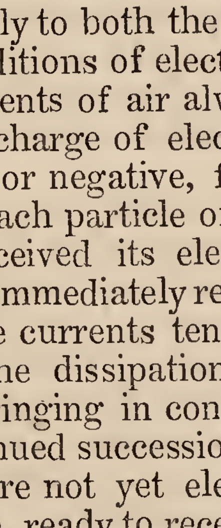
(94.) We have already had occasion to remark the great increase of intensity which the electric fluid acquires at the extremity of all elongated parts of conducting bodies; and the indefinite augmentation of this intensity which takes place at the apex of all projecting points. This high intensity will necessarily be accompanied with a powerful tendency in the fluid to escape; a circumstance which furnishes a natural and exact explanation of the rapid dissipation of electricity which takes place from all bodies of a slender and pointed form.

The following experiments illustrate these positions. Let the insulated conductor of a machine be furnished with a pair of pith-balls, suspended by a fine wire, and charged with either species of electricity; the divergence of the balls will indicate the presence and degree of this electricity. If a metallic rod with a ball at one end be held in the hand,

and the ball presented to the conductor, taking care not to bring it sufficiently near to draw a spark, the balls will be but little affected, and their divergence will continue for a considerable time. But if the rod terminate in a sharp point, instead of a ball, and the point be presented to the conductor at the same distance as the ball was in the former case, the electroscope will immediately collapse, showing that the electrical charge has entirely disappeared: it has, in fact, been rapidly drawn off by the pointed rod. It is quite immaterial to the success of the experiment whether we affix a point to the conductor itself, or whether we present to it a point held in the hand; the escape and dispersion of the electricity being equally promoted by the presence of a point, whether the fluid be given out or absorbed; for it is scarcely necessary to remark that the very same kind of reasoning applies equally to both the positive and negative conditions of electricity.

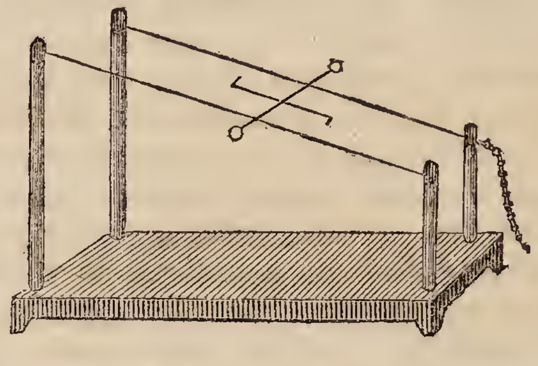
(95.) Currents of air always accompany the discharge of electricity, whether positive or negative, from pointed bodies; for each particle of air, as soon as it has received its electricity from the point, is immediately repelled by the body. These currents tend powerfully to increase the dissipation of the electricity, by bringing in contact with the point a continued succession of particles of air, that are not yet electrified, and are, therefore, ready to receive a charge. Many amusing experiments are founded on this principle. Let two cross wires, (*fig.* 21.) the ends of which terminate in points, bent in a similar direction with respect to the axis, be supported by means of a cap upon a fine point, and electrified by being placed upon the prime conductor of a machine. Each of the points will give off a stream of electricity: this will remove a part of the pressure which the fluid would have exerted on that side if no efflux had taken place; but as the pressure of the fluid on the opposite side of the wire, in the opposite direction, still operates in full force, the wire will be impelled in the direction of that force, that is, in a direction opposite to that of the stream; and this taking place at all the four points, the whole system will revolve backwards with considerable rapidity.

Fig. 21.



The following is another form in which this experiment may be made. Two wires, (*fig.* 22.) are stretched in the direction of a plane, slightly inclined to the horizon, between four insulating pillars. Across these wires, another wire is made to rest, terminating by small balls at each end, and having a cross wire fixed to it at right angles, with two bent points, as in the former experiment. When this system is electrified, the dispersion of the electricity from the points produces a revolution of the bars, which makes the transverse bar roll up the inclined plane.

Fig. 22.

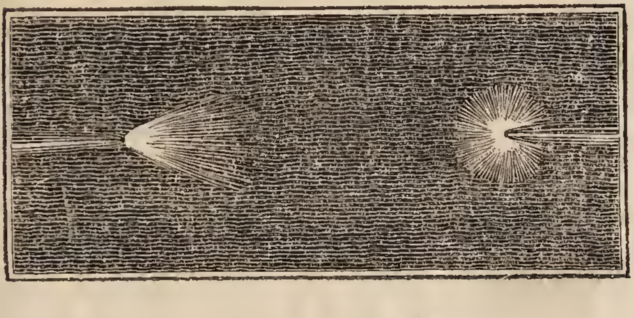


An apparatus consisting of wires terminating in points, and having balls annexed to them to represent the planets, may be constructed so as to revolve when electrified; and thus to imitate the planetary motions. Such an apparatus has been called an electrical orrery.

(96.) It should be observed, however, that a point loses its power of concentrating and dispersing electricity when it is surrounded by other parts of the conducting body which are equally prominent; as when it is placed between two balls, or inclosed in a tube, or when it does not rise above the general surface of the body. The effect of one point is much diminished even by the vicinity of another point; so that if several points placed near each other be presented to the conductor, the electricity is drawn off much less rapidly, and will be transferred by sparks instead of forming a continued stream.

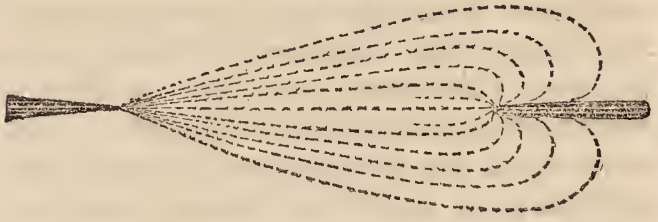
(97.) When the transfer of electricity takes place between smooth surfaces of a certain extent, no difference can be perceived in the nature and appearance of the spark, whichever be the position of the negative surface. But in the passage of electricity through points, the effect is considerably modified by the species of electricity with which the bodies are charged; or, in other words, by the direction in which the fluid moves. When the electric fluid is escaping out of a pointed conductor, the luminous appearance is that of diverging streams, as represented in *fig*. 23; forming what is termed a *pencil of light*, and resembling the filaments of a brush. When, on the contrary, the electric fluid is entering into the pointed body, the light is much more concentrated at the point itself, having a resemblance to a *star*, in which, if any streams appear, they are disposed like radii, and equally so in all directions. An approach to these different modifications may be remarked when sparks pass between balls of small diameter, especially if the charge is high. Thus the direction of the lateral ramifications sent out from the principal line, in the branched spark, *fig*. 20, is from the positive to the negative surface.

Fig. 23.



(98.) In describing the above appearances, we have, as usual, referred to the hypothesis of Franklin: but if we adopt that of the two electricities, we have only to consider the appearance of the pencil of light as arising from the double current of the vitreous electricity issuing from the point, and of the resinous electricity passing into it: while the star will be the effect of the irruption of the resinous, and the absorption of the vitreous electricities. But this remarkable difference in the phenomena produced, according to the particular species of electricity with which the point is charged, has always been urged as a convincing argument in favour of the Franklinian theory. They appear very strongly to indicate the emanation of some material fluid from the positive, and its reception by the negative point. The diverging lines on the one side, and their inflections on the other, represent exactly the paths of particles flowing out as from a pipe, and urged forwards by a force which gives them such a projectile velocity as to prevent their spreading out beyond a certain distance from the direct line of projection. But this very velocity will carry the particles that happen to have deviated most, somewhat beyond the point to which they are attracted: while the attraction to this latter point will tend to deflect them from the line of their path, and gradually turn them back, so that they will arrive at the point of attraction by very different paths, and some even by a retrograde motion. Hence, while in the first case they form a diverging cone of rays, in the latter they must be distributed on all sides of the point like the rays of a star. The annexed diagram, *fig*. 24, will sufficiently illustrate this explanation by representing the supposed course of the particles of electric fluid, passing through the air from the positive to the negative point. What weight the argument derived from this phenomenon may be allowed in deciding the question, will be discussed in the sequel.

Fig. 24.



(99.) The difference which we have now described in these two appearances, may be employed, on many occasions, as a useful criterion of the species of electricity, at least, which is passing from one conductor to another, if not of the absolute direction of its motion. For, if a needle be presented to an electrified body, the appearance of a star on the needle will show that the electricity of that body is positive; while, on the contrary, a luminous brush on the needle will indicate that the body is negative.

(100.) The influence of a point projecting a short distance from the surface of a body, is greater when that body is negative than when it is positive. Hence, a spark is more readily obtained in the latter case than in the former.

On this principle an instrument has been invented by Mr. Nicholson for distinguishing the negative from the positive electricity. It consists simply of two metallic balls fixed at the ends of two curved rods of glass, and moveable like branches on a joint, so as to admit of the balls being placed at different distances from each other, when held by a handle proceeding from the joint. A short point projects from one of the balls on the side adjacent to the other ball; and this point affords a spark at a shorter distance when positively, than when negatively electrified.

Chapter VII.

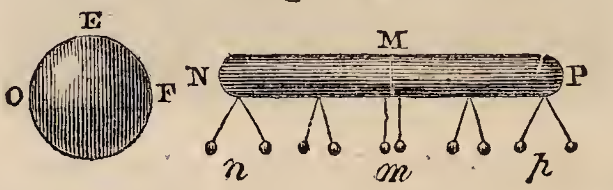
Development of the Law of Induction.

(101.) We have next to trace the consequences of that important law of electricity which has been called the Law of Induction.

Active electricity existing in any substance tends always to induce the opposite electrical state in the bodies that are near it. Now it is impossible, as we have already seen, to induce one electrical state in any body without at the same time producing the opposite state in the same body, or in the one which is immediately contiguous. According to the simpler theory, the accumulation of electricity in any one part can be effected in no other way than by withdrawing it from another part, nor can it be abstracted from the one without being received by another; so that there is always an equal degree of negative as of positive electricity, and *vice versâ*, in every case. According to the more complex theory, if we decompose the natural electricities residing in any body, we must at the same moment obtain equal quantities of both the vitreous and resinous electricities. It follows, therefore, that if the bodies subjected to the inductive influence are non-conductors, although the tendency to produce the opposite electricity still exists, yet in consequence of the immobility of the fluid, it can produce no visible change. In proportion as the body opposes less resistance to the passage of electricity, the operation of the disturbing force becomes sensible; and in order to fix our ideas, let us first take the case of a positively charged electric, acting by induction on an insulated conducting body. The redundant fluid in the former will tend to repel all the fluid contained in the latter: a portion of this fluid will, therefore, be driven from the side adjacent to the first body, towards the remoter side. The adjacent side will thus be rendered negative; the remote side, positive. But this will take place to a certain extent only: for there is a limit at which the repulsion of the fluid accumulated at the remote end, will just balance the repulsion of the fluid in the electric, added to the attraction of the under-saturated matter, in the near end; and when this limit has been attained, the flow of electric fluid from the near to the remote end of the body will cease, and an equilibrium will be established.

(102.) Experiment shows the perfect coincidence of theory with the actual fact. Let a cylinder of metal, NP, (see *fig.* 25,) of some length, with rounded ends, and furnished in different parts with pairs of suspended pith-balls, to serve as electroscopes, being previously insulated, be placed in the vicinity of an electrified globe of glass, E, taking care that it be not sufficiently near to receive any quantity of electricity by transference.

Fig. 25.



We shall find that every pair of balls, except those situated in a particular plane M*m*, about the middle of the cylinder, will immediately diverge, indicating the electrical states of the parts from which they are suspended. Those at either extremity of the body, *n*, *p*, diverge the most; and the divergence diminishes as we approach the middle plane before mentioned, at which the body is in the natural or neutral state. The position of this plane of neutrality, M*m*, varies according to the distance of the electric, and the relation which that distance bears to the length of the body itself. If we further examine the species of electricity residing in the different parts, we shall find it to be negative in all the parts nearer to the electric than the neutral plane, and positive in all those more remote. We may ascertain with much greater accuracy these electrical states by the employment of the proof plane and electrometer of Coulomb, than by the pith-balls; and the results are then found to correspond, with the most rigorous precision, with the deductions from the theory of electrical action.

(103.) These effects, it should be remarked, are simply the result of the action of electricity at a distance; for they depend upon no other circumstance.

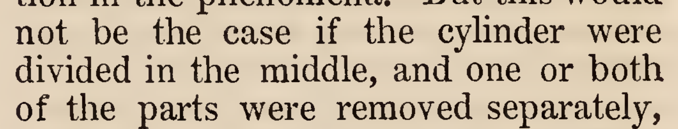
They take place in an equal degree whatever substance be interposed between the bodies which are exerting this action on one another, provided the interposed substance undergoes no change in its own electrical state; a condition which is fulfilled in electrics only. Thus, induction will take place just as effectually through a plate of glass, as if no such substance had intervened.

(104.) Let us now suppose that the acting body, E, is, instead of an electric, a conducting body, a globe of metal, for example, charged with positive electricity. The primary effects of this globe on the cylinder will be the same as in the former case; but the electrical state which the globe has induced on the cylinder will re-act upon its own electricity. The negative electricity, that is, the under-saturated matter at the nearer end of the cylinder N, exerts a tendency to induce positive electricity in the globe, and more especially upon the adjacent side, F: that is, it will tend, by its attraction for the fluid, to draw it to that side, and thus render it still more highly positive than it was before. This can only be done at the expense of the other side, O, from which the fluid must be taken, and which is, therefore, rendered less charged with fluid, that is, less positive than before. But this new distribution of the electric fluid in the globe, by increasing the positive state of the side, F, next to the cylinder, tends to augment its inductive influence on the fluid in the cylinder; that is, to drive an additional quantity of fluid from the negative to the positive end. This is followed, in its turn, by a corresponding reaction on the globe, and so on, constituting a series of smaller adjustments, until a perfect equilibrium is established in every part. When this has been attained, the electrical states will, it is evident, be of the same kind as those consequent upon the immediate actions, though somewhat increased in intensity by the series of reactions.

The following experiment is a practical illustration of the preceding reasoning. Furnish the metallic globe with electroscopes on its opposite surfaces; when the globe is insulated and alone, any electricity communicated to it will diffuse itself equally over the surface, and both the electroscopes will diverge equally. But no sooner do we bring near to it a conducting body, than the balls of the electroscope at the side most distant from that body begin to collapse, while those at the nearer side diverge to a greater degree than before; thus showing the nature of the reflex operation of the induced electricity of the conductor upon the body from which the induction originated.

(105.) It should be recollected that in all the changes we have thus traced as the effects of induction, there has been no transfer of electricity from either of the bodies to the other; as was sufficiently proved, indeed, by their taking place equally if a plate of glass be interposed. Another proof is afforded by the circumstance that the mere removal of the bodies to a distance from one another, is sufficient to restore each of them to their original state. The globe remains as positively electrified as before; the cylinder returns to its condition of perfect neutrality; nothing has been lost, and nothing gained on either side. The experiment may be repeated as often as we please, without any variation in the phenomena. But this would not be the case if the cylinder were divided in the middle, and one or both of the parts were removed separately, while they still remained under the influence of the globe. The return of the electric fluid from the positive to the negative end being thus prevented, each part will retain, after its separation, the electricity which had been induced upon it. The nearer portion will remain negative; the remoter portion, positive. If the division had been in three parts, the middle part only would have been neutral. The experiment may be made by joining two or more conductors endwise, as shown in *fig.* 26, so that they may act as a single conductor when placed near to the electrified globe, and after induction has thus been produced, removing them separately, and examining their electrical states. If E be positive, N will be found negative, P positive, and M neutral.

Fig. 26.

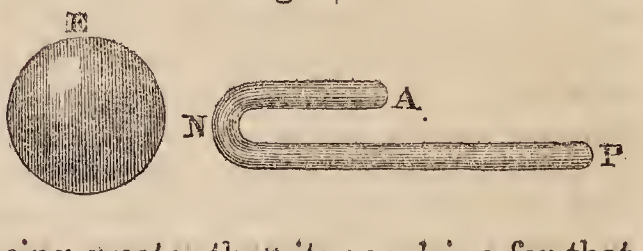


(106.) Another modification of effect will take place when an insulated conductor, rendered electrical at both ends by induction, is made to communicate with another conductor. Let us first suppose that a long metallic conductor is brought into contact with the remote end of the first cylinder P, (*fig*. 25), which has been rendered positive by induction. The fluid accumulated at this end will now pass into the conductor, and will remove to the most distant part of the conductor. The transit will now take place before actual contact, and will be manifested by the appearance of a spark when the bodies are brought within the striking distance. The removal of this fluid to a greater distance will occasion a disturbance in the equilibrium that had before been established. The repulsion which that fluid had excited, and which had contributed to prevent any more fluid from being propelled from the negative end N, is now considerably weakened by the greater distance at which it acts; and more fluid will leave the negative end, which end will consequently become more highly negative. This change of distribution will again occasion a further effect, by its reaction on the fluid in the globe whence the action originally proceeded; and another series of changes and adjustments will follow, until a new condition of equilibrium takes place, and then the fluid will be at rest.

(107.) Thus we learn that the effects of induction on a conductor are augmented by increasing its length; they would, therefore, be greatest of all, if we could give it infinite length: but the same condition is attainable by placing the conductor in communication with the earth, which will accordingly carry off all the fluid which the electrified body is capable of expelling from the nearest end. Accordingly, if we touch with the finger, or with a metallic rod held in the hand, the remote end of an insulated conductor under the influence of induction, we obtain a spark, more or less vivid according to the intensity of the electricity so induced; and the conductor so touched has now only one kind of electricity, namely, the one opposite to that of the electrified body which is acting upon it. The part touched is brought into a state, in which it appears to be neutral as long as it remains in the vicinity of the electrified body; because the actions of the redundant fluid, and unsaturated matter in the two bodies, exactly balance one another. But it all the while really contains less fluid than its natural share, in consequence of the repulsive tendency of the fluid in the body which produces the induction; and this negative state will readily become active, if the conductor that has been touched be again insulated, and then removed from the influence of the former. This peculiar condition of a body, in which its parts are really undercharged or overcharged with fluid, although, from the action of electrical forces derived from bodies in its vicinity, a state of equilibrium is established, and no visible effect results, has been denominated by Biot, *disguised electricity*.

(108.) It is also worthy of remark, that if the communication between the insulated conductor and another longer conductor, or the earth itself, be made at either end of the former, the same effect will result, and the electric fluid accumulated at its remote end will be carried off by the longer conductor, although, it will have, in one case, to pass round through the end nearest to the body which repels it. The operation which here takes place may be illustrated by the motion of a fluid in a syphon. A repulsive force is acting upon the fluid, both in the shorter and the longer column; but with regard to the motion of the fluid in the bent channel, the one force is in opposition to the other, and the tendency of the fluid in the longer column prevailing over that in the shorter, will draw off the latter, round the bend of the supposed syphon. Thus in the bent conductor A N P (*fig*. 27,), the repulsion exerted by the fluid in E for that in the longer column N P, being greater than its repulsion for that in the shorter column N A, the fluid in A will be carried over the bend N, notwithstanding its tendency to move from N towards A.

Fig. 27.



(109.) We have hitherto supposed the acting body to be positively electrified; but precisely the same effects would happen with regard to degree, although opposite as to the species of electricity, if it had been negatively electrified: and the same explanations will in every respect apply, with the requisite substitution of the terms negative for positive, and of attraction for repulsion, and *vice versâ*. A little reflection will also easily show the application of the theory of the double electricities to explain the same phenomena.

(110.) Another consequence of the induction of electricity must not be overlooked, namely, that the bodies between which it takes place, necessarily attract one another: for the action of the adjacent sides F and N (*fig*, 25), which are brought into opposite electrical states, is greater than the action of those sides which are in the same electrical states, F and P, and which are more distant: hence the attractive force always exceeds the repulsive. We have already seen that this circumstance sufficiently explains the fact that conducting bodies, previously neutral, are attracted by electrified bodies. Another fact, which appears more singular, and which cannot be accounted for on any other principle, is also a direct consequence of the law of induction. If a small body weakly electrified, be placed at a distance from another and a larger body, more highly charged with the same species of electricity, it will, as usual, be repelled; but there is a certain distance within which if it be brought, attraction will take place, instead of repulsion. This happens in consequence of the inductive influence producing so great a change in the distribution of electricity, as to give a preponderance to the attractive forces of the adjacent parts of the two bodies, over the repulsive forces that take place in the other parts, and which would have alone acted if the fluid had been immoveable.

(111.) From the principles now laid down, it will be easy to understand how induction may operate through a succession of conductors, which are all of them insulated, except the last; and which are separated from each other by distances greater than that at which a transfer of electricity would take place. If, under such circumstances, the first be electrified, alternate states of opposite electricities will be produced in the two ends of each conductor in succession. In all the ends nearest to the first body, the electricity will be of the opposite kind to that with which the first has been charged; in the other ends it will be of the same kind as that of the first body. The vicinity of these opposite electricities will tend powerfully to retain them in that condition, and will diminish their electric action on surrounding bodies. A large portion of the electricities so arranged and retained, is, therefore, in the condition designated by the term *disguised electricity.*

(112.) In proportion as the interruptions to the continuity of the line of conductors are more numerous, the more nearly will such a system approach to the condition of an imperfectly conducting body. The same principle admits of being extended, with some modifications indeed, to the constitution of electrics themselves, as we shall have occasion to notice in the sequel.

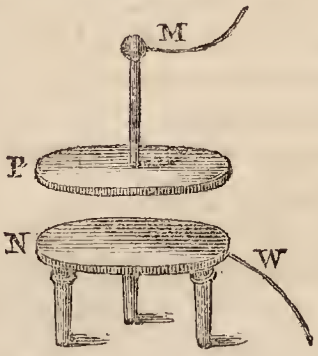
CHAPTER VIII.

Accumulation of Electricity by Induction.

(113.) THE most important application of the principle of induction is that by which a vast accumulation of electricity is obtained in a small space, while its intensity, or tendency to escape, is at the same time rendered exceedingly small. This condition exactly corresponds to that which has been termed *disguised electricity.*

(114.) Let two circular metallic plates P and N (*fig*. 28), be placed the one immediately over the other, but separated by a non-conducting medium, such as the air, or, what is still better, a plate of glass. Let the upper one P, communicate, by a wire M, with the prime conductor of the electrical machine; and let the lower one N, be insulated by resting upon three glass supporters. Let P be charged with a certain quantity of electric fluid. The fluid naturally contained in N, will be repelled by the fluid in P, and will quit the upper surface of N in order to occupy its lower surface. When this change has taken place, let N be touched by a wire W, establishing a communication between it and the ground. All the fluid which was accumulated in the lower surface of N, will be carried off by the wire, and the whole plate will thus be negative, or undercharged with fluid. The

Fig. 28



redundant matter in N will, by its attraction for the fluid, draw more of it into the upper plate, which will be supplied from the conductor of the machine through the wire M; and such an additional quantity will be accumulated in P, as will balance the increased attraction of the matter in N, and maintain it at the same intensity as the fluid in the prime conductor. That this is what really happens will be rendered evident by placing an electroscope upon the prime conductor; for the moment the plate N communicates with the ground, the balls of the electroscope collapse, showing that the intensity of the fluid in the prime conductor is suddenly reduced by the great quantity that has been absorbed by the plate P. The machine must now again be set in motion, in order to supply the electricity which has been thus abstracted from the conductor. The operation of each plate on the other may be considered as that of increasing its electrical capacity, or of rendering a large proportion of its electricity latent or disguised.

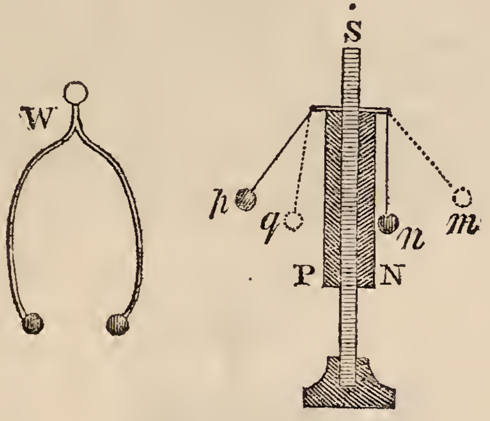
(115.) It is evident that the quantity of electric fluid driven out of the lower plate by the action of the fluid in the upper one, can never be quite equal to that of the fluid with which the upper one is itself charged, and the difference will be greater in proportion to the distance of the plates. When they are very close to each other, these two quantities approach very near to an equality; and this circumstance it was that misled Franklin into the belief that they were actually equal.

(116.) The capacity for accumulating electricity corresponding to a given intensity in the upper plate depends upon the distance between the plates, provided always that the intervening electric opposes a sufficient obstacle to the direct transfer of the electricity from the one to the other; and is in some inverse ratio to that distance. The lower plate, N, which communicates with the ground by the wire W, although strongly negative, is rendered, by the vicinity of the fluid in the upper plate P, neutral with respect to fluid in the wire W: that is, the attraction of its unsaturated matter, although nearer, is exactly balanced by the repulsion of the redundant fluid in the upper plate, which, although really stronger, is, from the greater distance at which it acts, only equal to the former. With reference to fluid in the wire M, however, the action of the redundant fluid in P, is not balanced by that of the unsaturated matter in N, which latter is both weaker in itself and more distant. Thus, while N is neutral with respect to the conductors which touch it, P is in a slight degree active, in consequence of this small preponderance of force, and a portion of its fluid tends to escape. Hence, if N be again insulated, by removing the wire W, and the wire M be now made to communicate with the ground, this portion of the fluid in P will pass off by it; but not any larger quantity, for the remaining portion is retained by the attraction of the unsaturated matter in N. P is, by this loss, rendered neutral, as N had before been, and it now no longer acts on the fluid beyond it in M. The influence of P on that fluid is greater than that of N in respect to its greater vicinity, but less in as far as regards the intensity of action, and the compensation is exact. But under these circumstances, N, which was before neutral, becomes in its turn active, and now that the repulsion of the fluid in P is diminished, will absorb a certain quantity of the fluid as soon as it is touched by W, after P has been again insulated. By this contact, N is again restored to the neutral state, a fresh portion of fluid in P is released from the attraction of N, and P is again active. By repeating these alternate contacts a sufficient number of times, we gradually deprive the plates of their whole charge of electricity; alternately imparting small portions to the negative plate, and taking away the like portions from the positive one, until they are both brought to their natural unelectrified state. The quantities of fluid which are thus successively added and abstracted were found, by the calculations of Laplace, to be in geometrical progression.

(117.) The most convenient mode of obtaining the accumulated electricity arising from induction is by the employment of *coated glass*, that is, of a plate of glass, on each side of which is pasted a sheet or coating of tin-foil. Care must be taken to leave a sufficient margin of glass uncovered by the metal, for preventing the transfer of electricity from the one coating to the other round the edge of the glass; and all sharp angles, or ragged edges in the coatings, should be avoided, as they have a great tendency to dissipate the charge.

(118.) The following experiment of Professor Richman, (the philosopher who fell a sacrifice to his zeal for electrical science by a stroke of lightning from his apparatus,) is very instructive. Let a pane of glass placed vertically, and seen edgewise in *fig.* 29, be coated on both sides, and furnished with two small electroscopes, *p*, *n*, consisting of two pith-balls, one attached to each of the coatings. Let the coating P be charged positively, while the coating N is made to communicate with the ground. The electroscope *p* will stand out from the plate, and *n* will hang down close to its coating, as long as N communicates with the ground. But in proportion as P loses electricity by gradual dissipation in the air, the ball *p* will gradually, but very slowly descend. If we now insulate N, *p* will fall down at first very speedily, and then more slowly, till it reaches *q*, about half its first elevation. The ball *n* will at the same time rise to nearly the same height; the angle between the two electroscopes continuing nearly the same as at first. When *n* has ceased to rise, both balls will very slowly descend, till the charge is lost by dissipation. If we touch N during this descent, *n* will immediately fall down, and *p* will as suddenly rise nearly as much; the angle between the electroscopes continuing nearly the same. Remove the finger from N, and *p* will fall, and *n* rise, to nearly their former places; and the slow descent of both will again recommence. The same thing will happen if we touch P, *p* will fall down close to the plate, and *n* will rise to *m*, and so on; and this alternate touching of the coatings may be repeated some hundreds of times before the plate is entirely discharged. If we suspend a crooked wire, bent, as shewn at W, (*fig.* 29,) having two pith-balls, from an insulated point, *s*, above the plates, it will vibrate with great rapidity, the balls striking the coatings alternately, and thus restoring the equilibrium by steps; each contact being attended by a spark.

Fig. 29.



(119.) If, instead of this gradual discharge, a direct communication is made between the two coatings by a metallic wire extending from the one to the other, the whole of the electric fluid which was accumulated in the positive coating rushes with a sudden and violent impetus along the conductor, and passes into the negative coating, thus at once restoring an almost complete equilibrium, and rendering every part very nearly, though not absolutely, neutral; for as there must always be some slight difference in the quantity of electrical charge in the two coatings, where one of them is in communication with the ground, there must always be a certain excess, however minute, of electricity, after the balance has been struck.

(120.) This sudden transfer of a large quantity of accumulated electricity is a real explosion; it gives rise to a vivid flash of light, corresponding in intensity to the magnitude of the charge. The effect of its transmission is much greater than that of the simple charge of the prime conductor of the machine; for while the latter gives a spark only, the former imparts what is called *an electric shock*, and the sensation it produces when passing through any part of the body is of a peculiar kind. We shall describe their effects in a future chapter; at present we must confine our attention to the purely electrical conditions of the phenomenon.

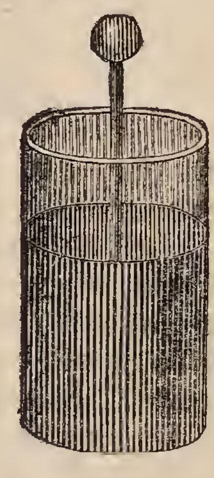
(121.) The presence of the coating is not absolutely essential to the charge and discharge for the two surfaces of the glass plate; for if the glass be furnished with moveable coatings, and charged in the usual manner, upon removing the coatings (taking care that they be touched only by electrics,) the greater part of the electricity will be found to have attached itself to the surfaces of the glass plate, where they are retained by their mutual inductive influence. In this state the charged plate of glass may be gradually discharged by making a communication between its several parts in succession. It cannot be discharged at once, for want of a common intermedium for the simultaneous transference of the electricity of the different parts of the surface. But if this be supplied by replacing the former coatings, or adding new ones, the complete discharge may be effected as before.

(122.) By peculiar management a charge may be given to a plate of glass independently of any coating whatever. For this purpose, it must be held by one corner, and passed before a ball, connected with the prime conductor of a machine, so that it may successively come in contact with every part of the middle of the plate of glass, while the finger, or any conducting body communicating with the ground, is held opposite to it on the other side. Thus the glass will be charged, and will be in the same state as the glass from which the coatings had been removed.

(123.) We often find, a short time after the discharge of coated glass, that it has acquired spontaneously a small charge, producing a faint spark when a second communication is made between the coatings by the discharging wire. This, which is called the *residual* charge, arises from two causes: first, a portion of the electricity adheres to the uncoated surface of the glass: and secondly, another part has penetrated from the coating for some little depth below its surface. Both these portions slowly return to the coatings after they have been deprived of their original charge, and give it a fresh charge. When a very large extent of coated glass is employed, this residual charge may even amount to a considerable quantity, and the experimenter should be cautious not to expose himself to the shock which he might thus receive, if he inadvertently touched the apparatus before he had properly discharged it. That charges are capable of penetrating even through the entire thickness of the glass is proved by the curious fact, that a coated, cylindrical jar may be discharged merely by keeping up for a sufficient time a continuance of the minute vibrations excited by rubbing it with the finger, or by making it ring. A discharge may also be effected by heating the glass, which renders it a conductor of electricity.

(124.) The most convenient form for coated glass for experimental purposes, is that of a cylinder or jar. In the earlier periods of electrical research, jars were filled with water, mercury, or iron filings, which furnished the interior coating, while the exterior coating was supplied either by water, in which the jar was immersed, or by the hand of the operator, who for that purpose grasped the outside of the jar: a rod of metal was employed to communicate the charge from the prime conductor of the machine to the inner coating. On making a communication between the exterior and interior coatings, by means of a circuit of conducting substances, the discharge took place, and the shock made to pass through the circuit thus formed. This instrument having been made known principally through the experiments of Kleist, Cuneus, and Muschenbroeck, at Leyden, the name of the *Leyden phial*, or *jar*, was generally applied to it. It is at present constructed as shewn in *fig*. 30, by applying coatings of tin-foil on both sides of the jar or bottle, leaving a sufficient space uncovered at its upper part to secure it from the risk of a spontaneous discharge, which might take place if the coatings were not separated by a sufficient interval. A metallic rod, rising two or three inches above the jar, and terminating at the top in a brass ball, which is often called the *knob* of the jar, is made to descend through the cover, till it touches the interior coating. It is through this rod that the charge of electricity is conveyed to the inner coating, while the outer coating is made to communicate with the ground. We have already seen, that if this last condition be not observed, the inner coating can receive no charge, and only a feeble spark will pass from the conductor to the knob.

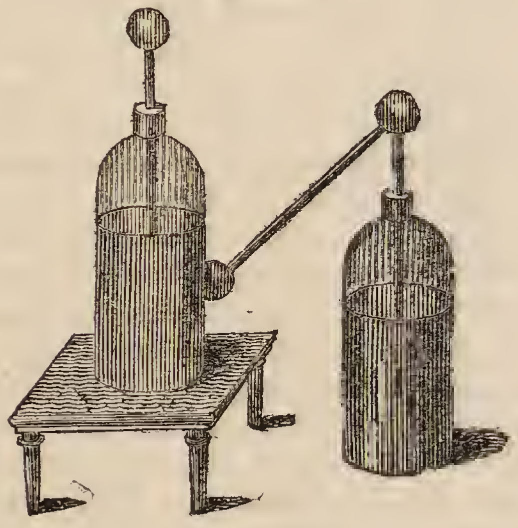
Fig. 30.



(125.) The outer coating may be made to communicate with the ground by holding it in the hand; and on presenting the knob of the jar to the prime conductor when the machine is in motion, a succession of sparks will pass between them, while at the same time nearly an equal quantity of electricity will be passing out from the exterior coating, through the body of the person who holds it, to the ground. If, instead of this, the jar be placed on an insulating stand, and a ball of metal, or the knuckle of the finger, be held near the outside of the jar, we have evidence of the escape of the electricity from the latter by a succession of sparks simultaneous with those that occur between the prime conductor and the knob of the jar.

(126.) If, instead of touching the outer coating of a jar supported on an insulating stand, we bring into contact with it the knob of a second jar, of which the outer coating communicates with the ground, as shewn in *fig.* 31, the electricity which is expelled from the outer coating of the first jar passes into the inner coating of the second jar, and thus both jars are charged. Thus may charges be given to a succession of jars, so placed as that the inner coating of each shall communicate with the outer coating of the one that precedes it in the series; taking care that the outer coating of the last jar communicates with the ground. All the jars will be found to be charged in a similar manner. It is evident, however, that the charge must diminish in intensity as it is conveyed from each jar to the next, because the quantity of electricity which is expelled from the exterior is never quite equal to that which passes into the interior.

Fig. 31.



(127.) For the sake of greater distinctness we have all along supposed the interior of the jar to be charged with positive electricity, but the very same effect would take place if the knob of the jar were charged negatively by communication with the negative conductor. A similar change in the electrical state of the coatings would result from placing the jar on an insulating stand, and then forming a communication between the outer coating and the prime conductor, while the knob is made to communicate with the ground. The only difference is, that the outer coating would then be active and the inner one neutral; but these conditions would again be reversed as soon as the knob was disconnected with the ground, and the outer coating touched with the hand.

(128.) If two jars, the one charged positively, the other negatively, be placed on two separate insulating stands, and their knobs then connected by a conductor, which is itself insulated, no explosion will take place, although the two coatings, which are thus brought into communication, are in opposite electrical states. But if the two outer coatings be at the same time connected, an explosion will take place, and both jars will be discharged.

(129.) Since the susceptibility of receiving a charge depends upon the proximity of the metallic surfaces, while the passage of the electricity from the one to the other is interrupted by the interposition of a non-conducting substance, it is evident that, in the construction of the Leyden jar, the thickness of the glass is an important consideration. The thinner the glass, the greater will be the power of taking a charge; but the power of retaining the charge will be less, on account of the diminished resistance which the glass will afford to the passage of the electricity through it. If the charge be higher than what the jar will bear, the glass will be broken by the violence with which the electricity forces a passage through its substance. Muscovy talc, even in very thin laminæ, resists much better than glass, and is, therefore, capable of receiving and of retaining a much higher charge. Another limit to the charge which a jar is capable of retaining, arises from the liability of the electricity to pass from one coating to the other, round the edges of the glass.

(130.) These spontaneous discharges, as they are called, are facilitated by the deposition of moisture on the glass, forming a chain of conducting particles in the very line which the electricity has a strong tendency to take. Hence, it is a requisite precaution to keep the apparatus in as dry a state as possible; and the deposition of moisture may be guarded against most effectually by covering the uncoated part of the glass with a layer of sealing-wax, or other resinous varnish. The liquid should be applied with a flat, camel-hair pencil, the glass being previously warmed.

On the other hand, it is a curious circumstance, that there is a degree of humidity in the inside of the jar, not only compatible with a high charge, but which even contributes to retain it. This effect was accidentally observed by Mr. Brooke, and afterwards by Mr. Cuthbertson, who states that a jar will take a much greater charge, namely, one-third more, if its inside be considerably damped by blowing into it with the mouth through a tube reaching to the bottom. The explanation of this remarkable fact has been given by Professor Robison on the principles formerly explained, namely, that there is no electric intensity so great, but that it may be insulated by the least imperfect conductor, provided the latter be long enough, and so constituted as that the intensity of the electricity it contains shall diminish by sufficiently gentle gradations. An uniform dampness, indeed, will not do this; but it will diminish the abruptness of the variations of intensity, and thus give security against a spontaneous discharge. A similar protection against the breaking of the glass is afforded by placing a layer of paper between the glass and the tin-foil, and making it extend also an inch beyond the coating.

(131.) Glass balloons of a spherical shape, being of more uniform thickness than jars, would be much preferable for the construction of an apparatus of this kind, were it possible to apply an uniform coating to the inside. Professor Robison recommends the following construction for a portable jar, which he found to answer exceedingly well. A long-necked phial was made of sheet tin, and then coated entirely on the outside with fine sealing-wax, one thirtieth of an inch thick. The sealing-wax was then coated with tin-foil, all but the neck. It is evident, that the wax here acts the part of the glass in the common jar, the tin plate corresponding to the inner coating and wire, and the tin-foil to the outer coating. The dissipation is almost nothing if the neck be very small; and it only requires a little caution to avoid bursting by too high a charge. Even this may be prevented by coating the sealing-wax so near to the end of the neck, that a spontaneous discharge must happen before the accumulation is too great. Alternate layers of tin-foil and hard varnish form also a very compendious battery. It admits of a surprising accumulation, without shewing any vivid electricity; but it must be used with more caution, lest it should be spoiled by a spontaneous discharge, in which case we cannot discover where the flaw has happened, and the whole is rendered useless.

(132.) By combining together a sufficient number of jars we are able to accumulate an enormous quantity of electricity: for this purpose all the interior coatings of the jars must be made to communicate by metallic rods, and a similar union must be established among the exterior coatings. When thus arranged, the whole series may be charged, as if they formed but one jar; and the whole of the accumulated electricity may be transferred from one system of coatings to the other, by a general and simultaneous discharge. Such a combination of jars is called an *Electrical Battery.*

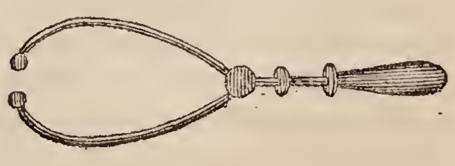
(133.) It is evident, that an apparatus of this kind, consisting of a great number of parts, must be more liable to derangement than a single jar: for if any one of the jars should happen to break by a spontaneous explosion, the whole battery would be rendered useless, until the broken jar be removed. It is prudent, therefore, to secure the adjacent jars from actual contact, by fixing them in a box having thin partitions; the coated bottoms of the jars resting on a trellis of wire, or on a sheet of tin-foil, which may establish a general communication between them; while the rods from the interior coatings are connected above by cross wires, having balls at their extremities in order to obviate the dissipation of the electricity. On the other hand, by limiting the communications to a certain number of jars, we have it in our power to charge only a part of the battery, without employing the whole.

CHAPTER IX.

Management of Electrical Jars and Batteries.

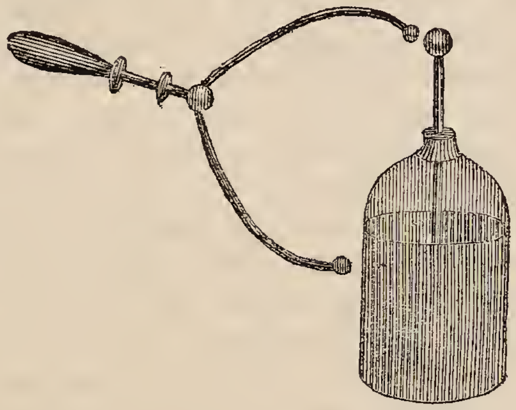
(134.) FOR the purpose of making the direct communication between the inner and outer coating of a jar or battery, by which a discharge is effected, the instrument shown in *fig.* 32, and which is

Fig. 32.



called the *Discharging Rod* or *Jointed Discharger*, may be conveniently employed. It consists of two bent metallic rods, terminated at one end by brass balls, and connected at the other by a joint, which is fixed to the end of a glass handle, and which, acting like a pair of compasses, allows of the balls being separated at different distances. When opened to the proper degree, one of the balls is made to touch the exterior coating, and the other ball is then quickly brought into contact with the knob of the jar, as represented in *fig*. 33, or with any part of the system of the interior coatings, and thus a discharge is effected; while the glass handle secures the person holding it from the effects of the shock.

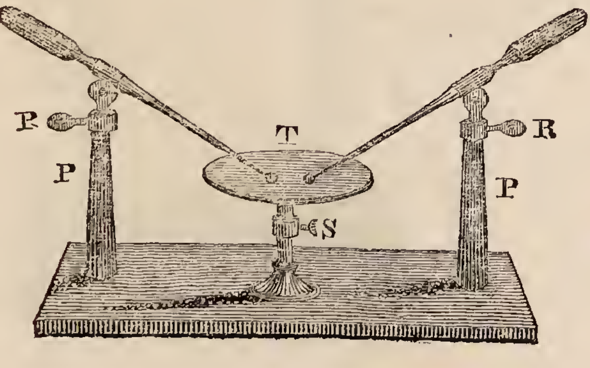
Fig. 33.



(135.) If we wish to send the whole charge of electricity through any particular substance which may be the subject of experiment, we must so arrange the connecting conductors, as that the substance shall form a necessary part of the *circuit of the electricity*, as it is termed. With this view, we must place it between two good conductors, one of which is in communication with the outer coating; and the circuit may then be completed by connecting the other conductor with the inner coating by means of a discharging rod, to one branch of which, if necessary, a flexible chain may be added.

(136.) In order to direct the charge with more certainty and precision, an apparatus, called the *Universal Discharger*, was contrived by Mr. Henley, and is represented in *fig*. 34. It consists of a wooden stand with a socket fixed in its centre, to which may be occasionally adapted a small table T, having a piece of ivory (which is a non-conductor) inlaid on its surface. This table may be raised and kept at the proper height by means of a screw S. Two glass pillars P, P are cemented into the wooden stand. On the top of each of these pillars is fitted a brass cap, having a ring R attached to it, and containing a joint, moving both vertically and horizontally, and carrying on its upper part a spring tube, admitting a brass rod to slide through it. Each of these rods is terminated, at one end, either by a ball, a point, or a pair of forceps, and is furnished at the other extremity with a handle of solid glass. The body through which the charge is intended to be sent, is placed on the table, and the sliding rods, which are moveable in every direction, are then, by means of their insulating handles, brought in contact with the opposite sides, and one of the brass caps being first connected with the outside of the jar or battery, the other may be brought in communication with the inner coatings, by means of the discharging rod above described. For some experiments it is more convenient to fix the substance, on which the experiment is to be made, in a mahogany frame, consisting of two boards, which can be pressed together by screws, and which may then be substituted for the table T. In either of these ways the charge can be directed through any part of the substance with the greatest accuracy.

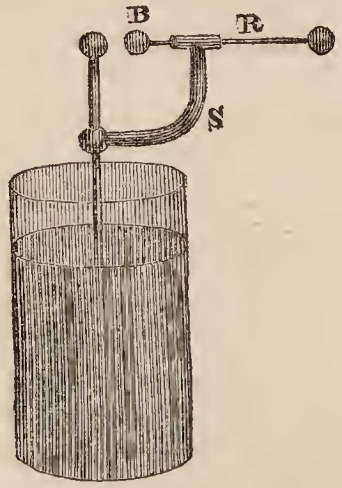
Fig. 34.



(137.) The quantities of electricity which can be accumulated in any given extent of coated glass, are in the inverse proportion to the thickness of the glass. Different jars or batteries, therefore, will, according to the thinness of their sides, and the quantity of coated surface they contain, have different capacities of holding charges of electricity. But in any given instrument of this kind, the quantity of the charge communicated to it by a machine may be measured by the intensity of the electricity in the prime conductor, which communicates with the interior coating. Some estimate of the intensity may be obtained by the employment of Henley's quadrant electrometer already described, (§ 72,) the index of which rises very slowly while the battery is charging, till it reaches a certain elevation, corresponding to the capacity of the battery. If the electricity be accumulated beyond this limit, a spontaneous discharge takes place, and the process must then be renewed in order to obtain a full charge. It is more prudent, however, to stop before this degree of accumulation is attained: and one great advantage of Henley's electrometer is, that it shows us the progress of the charge, and how far we may proceed with safety.

(138.) But the most effectual security against fracture from a spontaneous discharge, is to form an interrupted circuit, of which the parts, where the interruption occurs, terminate by metallic balls, placed at a certain distance from each other. By varying the interval between them, we may regulate the quantity of electricity which we shall allow to accumulate in the battery; for the moment it exceeds the quantity of which that interval is the *striking distance* (§ 87,) an explosion happens, by the electricity forcing its way through the air from one ball to the other. If the balls be brought very near each other, a discharge will take place with a comparatively small accumulation: when farther separated, a greater charge will be retained, because a higher intensity of electricity is required in order to pass through the larger intervening space. It is on this principle that the instrument, called *Lane's Discharging Electrometer*, is constructed. It consists of a brass ball, B, *fig*. 35, placed at the end of a short metallic rod R, which moves through a tubular piece, supported by a bent glass stand S. This stand is made so as to be capable of being fixed, by its other extremity, to the rod passing up from the interior coating, and adjusted so that the ball B is immediately opposite to the knob of the jar, and may be brought to the exact *striking distance* from it which may be required: the other end of the moveable rod must be connected, by means of a chain or wire, with the outer coating.

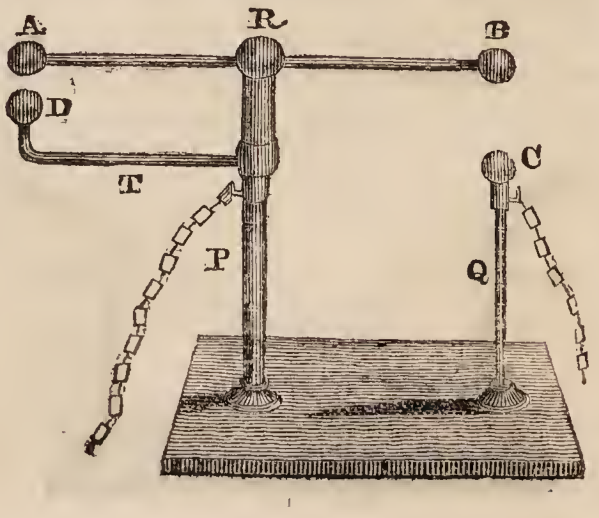
Fig. 35.



The chief use of this instrument is to allow a jar to discharge itself spontaneously through any previously arranged circuit, without employing a discharging rod, or moving any part of the apparatus; and also to produce successive explosions nearly of the same strength. The magnitude of the charge is measured by the distance at which the balls are placed; and the power of the machine may be estimated by the number of explosions, which, at any given distance, take place in equal times. In Mr. Lane's experiment the shocks were twice as frequent when the interval between the balls was 1-24th of an inch, as when twice as much: hence he concluded that the quantity of electricity required for a discharge is in exact proportion to the distance between the surfaces of the balls. But the indications of this instrument are in reality subject to great fallacy, on account of the variable state of the atmosphere, which affects its conducting power; the quantity of dust which, even during the course of an experiment, is liable to be attracted, and to collect upon the balls; and also from the roughening and tarnishing of the metallic surfaces produced by frequent electric explosions. This last imperfection is one to which brass balls are particularly exposed; and might, if it were worth while, be remedied by having the balls made of fine silver.

(139.) Another contrivance for regulating the amount of the charge which we may wish to send through any substance, is that invented by Cuthbertson, and termed the *Balance Electrometer*. It consists of a metallic rod, R, *fig*. 36, terminated by two equal balls A, B, and balanced, like a scale-

Fig. 36.



beam, upon knife-edged centres. One of the arms of this beam is graduated, and carries a slider, which, when set at different distances from the centre of motion, acts on the lever with a proportionate weight from one grain to sixty. The ball A, at the extremity of this loaded arm, rests on a similar ball D, below it, which is supported by a bent metallic tube T, proceeding from the same stand as that which supports the rods; the whole being insulated by a glass pillar P. At a little distance below the ball B, at the other extremity of the beam, another ball C, insulated by the glass pillar Q, is placed; this last ball is to be connected by a chain with the outer coatings of the battery, while the metallic support of the balance is connected with the inner coatings. When a charge is communicated to the battery, the two balls A and D, which are in contact, become repulsive of each other; and when the force of this repulsion is sufficient to raise the weight on the loaded arm of the beam, the other arm will be forced down, and the ball B coming in contact with the ball C, the circuit will be completed and a discharge take place. As the force of the repulsion depends upon the intensity of the charge, the weight it has to overcome affords a measure of this intensity, and enables us to regulate its amount.

The practical application of accumulated electricity to various purposes of experiment, involves considerations which relate to the laws observed by electricity in its movements, and which more properly belong to the subject of the ensuing chapter.

Chapter X.

Of the Motion of accumulated Electricity.

(140.) In forming arrangements for directing the passage of accumulated electricity, it should be borne in mind that the electric fluid will, on these occasions, always pass through the best conductors, although they may be more circuitous, in preference to those which are more direct, but have inferior conducting power: and it must also be recollected, that when different paths are open for its passage, along conductors of equal power, the electricity will always take that which is the shortest. Thus if a person, holding a wire between his hands, discharges a jar by means of it, the whole of the fluid will pass though the wire, without affecting him: but if a piece of dry wood be substituted for the wire, he will feel a shock; for the wood, being a worse conductor than his own body, the charge will pass through the latter, as being the easiest, although the longest circuit. During its transit through the human body, in like manner, the shock is felt only in the parts situated in the direct line of communication; and if the charge be made to pass through a number of persons who take one another by the hand, and form part of the circuit between the inner and outer coatings of the jar, each will feel the electric shock in the same manner and at the same instant; the sensation reaching from hand to hand, directly across the breast. By varying the points of contact, however, the shock may be made to pass in other directions, and may either be confined to a small part of a limb, or be made to traverse the whole length of the body from head to foot.

(141.) By accurate experiments it appears that the force of the electric shock is weakened, that is, its effects are diminished, by employing a conductor of great length for making the discharge. But it is difficult to assign a limit to the number of persons through which even a small charge of electricity may be sent, so that all shall experience the shock; or to the distance along which it may be conveyed by good conductors. At an early period of electrical inquiries, much interest was attached to the determination of these points. The Abbé Nollet passed an electrical shock from a small phial through a hundred and eighty of the French guards in the presence of the king; and at the Carthusian convent in Paris, the monks were formed into a line of above a mile in length, by means of iron wires held between them: on the discharge of the phial, the sensation was felt at the same moment by all the persons composing this extensive circuit. Many experiments were made both by the English and French electricians with a view to ascertain the space which a discharge can be made to traverse, and the velocity with which it is transmitted. Of these the most ingenious and satisfactory were the experiments planned and executed by Dr. Watson, with the assistance of the leading members of the Royal Society. A circuit was formed by a wire which extended the whole length of Westminster bridge, at a considerable height above the river: one end of this wire communicated with the outer coating of a charged phial, the other being held by a person on the opposite side of the river, who formed a communication with the water by dipping into it an iron rod held by the other hand. The circuit was completed by another person, who stood near the phial, and who likewise dipped an iron rod into the river with one hand, and was enabled, by means of a wire held in the other, to effect a contact with the knob of the phial. Whenever the discharges took place, the shocks were felt by both persons; thus proving that the electric fluid must have been in motion along the whole line of the circuit, including both the wire above and the river below.

In another experiment, made on Shooters'-hill, at a time when the ground was remarkably dry, the electricity was made to perform a circuit of four miles; being conducted for two miles along wires supported upon baked sticks, and for the remaining distance, also of two miles, through the dry ground. As far as could be ascertained, by the most careful observation, the time in which the discharge was transmitted along that immense circuit was perfectly instantaneous: nor has any other trial that has yet been made afforded the least approach to a measurement of the velocity with which electricity moves.

(142.) On this subject, however, an important distinction should be made between the actual movement of each individual particle of electric fluid, and the transmission of an impulse along a series of such particles, for the one may bear hardly any proportion to the other: just as we find that sound proceeds with a velocity incomparably greater than that of the particles of air which are concerned in its propagation. In like manner the portion of blood, which raises the artery at the wrist, where the pulse is felt, is not the identical portion of blood which is thrown out from the heart by the contraction of that organ producing that pulsation: the impulse, in all these cases, being propagated like a wave, from one particle to another. There is, therefore, no reason to suppose that the same particles of electric fluid, which enter at one part, have traversed from one end to the other the whole line of conducting substances which form the circuit.

(143.) If we conceive the conducting bodies which compose the circuit to be divided into an indefinite number of filaments, every one of which is capable, in an equal degree, of conveying the electric fluid, it is evident that the united power of these filaments, or what is the same thing, the capability of the body itself to convey a charge of electricity, is in proportion to the number of these elementary filaments which it contains, that is, to the magnitude of its transverse section, without any relation to its form. Thus, the same metallic rod will conduct a charge equally well, whether it be flattened, or divided into several smaller wires, or whether it consist of a single cylinder of the same area.

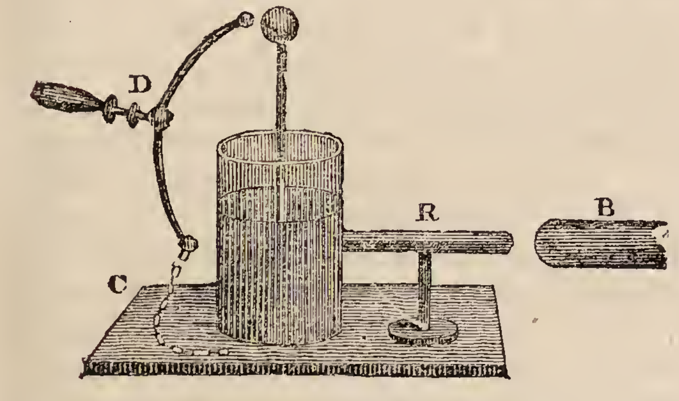
(144.) If the size of the conductor be sufficiently great, the whole charge may be conveyed without any sensible obstruction or retardation, and therefore without any tendency to deviate from the direct line of its course. But it is otherwise when the conductor is too slender to afford a ready passage to the fluid which is pressing onwards: and it is important to inquire into the consequences to which these obstructions may give rise.

(145.) The first effect of an impediment to the free passage of accumulated electricity must be a retardation of its motion. It is reasonable, therefore, to expect that with a circuit composed either of bad conductors, or of conductors of inadequate size, although good, the discharge will not be effected so instantaneously, nor so completely; and that the shock which accompanies it will be diminished in its violence. This principle may find its application on occasions where it is desirable to soften the intensity of the shock, as in the medical employment of electricity, where imperfect conductors are on this account sometimes preferable, both for taking sparks and shocks.

(146.) A second effect resulting from an obstruction to the flow of electricity, is a tendency in the fluid to diverge from the direct line of its course, and to fly off to different objects in the vicinity. This is frequently exemplified in the case of lightning, which, on striking a building, is apt to take a very irregular and seemingly capricious route, darting towards conducting bodies which may happen to attract it, although at some distance from the immediate direction it was pursuing. The position of such conducting bodies would appear to have a material influence in determining the striking distance. It was remarked by Dr. Priestley, that the explosion from a large battery extends to a greater distance over the surface of water than in air alone.

(147.) An effect which seems to depend upon this tendency in the fluid to divergence in consequence of obstruction, although it has by some been referred to a different principle, is that which has been termed *the lateral explosion.* When a large jar or battery is discharged by a metallic wire which is held in the hand without the protection of any glass or other insulating handle, it often happens that a slight shock is felt in the hand that grasps the wire, especially if the charge of electricity be very considerable. This apparent divergence or overflow of electric fluid, when rushing in large quantities through a narrow space barely sufficient to contain it, may also be rendered visible in other ways. If one end of a chain be connected with the outer coating of a charged jar, while the remainder of the chain is lying loosely upon a table, on discharging the jar in a darkened room, by a discharging rod, in the usual way, it will be found that the chain, although it makes no part of the circuit, is rendered luminous by the passage of sparks from one link to another. The following experiment, made by Dr. Priestley, may also be regarded as a case of lateral explosion. Let a thick metallic rod R, *fig*. 37, be supported on an insulating stand, and placed with one of its ends in contact with the outer coating of a Leyden jar; and at a distance of half an inch from its other extremity place a long conducting body B, of at least six or seven feet in length, and only a few inches in breadth. Let a chain C, be now placed upon the table, so that one of its ends may be about an inch and a half distant from the outer coating of the jar, and apply one end of the discharging rod D, to the other extremity of the chain. As soon as the other ball of the discharging rod is made to touch the knob of the jar, so as to effect a discharge, a brilliant spark is seen to extend between the insulated rod R, and the adjacent conductor B. This lateral spark has the same length and brilliancy whether it be received on flat or smooth surfaces, or on sharp points.

Fig. 37.



It is stated by Dr. Priestley, that the effect we have been describing takes place without any apparent change in the electrical state of the conductor B; and hence Cavallo conceived that the lateral spark was sent out from the jar, and returned to it almost at the same instant, allowing of no perceptible time for an electrometer to be affected. Dr. Robison, however, always observed, on repeating the experiment, that a very delicate electrometer was affected under these circumstances: and the same observation is confirmed by Biot.

The phenomena of the lateral explosion have been attempted to be explained by the electricity exerting, during its passage, an inductive influence, of which the effects may be expected to cease the moment the cause is removed. But this explanation appears to be less satisfactory than the one which attributes the phenomena to an expansive propulsion, followed by an immediate recession of electric fluid, produced by obstructions to its free passage in the circuit of conductors.

Chapter XI.

Effects of Electricity upon Bodies.

(148.) Having considered the circumstances attending the motion of electricity with reference chiefly to the fluid itself, we next proceed to give an account of the effects which it produces upon bodies by its passage through them.

(149.) Independently of electrical attraction and repulsion, it does not appear that the simple accumulation of electricity in any quantity in bodies, as long as it remains quiescent, produces the least sensible change in their properties. A person standing upon an insulating stool may be charged with any quantity of electricity from a machine, without being perceptibly affected, until the equilibrium of the fluid is disturbed, by drawing sparks from his body, or from the prime conductor with which he may be in communication.

We have already seen, indeed, (§ 78, 79, 80,) that it is only a very small part of an electrified body, namely, the mere surface, that is in an active state, either of positive or negative electricity, and that the rest of the substance of the body is in a state of perfect neutrality.

(150.) It also appears that the uninterrupted passage of any quantity of electricity through a perfect conductor, such as a rod of metal which is of sufficient thickness to convey it, occasions no perceptible alteration in the mechanical properties of the conducting body.

(151.) On the contrary, very considerable effects are produced when a powerful charge is sent through a wire, which from the smallness of its size will not admit of the whole quantity to pass with perfect freedom; or through a substance which, although large, is deficient in conducting power; or, in other words, which opposes a degree of resistance to the passage of electricity. Thus, an iron conductor will carry off the whole electricity of a thunder-cloud in safety and in silence, while a beam of wood, or a tree, struck by lightning, is shivered into a thousand fragments.

(152.) When electricity thus changes the physical properties of bodies, its operation may, in general, be referred to that of separating their particles in the line of its course. This separation is effected with more or less violence, according to the intensity and quantity of the charge, and is frequently attended by the evolution of heat and light. The mechanical effects of electricity resemble those which would be produced by a material agent driven with great velocity and force through the substance of the body. Some of these effects, on the other hand, seem to be the consequences of the expansion produced by heat; but many of the changes induced by electricity are of a chemical nature, and such as mechanical agencies alone are insufficient to explain. We proceed to describe these several effects more particularly.

§ 1. *Mechanical Effects of Electricity.*

(153.) The cohesion of the particles of solid bodies may be conceived to oppose some resistance to the tendency of electricity to separate these particles from one another; for we find that fluids are more violently acted upon than solids, by the passage of the electric discharge. If the stem of a capillary tube, such as is employed for making thermometers, be filled with mercury, and placed so that the filament of this metal forms part of the circuit; on the discharge being made, the glass tube will be burst, and its fragments, together with the mercury, will be completely dispersed. If a fluid of inferior conducting power, such as water, be contained in a tube of larger diameter than in the preceding experiment, the passage, even of a moderate charge, will be sufficient to break the tube, and scatter its contents. Oil, alcohol, and ether, oppose still greater resistance than water to the passage of electricity, and they are expanded and scattered with still greater violence by a discharge being made to pass through them.

(154.) Beccaria introduced two wires through holes in the opposite sides of a perforated ball of solid glass of two inches diameter, the ends of the wires being separated by a drop of water, which occupied the centre of the perforation. On passing a shock through the wires and intervening drop, the ball was shattered with great violence. By a similar arrangement, Mr. Morgan succeeded in breaking green glass bottles filled with water, when the distance of the wires between which the explosion passed exceeded two inches. In this way, also, glass tubes, half an inch thick, with a bore of the same diameter, were burst with a very moderate charge, in Mr. Singer's experiments. If a cup-like cavity be turned in a piece of ivory, capable of receiving the half of a light wooden ball, with a small conical cell at the bottom of the cavity, and two wires be inserted into it through the sides of the ivory; on putting a drop of water, alcohol, or ether between the wires, and placing the ball over them in its cavity, and sending a charge through the drop of fluid, part of it will be suddenly converted into vapour, and the ball will be propelled with great violence. Even a common drinking glass, filled with water, may be broken by the explosive force with which vapour is formed at the point where the electricity passes. Beccaria constructed a small mortar with a ball, behind which a drop of water was placed, so as to be between the two wires that passed through the sides of the mortar. The charge being sent through the two wires, the drop of water was expanded with such force, as to drive out the ball with great velocity Mr. Lullin, of Geneva, found that, by using oil instead of water in this experiment, the ball was projected with still greater force.

(155.) If two wires be introduced into a soft piece of tobacco-pipe clay, so that their ends be near each other, and a shock passed through them, the clay will be curiously expanded in the interval between the wires. The experiment will not succeed if the clay be either too dry or too moist. If the clay be too dry, or the shock too powerful, the mass will be shivered into innumerable fragments. If the clay be placed in the tube of a tobacco-pipe, or in a glass tube, the expansion of the clay will be so considerable as to shatter the tube which con tains it.

(156.) The expansion of air by the passage of the electrical fluid, either in the form of sparks or shocks, is shown in the following experiment of Kinnersley, the apparatus for which has been called the *Electrical Air Thermometer.* It consists of a glass tube closed at both ends by air-tight brass caps, through which two wires slide in the direction of the axis of the tube. These wires are terminated by brass balls, which are made to approach within the striking distance. To an aperture in the bottom of the lower cap is fitted a bent tube of glass which turns upwards, and is open at both ends; the bent part is filled with mercury, or with a coloured fluid, which may indicate by its rising or falling in the tube any dilatation or contraction that may take place in the air within the vessel. It is found that every time a spark passes between the brass balls, the fluid suddenly rises, but descends again to its former level immediately after each explosion; thus showing that the dilatation of the air, produced by the abrupt passage of electricity, is but of momentary duration.

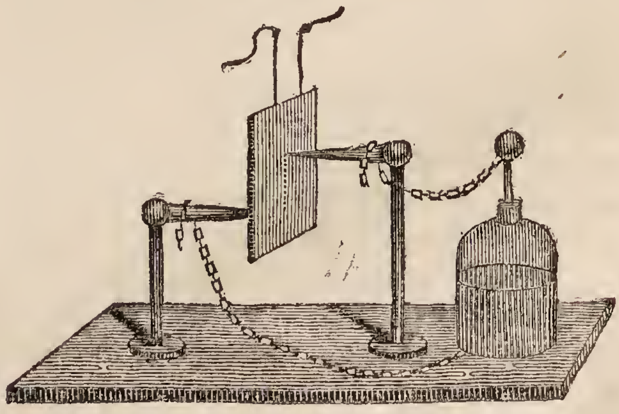
(157.) When a strong electrical charge is sent through a very confined portion of air, the explosive effects produced by it are as considerable as those we have seen exhibited by denser fluids. Thus if a piece of plate glass of the size of a square inch, and half an inch in thickness, be laid flat upon the small table of Henley's universal discharger, (§ 136,) and pressed down by a weight, and the points of the sliding wires be set opposite to each other and against the under edge of the glass, so that the electricity may pass beneath it, the charge of a large jar transmitted in this way will break the glass into innumerable fragments, and even reduce a portion into an impalpable powder. If the mouth of a small mortar made of ivory, with a cavity of half an inch diameter and an inch deep, be stopped by a cork, fitted so as to close the aperture accurately, yet without much friction, and if two wires be inserted through the sides of the mortar so that their points within the cavity be separated by an interval of about a quarter of an inch, a strong charge being sent through the wires will expand the air within the cavity so suddenly as to project the cork to some distance.

(158.) Solid bodies of a porous texture, such as wood, are easily torn asunder by an electric charge. If two holes be drilled in the opposite ends of a piece of wood, about half an inch long, and a quarter of an inch thick, and the ends of two wires inserted in the holes, so that their points may be at the distance of a quarter of an inch; on passing a strong charge through them, the wood will be split in pieces. Stones, loaf-sugar, and other brittle and imperfectly conducting substances, may be broken in a similar way.

Place a piece of dry writing paper upon the table of the universal discharger, and having removed the balls from the ends of the sliding wires, press the points of the wires against the paper at the distance of two inches from each other; if a powerful shock be now sent through the wires, the paper will be torn in pieces. If a number of wafers be placed on the table, instead of paper, they will be dispersed in a curious manner, and many of them broken into small fragments.

(159.) A singular result is obtained by the following variation in the circumstances of the last experiment, which was made by Mr. Lullin. Suspend a varnished card by silk threads, (see *fig.* 38,) in such a manner that two blunt wires proceeding from the two sides of a jar or battery, may be in contact with

Fig. 38.



the opposite sides of the card, but at the same time half an inch distant from each other; when the discharge is made between the wires, and along the surface of the card, the latter is found to be perforated, but always at the point where the wire communicating with the negative side of the battery had touched it. The same perforation takes place at this point, even when a hole has been previously made at the point, where it is touched by the positive wire.

The course of the electric fluid may be traced with more precision, by having both sides of the card coloured, previously to the experiment, with vermilion, for it will then leave on the card a well defined black line extending from the point of the positive wire to the perforation; and a diffused black mark on the opposite side of the card, around the perforation, and next to the negative wire.

(160.) When the electrical discharge is made to pass in a perpendicular direction through the thickness of a card, which may be effected by placing it against the outer coating of a Leyden jar, and setting the lower ball of the discharging rod against the other side of the card, so that its thickness may be interposed between it and the tin-foil, and making the explosion in the usual way, as represented in *fig.* 33, (§ 134,) the card will be perforated. At the edge of the perforation, on each side of the card, there will be a small bur or protrusion, which is always larger on the side next to the jar, than on that next to the discharging rod; the former being the negative, and the latter the positive side. By passing the shock through a quire of paper, instead of a single card, the progress of this effect at different depths from the surface may be accurately analysed. Mr. Symmer, who devised this experiment, observed that the ragged edges were for the most part directed outwards from the body of the quire. Upon examining the leaves separately, however, he found that the edges of the holes were bent regularly two different ways, and more remarkably so about the middle of the quire; one edge of each hole being throughout its course forced one way, and the other edge in the contrary direction, as if the hole had been made in the paper by drawing two threads through it in opposite directions.

(151.) The following variation of the experiment illustrates the nature of the mechanical impressions made by electricity. Let a sheet of tin-foil be placed in the middle of a quire of paper; on making the discharge through it, the tin-foil is found to have received two indentations in opposite directions, and the leaves of paper are rent in such a manner, that on both sides of the tin-foil the burs point towards the outsides of the quire; but the indentations upon the tin-foil, and the burs on the paper, are in opposite directions. If another quire of paper be taken, and two sheets of tin-foil be placed within it, so that they are separated by the two middle leaves of the quire, the result will be that all the leaves will be perforated, excepting the two within the tin-foil, and in these two leaves there will be two impressions or indentations in opposite directions.

(162.) The mechanical effects we have just described have been often adduced, not only as proofs of the materiality of the electric fluid, but also as positive indications of the direction of its motions, according as either the one or the other of the two theories of electricity is adopted. But this is a subject which we reserve for future discussion.

(163.) The fracture of glass by the electrical explosion has already been adverted to, (§ 129;) but there are still a few circumstances attending it which deserve to be noticed. The edges of the fractured portion appear well defined on the positive side; while on the negative side they are splintered, as might be expected from the passage of a material agent from the former to the latter. It is remarkable also, that a perforation may be made in glass by a very moderate discharge, when the glass is in contact with oil or sealing-wax. Thus if a small phial, or glass tube, closed at one end, be filled with olive oil, and a pointed wire, bent at right angles, and passing through a cork fitted to the mouth of the phial or tube, be introduced into it, so that the point may touch any part of its inside beneath the surface of the oil; on suspending the vessel by its wire to the prime conductor of an electrical machine, and applying to the outside, either the knuckle, or a brass ball, exactly opposite to the point of the wire within, so that a spark may pass between them, it will be found to have made a small perforation through the glass; by bringing the wire in contact with different parts of the glass, a great number of holes may thus be made in it. The effect of the oil appears to be that of controlling the tendency of the electric fluid to diverge, and of concentrating the whole power of the charge into a single point.

(164.) This repulsive tendency is also well illustrated by the following experiments made by Dr. Priestley. If a clean brass chain, previously dipped in melted resin, be laid upon paper, and the charge of a battery of at least 32 square feet be sent through it, the resinous coating will be thrown off from every part of the chain, which will be left perfectly clean, and free from resin. If a brass chain be laid upon a piece of glass, and a similar charge passed through it, the glass will be marked in a beautiful manner on every part of its surface, where it had been touched with the chain, every spot having the width and colour of the link. The metal may be scraped off the glass at the outside of the marks, but in the middle part it is forced within the pores of the glass. Dr. Priestley communicated a similar tinge to glass, by means of a silver chain, and small pieces of other metals; but he could not succeed with large pieces.

(165.) The effects of accumulated electricity upon metallic bodies, are referable, for the most part, to the agency of the heat produced by its passage through them; yet the phenomena, in many cases, indicate also the operation of other forces. By the transmission, through a piece of metal, of repeated shocks, which are not powerful enough to effect its fusion, or even ignition, a permanent alteration may be produced in its form, such as would not have resulted from heat alone. Dr. Priestley and Mr. Nairne found by experiment, that a chain through which an electrical charge had passed, undergoes a diminution in its length. A piece of hard drawn iron wire, ten inches long and one hundredth of an inch in diameter, was found, after fifteen discharges, to have lost one inch and one tenth of its length; and the increase of thickness seemed to be in proportion to this longitudinal contraction, for the wire had not perceptibly lost any of its weight during the experiment. A copper wire plated with silver, of the same dimensions as the former, underwent, by the same treatment, a diminution of length two thirds as great as that of the iron wire.

On the other hand, if the shocks be transmitted through a wire which has a weight suspended by it, so as to give it considerable tension, the length of the wire becomes increased instead of diminished, as in the above experiment. This is evidently owing to the influence of the heat which accompanies the passage of the electricity, and which diminishes the cohesion of the particles of the metal, and disposes them to yield to the extending force which the weight supplies.

§ 2. *Evolution of Heat by Electricity.*

(166.) The ignition and fusion of metals by the electric discharge, are phenomena which have been long observed. Thus by passing a strong charge through slender iron wires, they are ignited, and partly melted into globules. It was formerly believed that very large batteries were necessary for obtaining this effect; but if the wire be sufficiently fine, the electricity accumulated in a single jar of moderate size will suffice for its production. The best material for exhibiting this effect, is the finest flatted steel sold at the watchmakers' tool shops, under the name of watch pendulum wire. Van Marum has given a statement of the lengths of wires of different diameters, and of different metals, which his powerful machine enabled him to melt; when they were drawn to the thirty-second part of an inch in diameter, he found that he could fuse 120 inches of lead wire, and the same quantity of tin wire; five inches of iron wire; three inches and a half of gold wire; and only one quarter of an inch of wires of silver, copper, or brass.

(167.) From the experiments of Brooke and of Cuthbertson, it has been inferred that the length of wire which is thus melted by the electric discharge, varies as the square of the quantity of accumulated electricity which is sent through it; thus a combination of two jars, charged to an equal degree, will melt four times the length of wire which one jar will melt.

(168.) While the electric battery thus effects the fusion, and even in some cases the volatilization of metals, the phenomena appear also to indicate the action of propelling or dispersive forces, as if the agent concerned in their production was endowed with great mechanical momentum. Thus the densest metals are rent and dispersed with violence by the passage of accumulated electricity If a slip of gold or silver leaf be placed on white paper, and a strong shock passed through it, the metal will disappear with a bright flash, and the impulse with which its particles are driven against the paper will produce a permanent stain of a purple or grey colour. Franklin found that if the metallic leaf be placed between two panes of glass firmly tied together, the explosion, provided the glass withstands the concussion, will leave on each of its surfaces an indelible stain, in consequence of some of the metallic particles being actually forced into the substance of the glass, and being then inaccessible to the action of chemical solvents applied to the surface of the glass. Sometimes it is found that these metallic stains extend to a greater distance than the breadth of the piece of metal. It often happens, however, that the pieces of glass themselves are shattered to pieces by the discharge.

(169.) The colours produced by the electric explosion of metals have been applied to impress letters or ornamental devices on silk and on paper. For this purpose Mr. Singer directs that the outline of the required figure should be first traced on thick drawing paper, and afterwards cut out in the manner of stencil plates. The drawing paper is then placed on the silk or paper intended to be marked; a leaf of gold is laid upon it, and a card over that; the whole is then placed in a press or under a weight, and a charge from a battery sent through the gold leaf. The stain is confined by the interposition of the drawing paper to the limit of the design, and in this way a profile, a flower, or any other outline figure may be very neatly impressed.

(170.) The heat evolved by electricity, like most other of its effects, is in proportion to the resistances opposed to its passage. The less the conducting power of a metal, the greater is the portion of it which the same shock can ignite or destroy. A rod of wood of considerable thickness being made part of the circuit, has its temperature sensibly raised by a very few discharges. Most combustible bodies are capable of being inflamed by electricity, but more especially if it be made to strike against them in the form of a spark or shock obtained by an interrupted circuit, as by the interposition of a stratum of air. In this way may alcohol, ether, camphor, powdered resin, phosphorus, or gunpowder be set fire to. The inflammation of oil of turpentine will be promoted by strewing upon it fine particles of brass filings. If the spirit of wine be not highly rectified, it will generally be necessary previously to warm it, and the same precaution must be taken with other fluids, as oil and pitch; but it is not required with ether, which usually inflames very readily. But, on the other hand, it is to be remarked that the temperature of the body which communicates the spark appears to have no sensible influence on the heat produced by it. Thus the sparks taken from a piece of ice are as capable of inflaming bodies as those from a piece of red-hot iron. Nor is the heating power of electricity in the smallest degree diminished by its being conducted through any number of freezing mixtures which are rapidly absorbing heat from surrounding bodies.

(171.) Light, as well as heat, is emitted during the electric discharge at every point where the circuit is either interrupted, or is occupied by bodies of inferior conducting powers. A moderate charge will produce a bright spark when made to pass through water, and the spark is still more luminous in oil, alcohol, or ether, which are worse conductors than water: on the contrary, in fluids of greater conducting power there is greater difficulty of eliciting electric light. Thus a much higher charge is required to produce a spark in hot water than in cold; a still higher in saline solutions; and in concentrated acids, light can be obtained only when their volume is very small; so that it is necessary for that purpose, to draw a line of the acid upon a plate of glass with a camel's hair pencil. This is illustrated by the following experiment mentioned by Singer. Draw a line with a pen dipped in water on the surface of a slip of glass; place one extremity of the line in contact with the coating of a Leyden jar, and at six inches distance upon the line place one knob of the discharging rod; when the jar is fully charged, bring the other ball of the discharger to the knob of the jar, and the discharge will take place luminously over the six inches of water. Next, trace a line with a pen dipped in sulphuric acid on a slip of glass, as in the former experiment, and place one extremity of it in contact with the outside of the jar; the ball of the discharger may then be placed on the glass at twelve inches distance, and the electric fluid will pass as brilliantly over that interval as over the six inches of water. In either of these experiments, if the line of fluid be wider in any particular part, the light of the discharge will appear less brilliant in passing that portion; this must arise from the greater division of the fluid when passing over an extended conductor than over one that is narrow.

§ 3. *Chemical Effects of Electricity.*

(172.) Electricity exerts a most extensive and important influence in effecting changes in the chemical composition of bodies; but as this influence is most conspicuously exerted in that particular mode of agency, which is known by the name of *Galvanism,* this subject will more properly be considered in the treatise on that branch of electrical science. For the present, we must content ourselves with adducing a few instances, illustrative of the chemical effects of electricity in the forms under which it has now been presented to our notice.

(173.) Some of the chemical changes consequent on powerful electrical explosions, appear to be merely the effects of the heat which is evolved in that process. The surfaces of metallic bodies through which accumulated electricity is made to pass are frequently oxidated; this is seen more especially in the case of wires that have been fused or volatilized by the electric discharge. It is known that metals intensely heated are disposed to combine with the oxygen of the atmosphere, and, consequently, to assume the state of oxides; it is simpler, therefore, to ascribe this effect in the present case to a cause which is known to be in operation at the same moment, than to any peculiar or determining agency of electricity. A multitude of experiments are on record in which the partial oxidation of metals has been effected by electric explosions. This subject was prosecuted with minute and laborious attention by Van Marum, by Cuthbertson, and more lately by Singer. It is remarked by this last experimentalist, that the oxides of metals produced in this way appear to consist of several distinct portions of different degrees of fineness; when a wire is exploded in a receiver, part of the oxide immediately falls to the bottom, but another portion remains suspended in the air for a considerable time, and is at length gradually deposited. It is probable that this circumstance may in part account for the different colours of oxides produced in close receivers and in the open air, for in the latter case a portion of the oxide is always lost.

(174.) Under other circumstances, electricity is found to exert a power the reverse of the former; for it decomposes metallic oxides, extricating their oxygen, and restoring them to the metallic state. This deoxidating power was known to several of the earlier electricians. Beccaria reduced the oxides of tin and of mercury to their metallic state by electricity. In order to effect this change, a quantity of the oxide may be introduced into a glass tube, and pointed conducting wires inserted through corks at the opposite ends of the tube, so that a portion of the oxide may lie between them. This apparatus is then to be placed on the table of the universal discharger, (§136,) and repeated shocks are to be sent through the oxide until its partial or total reduction is accomplished. Vermilion, which consists of sulphur and mercury, is very easily decomposed by this process, and by a very moderate charge.

(175.) When a succession of electric discharges from a powerful electric machine are sent through water, a decomposition of that fluid takes place, and it is resolved into its two elements of oxygen and hydrogen, which immediately assume the gaseous form. This fact was discovered in 1789, by Messrs. Dieman, Paetz, and Van Troostwyck, who had formed themselves into a society for experimental research in Holland; and it completed the chain of evidence by which the great discovery of the composition of water, made five years before by Cavendish, is established. The abovementioned Dutch chemists being occupied, in conjunction with Mr. Cuthbertson, in investigating the effects of electricity when passed through different bodies, were desirous of ascertaining its effect on pure water. They employed for this purpose an apparatus consisting of a glass tube, twelve inches long and one-eighth of an inch in diameter, through one end of which a gold wire was inserted, projecting about an inch and a half within the tube; that end was then hermetically sealed. Another wire was introduced at the other end of the tube, which was left open, and passed upwards, so that its extremity came to a distance of five-eighths of an inch from the end of the first wire. The tube was then filled with distilled water, which had been freed from air by an excellent air-pump, and inverted in a vessel containing mercury. A little common air was let into the top of the tube, in order to prevent its being broken by the discharge. Electrical shocks were then passed between the two ends of the wires through the water in the tube by means of a Leyden jar, which had a square foot of coated surface. This jar was charged by a very powerful double plate machine, which caused it to discharge twenty-five times in fifteen revolutions. At each explosion bubbles of air were formed, and rose to the top of the tube. As soon as a sufficient quantity had collected to leave the upper end of the wire uncovered by the water, so that the shock had now to pass through a portion of the mixed gases, they were instantly kindled; a reunion of the elements took place; water was again formed, and the space they had occupied was immediately filled with fluid from below, so as to restore every thing precisely as at the outset of the experiment. It was ascertained by the most decisive chemical tests, that the gases thus obtained consisted of a mixture of oxygen and hydrogen gases.

(176.) It may appear somewhat paradoxical that the same agent should, in the course of the same experiment, produce at one time decomposition, and at another combination of the same elements. The simplest way of reconciling this apparent discordance, is to suppose that the combination of the gases is the effect of the heat evolved during its forcible transit through an aëriform fluid that opposes considerable resistance to its passage; while the decomposition of the liquid is the direct consequence of the agency of electricity when not interfered with by heat.

(177.) Until lately, it was thought necessary to employ powerful machines and large jars in order to effect the decomposition of water by electricity, and that mere sparks from a common machine were inadequate to accomplish this purpose. That there is in this respect, however, no essential distinction in the operation of these two forms of electricity has been satisfactorily shown by Dr. Wollaston. This distinguished philosopher, perceiving, with his accustomed sagacity and penetration, that the decomposition would depend on duly proportioning the strength of the charge to the quantity of water, and that the quantity exposed to its action at the surface of communication depends on the extent of that surface, inferred that by reducing the surface of communication the decomposition of water might be effected by smaller machines, and with less powerful excitation than had hitherto been applied to this object. Having procured a small wire of fine gold, and given to it as fine a point as possible, he inserted it into a capillary glass tube; and after heating the tube, so as to make it adhere to the point, and cover it in every part, he gradually ground it down, till, with a pocket lens, he could discern that the point of the gold was exposed. When sparks from a prime conductor of an electrical machine were made to pass through water by means of a point so guarded, a spark, extending to the distance of one-eighth of an inch, would decompose water when the point exposed did not exceed one 700th of an inch in diameter. With another point, estimated at one 1500th, a succession of sparks one-twentieth of an inch in length afforded a current of small bubbles of air. With a still finer filament of gold, the mere current of electricity, without any perceptible sparks, evolved gas from water.

(178.) When a solution of sulphate of copper was subjected to the action of electricity by means of these slender conducting wires, the metal was revived around the negative wire; but upon reversing the direction of the current of electricity, so that the same wire now became positively electrified, the copper which had collected around it was redissolved, and a similar precipitate was deposited on the opposite wire, which was now the negative one. Similar experiments made with other metallic solutions were attended with analogous results; the negative wire always separating oxygen from its combinations, the positive wire always attracting it, and effecting its union with the bases presented to it. With solutions of neutral salts, the alkaline or earthy bases were attracted by the negative, while the acids were attracted by the positive wire. The experiments of Sir Humphry Davy have confirmed these results as far as concerns the chemical action of common electricity; but as this is a subject which bears more immediate relation to chemistry and to galvanism, it would not be right to enlarge upon it in the present treatise.

(179.) The magnetic effects of electricity will likewise form the subject of a distinct treatise, as they now constitute a new branch of science, under the title of Electro-Magnetism.

§ 4. *Effects of Electricity upon Animals.*

(180.) Having seen that the effects of electricity on inanimate matter are of various kinds, we should be led to expect that its operation on living bodies would be still more complicated; for in addition to its mechanical and chemical agencies, it can hardly fail of exerting considerable influence on the living powers, and more especially on the functions of the nervous system. It is unnecessary to describe the sensations excited in the body by receiving electric sparks or shocks, since most persons in the present day are familiar with them. It is curious, however, to take a retrospective view of the mode in which the effects of the Leyden phial were announced to the world, on their first discovery. The philosophers who first experienced, in their own person, the shock attendant on the transmission of an electric discharge, were so impressed with wonder and with terror by this novel sensation, that they wrote the most ridiculous and exaggerated account of their feelings on the occasion. Muschenbroek states, that he received so dreadful a concussion in his arms, shoulder, and heart, that he lost his breath, and that it was two days before he could recover from its effects; he declared also, that he should not be induced to take another shock for the whole kingdom of France. Mr. Allemand reports, that the shock deprived him of breath for some minutes, and afterwards produced so acute a pain along his right arm, that he was apprehensive it might be attended with serious consequences. Mr. Winkler informs us, that it threw his whole body into convulsions, and excited such a ferment in his blood, as would have thrown him into a fever, but for the timely employment of febrifuge remedies. He states, that at another time it produced copious bleeding at the nose; the same effect was produced also upon his lady, who was almost rendered incapable of walking. These strange accounts naturally excited the attention and wonder of all classes of people; the learned and the vulgar were equally desirous of experiencing so singular a sensation, and great numbers of half-taught electricians wandered through every part of Europe to gratify this universal curiosity.

(181.) As it is probable that the electric fluid meets with greater impediment in passing from the surface of one bone to another, at the parts where the continuity of substance is interrupted by the joints, this circumstance explains why the shock is often more especially felt at the joints than in any other part of a limb. But if the shock be directed more particularly through muscles, its effects are chiefly shown by exciting a convulsive and involuntary action of those muscles. This is often observed to take place in a paralysed limb, when electric shocks are sent through it, although the nerves of the limb are at the time incapable of conveying the impressions which produce sensation. Mr. Morgan states, that if the diaphragm be included in the circuit of a coated surface of two feet in extent, fully charged, the sudden contraction of the muscles of respiration will act so violently upon the air in the lungs, as to occasion a loud and involuntary shout; but if the charge be small, a fit of convulsive laughter is induced, presenting a most ludicrous exhibition to the by-standers.

(182.) It is on the nervous system, however, that the most considerable action of electricity is exerted. A strong charge passed through the head, gave to Mr. Singer th sensation of a violent but universal blow, and was followed by a transient loss of memory and indistinctness of vision. If a charge be sent through the head of a bird, its optic nerve is usually injured or destroyed, and permanent blindness induced: and a similar shock given to larger animals, produces a tremulous state of the muscles, with general prostration of strength. If a person who is standing receive a charge through the spine, he loses his power over the muscles to such a degree, that he either drops on his knees, or falls prostrate on the ground; if the charge be sufficiently powerful, it will produce immediate death, in consequence, probably, of the sudden exhaustion of the whole energy of the nervous system. Small animals, such as mice and sparrows, are instantly killed by a shock from thirty square inches of glass. Van Marum found that eels are irrecoverably deprived of life when a shock is sent through their whole body; but when only a part of the body is included in the circuit, the destruction of irritability is confined to that individual part, while the rest retains the power of motion. Different persons are affected in very different degrees by electricity, according to their peculiar constitutional susceptibility. Dr. Young remarks, that a very minute tremor, communicated to the most elastic parts of the body, in particular to the chest, produces an agitation of the nerves, which is not wholly unlike the effect of a weak electricity.

(183.) The bodies of animals killed by electricity, rapidly undergo putrefaction, and the action of electricity upon the flesh of animals is also found to accelerate this process in a remarkable degree. The same effect has been observed in the bodies of persons destroyed by lightning. It is also a well-established fact, that the blood does not coagulate after death from this cause.

(184.) It has not been determined with any degree of certainty, whether electricity, in its ordinary mode of application, exerts any sensible influence on the functions of the animal system. The Abbé Nollet persuaded himself, from the experiments he made on man and animals, that the perspiration was increased during the time they were electrified; and De Bozes had noticed that the pulse was quickened under the same circumstance. But Van Marum, on repeating these experiments in a variety of ways, met with such variable and contradictory results, that he could deduce from them no satisfactory conclusion respecting the real operation of electricity; and, indeed, if we take into account the powerful influence which the imagination exerts on most persons who are the subjects of such experiments, as well as on those who witness them, there appears but little chance, amidst such multiplied sources of fallacy, of arriving at the truth. The only general fact, perhaps, which appears to be established, is that electricity acts as a stimulant both to the muscular and the nervous systems.

(185.) When the energetic effects of the shock from the Leyden phial were first made known, the most sanguine expectations were immediately raised, that electricity would prove an agent of considerable power in the cure of diseases. It was supposed that as a stimulant, it would have many advantages over other remedies; for it can be administered in various degrees of intensity, which may be regulated with great exactness; and its application can be directed especially to the organ we wish to affect, and can be limited to that organ, so as not to interfere with the functions of the general system. Accordingly we find, that at one period electricity was in great repute as an efficacious remedy in a number of diseases; but at present it is seldom employed except in a very few. It is not unfrequently had recourse to in palsy, contractions of the limbs, rheumatism, St. Vitus's dance, and some kinds of deafness, and impaired vision; it has also been applied to discuss tumours, to remove obstructions, and to relieve pain.

(186.) Electricity may be administered medicinally in four different ways. The first and most gentle is under the form of a continued stream, or *aura* as it is termed, derived from a wire or pointed piece of wood connected with the prime conductor of the machine, held by an insulated handle, at the distance of one or two inches from that part to which it is to be directed; an impression is felt similar to a current of air; and in this way it may be borne by parts of great sensibility, such as the eye. The second mode is by directing sparks of various sizes to the affected part, by means of a metallic ball at the extremity of a brass rod, which is within a moderate distance from the part; or else by placing the patient on an insulating stool, and while he is in communication with the prime conductor of the machine, taking sparks from him by another person with a metallic ball at the end of a rod which he holds in his hand. The size and intensity of the spark will, of course, be regulated by the distance at which the ball is placed from the body, provided the machine be steadily worked. The third mode is that by shocks from the discharge of a Leyden phial, which is, of course, the most severe and painful method of applying electricity. Great caution is required against the indiscriminate application of this last method, which is not wholly free from danger. The fourth mode is by Galvanism, hereafter to be noticed.

§ 5. *Effects of Electricity upon Vegetables.*

(187.) It has also been imagined that electricity acts as a stimulus to vegetable life: and many fanciful projects of improvements in horticulture by the aid of artificial electricity have been entertained. It is needless, however, to enlarge upon these visionary speculations, the fallacy of which has been sufficiently shown by the late Dr. Ingenhouz, who, upon the most accurate inquiry, found that the vegetation of plants was in no sensible degree either promoted or retarded by common electricity. The experiments of Van Marum, however, in which be found that electricity increased the evaporation of plants, appear to be entitled to some confidence; but still the effect observed, may, as he himself remarks, have been occasioned by the increased current of air from the parts of the electrified leaves. His observations on the influence of electricity on the sensitive plant, (*Mimosa pudica,*) deserve also to be noticed. The mere approach of an electrified conductor, whether charged with positive or negative electricity, produced no effect upon the plant; but when sparks were taken from it, the leaves collapsed, just as they would have done by concussions of a mechanical nature, and in other respects the plant underwent no change. In the *Hedysarum gyrans*, a plant remarkable for the continual rotatory motions of its leaves, electricity appeared to have no sensible influence either in accelerating or retarding these movements.

(188.) The passage of shocks through living plants immediately destroys the vitality in the parts through which the shock has been sent. It is, indeed, very easy to kill plants by means of electricity. A very small shock, according to Cavallo, sent through the stem of a balsam, is sufficient to destroy it. A few minutes after the passage of the shock, the plant droops, the leaves and branches become flaccid, and its life ceases. A small Leyden phial, containing six or eight square inches of coated surface, is generally sufficient for this purpose, which may even be effected by means of strong sparks from the prime conductor of a large electrical machine. The charge by which these destructive effects are produced, is probably too inconsiderable to burst the vessels of the plant, or to occasion any material derangement of its organization; and, accordingly, it is not found, on minute examination of a plant thus killed by electricity, that either the internal vessels or any other parts have sustained perceptible injury.

(189.) It appears from the experiments of Mr. Achard, that the fermentation of vegetable matter is accelerated by electricity.

(190.) The general conclusion deducible from these inquiries is, that feeble electricity exerts no perceptible influence on either animal or vegetable life: but when transmitted in powerful shocks, its destructive effects are similar to those which are produced by lightning.

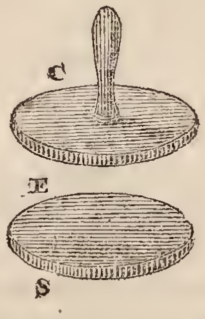
Chapter XII.

Instruments adapted to collect weak Electricity.

(191.) Before we proceed to consider the developement of electricity under various circumstances, it will be proper to give a description of several instruments which have been contrived for the purpose of collecting and exhibiting weak degrees of electricity, that would otherwise escape detection. All these instruments derive their efficacy from the principle of electric induction; and their mode of operation will be best understood by previously directing our attention to the electrophorus.

(192.) The instrument termed the *Electrophorus* was invented about the year 1774 by Professor Volta, a name which is associated with many important discoveries in the science of electricity. It consists of three parts: the essential part, which supplies the electricity, being a cake of some electric substance, (E, *fig.* 39,) such as sulphur, gum lac, sealing-wax, pitch, or other resinous composition; this is melted on a conducting plate S, called *the sole,* which is formed with a rim to contain it, and the fluid then allowed to congeal. The third part of the apparatus consists of a circular metallic plate C, provided with an insulating handle fixed upon its upper surface. This is called *the cover;* and is sometimes made of wood, covered on all sides with tin-foil well rounded at the edges to prevent the dispersion of electricity. In order to bring the apparatus into a state of activity, the surface of the cake is excited by friction with fur or flannel, and is thus rendered negatively electrical. The cover, held by its insulating handle, must now be placed

Fig. 39.

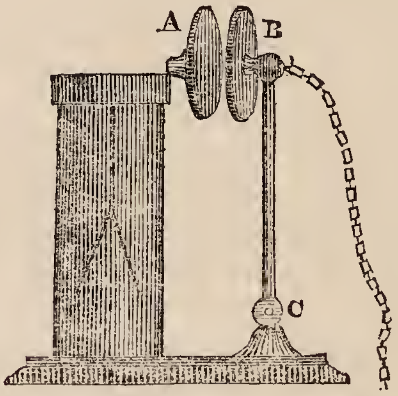


on the cake: in this situation it does not come sufficiently in contact with the cake to receive its electricity, but acquires by induction an opposite state at its lower surface, and a similar state at its upper; that is, the cake being negative, the under side of the cover will be positive, and the upper side negative. If, while in this state, the upper negative surface be touched with the finger, or with any other conductor communicating with the earth, a spark will pass from the latter to the cover, so as to restore the electric equilibrium; the quantity of electricity thus superadded being retained in the cover by the inductive influence of the cake. But when the plate is raised, provided it be held by its insulating handle, the action of the cake being withdrawn, the cover is found to be charged with positive electricity, which may be imparted to an insulated conductor, or to a Leyden jar. This operation may be repeated an indefinite number of times, since the electricity of the cake continues unimpaired during the process, and thus may a charge be communicated to the jar of an intensity equal to that of the cover of the electrophorus when raised. The instrument has been known, indeed, to retain its power undiminished for months, and may therefore be regarded as a sort of magazine of electricity. It is obvious, that if the cover were simply placed on the cake, and again raised without previously touching it, it would then exhibit no sign of electricity. If the sole of the electrophorus be insulated, a spark may be obtained from it, when the cake has been excited; and if while placed on the cake the cover be touched with the finger, and at the same time the sole be touched with the thumb, a sensible shock will be felt in that part of the hand.

(193.) Volta is also the inventor of an instrument acting on the same principle as the electrophorus, and which he termed *the condenser*, of which the purpose is to collect a weak electricity, spread over a large surface, into a body of small dimensions, in which its intensity will be proportionably increased, and therefore become capable of being examined. A small metallic plate, connected with the substance of which the electricity is to be determined, is brought within a very small distance of another plate communicating with the earth. The small portion of electricity received from the substance to be tried by the first plate, acts by induction on the second plate, and occasions it to acquire the opposite electrical state: this latter state reacts upon the first plate, increasing its capacity for the electricity which it had first received, and tends to accumulate a larger quantity in it, which quantity it must derive from the substance with which it communicates. This mutual action and reaction continues till an equilibrium is attained. If the communication between the substance tried and the first plate be broken off, and the plate thus insulated be removed from the contiguity of the second plate, the accumulated electricity with which it is charged will become evident upon its application to an ordinary electroscope, such as those described in § 13 and 14.

(194.) Various have been the forms given to the condenser, according to the fancy of electricians, without any change in the principle on which it acts. In general, the two plates are merely separated by a thin stratum of air. Sometimes their surfaces are covered with a non-conducting varnish, which prevents any communication of electricity from the one plate to the other, while it allows of a very near approach of the plates to each other; but this method is liable to objection, from the permanent electricity which the varnish sometimes contracts by friction, and which may interfere with the regular operation of the instrument. One of the most convenient forms is that of the condensing electrometer, (*fig.* 40,)

Fig. 40.



in which the first plate of the condenser A, is fixed to the cap of the gold-leaf electroscope; the second plate B, which communicates by a chain with the ground, being moveable round a joint C, and thus capable of being turned back and removed from the first plate, so as to allow its electricity to be manifested by the divergence of the gold leaves.

(195.) The instruments called *Doublers* are so contrived, that by executing certain movements, very small quantities of electricity communicated to a part of the apparatus may be continually doubled, until it becomes perceptible by an electroscope. The first invention of this kind was that of Mr. Bennet, which consists of three brass plates, which we shall call A, B, and C. The plate A has an insulating handle fixed in its centre, while the plate B has a similar handle fixed in its circumference. The under side of A, and both sides of B are covered with varnish. The third plate C is also of brass, and is only varnished on its upper side, the lower side communicating with the gold-leaf electroscope. The body whose electricity is to be tried, is made to communicate with the under side of the plate C, which touches the electroscope, while B is placed upon C, and then touched with the finger: the communication with the electrified body is then removed, and B is lifted up by its insulating handle. A is then placed, by means of its handle, upon B thus elevated. A is then touched, and, after withdrawing the finger, is separated from B. In this process B acquires an electricity contrary to that of C; and A an electricity contrary to that of B, that is, the same as that of C. If the plate A, thus electrified, be next applied to the under surface of C, and B be again applied over C, and touched with the finger as before, it will be acted upon by the electricities contained both in C and A, and thus acquire, by induction, nearly double the quantity which it had done in the first operation. The consequence of this will be that nearly all the free electricities of A and C will be concentrated in C. A may now be removed, and after withdrawing the finger from B, B may also be removed, and C will be left with double the quantity of electricity which it had received from the body with which it was originally made to communicate.

If after this duplication the electricity of the plate C be still too feeble to be indicated by the electroscope, the same series of operations must be repeated ten or even twenty times; when by doubling it every time, the smallest conceivable quantity of electricity must at last be rendered sensible; since, at the end of the twentieth operation, it will be augmented more than 500,000 times. Although the frequent repetition of the operations may appear tedious, yet, by a little practice, the art is readily acquired, and the whole process need not occupy a minute. Great care must be taken in conducting these experiments, not to excite any electricity by the friction of the finger, or by any other means, in the varnished sides of the plates. In order to obviate this source of error, Cavallo contrived a form of the instrument, that enabled the plates to be brought within a very small distance of one another, yet without actual contact, so as to enable him to dispense altogether with the employment of varnish. But notwithstanding every precaution of this kind, it is always found that the instrument exhibits electricity of itself, although none has been previously communicated to it: so that its indications cannot be at all depended upon for the detection of very minute quantities of electricity. It is unnecessary, therefore, to describe the particular mechanisms invented by Dr. Darwin, and improved by Nicholson, for bringing the plates into the requisite positions, and effecting in succession the necessary contacts, by the simple rotation of a winch, aided by wheel-work: instruments which have gone by the names of the *moveable*, or *revolving doubler*, and the *multiplier of electricity*, and which are now superseded in practice by instruments more sensible and certain in their operation.

Chapter XIII.

Developement of Electricity by Changes of Temperature and of Form.

(196.) There are certain mineral bodies, which, from being in a neutral state at ordinary temperatures, acquire electricity simply by being heated or cooled. This property is possessed only by regularly crystallized minerals; and of these the most remarkable is the tourmalin, which is a stone of considerable hardness, found in many parts of the world, and particularly in the island of Ceylon. The Dutch, who first became acquainted with it in that island, gave it the appellation of *Aschentrikker*, from its property of attracting ashes when it is thrown into the fire. It appears from the researches of Dr. Watson, that its attractive properties were known to Theophrastus, who describes it under the name of *Lyncurium*. Linnæus has termed it the *Lapis Electricus*, (Electric stone.) The form of its crystals is generally that of a nine-sided prism, terminated by a three-sided pyramid at one end, and by a six-sided pyramid at the other. Lemery noticed its electric properties in the year 1717; but the first scientific examination of them was made by Æpinus in 1756, and published in the Memoirs of the Berlin Academy. He found that when a crystal of tourmalin has its temperature raised to between 100° and 212° of Fahrenheit, one extremity, which is that terminated by the six-sided pyramid, becomes charged with positive electricity, while the other extremity is negative; so as to be capable of affecting a delicate electroscope. When the stone is of considerable size, flashes of light may be seen along its surface. Mr. Wilson, who made many experiments on this subject, observed that a flat tourmalin retained its electricity without diminution, after exposure to intense heat for half an hour; but Canton, upon repeating these experiments, did not obtain the same result. Hauy states, that very high degrees of heat destroy the electricity of the tourmalin. After this has been effected, it recovers its electricity as it gradually cools: but in that case the electric states are generally reversed; that extremity, or *pole*, as it has been called, which was before positive, is now negative, and *vice versâ*. It is only at the summits of the pyramids, by which the crystal is terminated, that the electricity is manifested; the intermediate portions exhibiting no sign of electrical excitation, unless the stone be broken in pieces; and then each fragment is found to possess a positive and a negative pole, like the entire crystal. This fact bears a striking analogy to a corresponding property in magnets. At the ordinary temperature of the atmosphere, the tourmalin may be rendered electrical by friction.

(197.) There are several other gems and crystallized minerals which possess the same property as the tourmalin. The luminous appearance of some diamonds, when heated, is ascribed by Sir Humphry Davy to their electrical excitation. The substance called the *Boracite*, composed of borate of magnesia, which crystallizes in cubes, having its edges and angles defective, becomes electrical by heat, and in one variety presents no less than eight sides, alternately in different states; that is, four positive and four negative; the opposite poles being in the direction of the axes of the crystal. In those varieties in which only four of the angles of the crystal are truncated, that is, cut off by planes, while the rest are either entire, or are replaced by more than one plane, it is always the former of these angles that become positive, and the latter negative.

(198.) Similar properties are possessed by the Topaz, which consists of siliceous fluate of alumina; its electric poles are situated upon the two opposite summits of the secondary crystal. In some varieties, Hauy found a series of consecutive poles alternately positive and negative. Axinite, Mesotype, and Prehnite, become electrical by the application of heat: as also the two following metallic oxides, namely Calamine, which is an oxide of zinc, and Sphene, or calcareo-siliceous oxide of titanium. Mr. Dessaignes has lately shown that all metallic bodies are capable of a feeble electric excitation by changes of temperature. It results from the researches of Hauy, that this electrical property in mineral bodies is intimately related to the laws of their crystallization, and also to the direction in which the light is most readily transmitted through them.

(199.) There are a great many substances which become electrified on passing from the liquid to the solid form. This happens to sulphur, gum lac, bee's wax, and in general all resinous bodies. Unless proper precautions be taken, however, we frequently obtain no indications of this electricity, because it is usually *disguised*, that is, rendered inactive by the opposite electricity of the contiguous substances. Thus, if sulphur be melted over the fire in an iron ladle, and then set by to cool and harden, it exhibits no sign of electricity; because the negative electricity of the sulphur is exactly counterbalanced by the positive electricity accumulated in the iron vessel which contains it. But if the sulphur be removed from the vessel, which may be done by again heating it for a short time, so as just to melt the surface in contact with the iron, and allow of its being detached when the ladle is inverted, on suffering the sulphur to cool in this situation, its electricity becomes very apparent. If sulphur be melted in a wine glass, the conical shape of which admits of its being taken out when cold, the opposite electricities of the two surfaces will then manifest themselves, that of the sulphur being negative, and that of the glass positive; but when the sulphur is replaced in the glass, all indications of electricity disappear. The electricity developed by the process of cooling was called by Wilke, who first observed it, *spontaneous electricity*, in contradistinction to that which, originating from friction, he called *excited electricity*. Van Marum, however, attributes the electricity developed by the separation of the two substances, to a species of friction; for he remarks, that the electricity does not manifest itself till the sulphur begins to contract in the act of congelation, and that it attains its maximum at the point of the greatest contraction.

(200.) It is reasonable to suppose that whatever change was produced in the electrical state by congelation, the reverse would be produced by liquefaction. We are not aware of any experiments which bear directly upon this question.

(201.) The conversion of bodies into the state of vapour, as well as the condensation of vapour, is generally attended by some alteration of their electrical condition; and the bodies in contact with the vapour are thereby rendered electrical. Thus, if a plate of metal strongly heated be placed upon a gold-leaf electroscope, and water be dropped upon the plate, at the moment the vapour rises the leaves of the electroscope diverge with negative electricity. The general fact was noticed by Laplace, Lavoisier, and Volta, in the year 1781; and was found to extend both to solids and to liquids passing into a gaseous form. De Saussure made an extensive series of experiments on the ebullition of water and other fluids, with a view to ascertain the degree and kind of electricity developed during this process. But investigations of this kind are attended with great difficulty, from the multitude of minute circumstances which are liable to affect the results; and we accordingly find, that different experiments of the same kind often afford the most opposite conclusions.

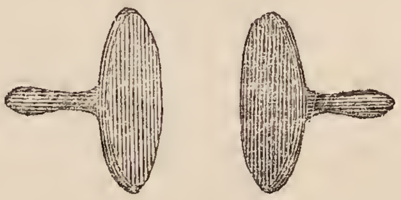
(202.) In general it is found, that the vaporization of water by simple ebullition produces negative electricity in the remaining fluid, or vessel which contains it: the vapour itself being positive. On the contrary, when aqueous vapour is condensed into water, it becomes negative, leaving the bodies with which it was last in contact in a state of positive electricity. Yet in some of De Saussures's experiments, when the heat was communicated to a quantity of water contained in an insulated metallic vessel, by throwing into it a mass of red-hot iron, the electricity was very strongly positive. This difference in the result was probably owing to the chemical decomposition of the water in the latter experiment, a circumstance which, as we shall presently see, is itself a source of electricity. It is principally on account of the interference of chemical actions with the regular operations of temperature, and of the complications introduced by electric induction, that experiments on this subject have hitherto presented such anomalous, and, apparently, discordant results.

Chapter XIV.

Developement of Electricity by Contact, Compression, and other mechanical Changes in Bodies, and also by their Chemical Action.

(203.) It had long been suspected, rather than proved, that a feeble degree of electricity is evolved by the contact or collision of different metals: but this important fact was established in the clearest manner by Volta, about the year 1801. The apparatus he employed in his investigations on this subject consisted of two discs, the one of zinc, the other of copper, (*fig.* 41,) rather more than two inches in diameter, ground perfectly plane, and having in their centres insulating handles perpendicular to their surfaces, by means of which the plates could be brought into contact, without being actually touched with the hand. With this precaution the discs were made to approach till they touched one another; they were then separated, by keeping them parallel as they were drawn back. The electricity they possessed after this separation was then examined by means of the condenser; and, that the effects might be rendered more distinct, the electricity produced by a number of successive contacts, (taking care to restore the discs to the neutral state after each contact,) was accumulated in the same condenser. It was constantly found that the copper disc charged the condenser with negative, and the zinc disc with positive elec-

Fig. 41.



tricity. Thus it was established as a general fact, that these two metals, insulated, and in their natural state, are brought, by mutual contact, into opposite electrical states; the zinc acquiring positive electricity, and the copper becoming, in an equal degree, negative.

(204.) No explanation has yet been given of this curious fact, which seems to be at variance with all the previously ascertained laws of electric equilibrium. The transfer of electricity from one metal to the other during their contact, implies the operation of some new force which no theory has yet embraced. While the contact is preserved, neither of the metals gives any indication of its electrical state, the electricity being *disguised;* as would be the case of that of the coatings of a Leyden jar, if we could suppose them both in actual contact, but yet incapable of allowing any transfer of the electricity from the one to the other, so as to restore both to the state of neutrality.

We shall have occasion to resume the consideration of this curious subject in the treatise on Galvanism, with the theory of which it appears to have an intimate relation.

(205.) There are some bodies which are rendered electrical by pressure. This property is possessed in the most remarkable degree by that transparent variety of carbonate of lime which is known by the name of *Iceland spar.* According to Hauy, if a crystal of this spar, which has the form of a rhomboid, be held in one hand by two of its opposite edges, and if at the same time two of its parallel planes be lightly touched by two fingers of the other hand, and then brought near to the small needle of the electroscope, (§ 12) a decided attraction will be perceptible. By applying a more powerful pressure, the electrical effects will be still more considerable; the electricity being in all cases positive. Hauy observes that this property resides principally in those crystalline minerals that are capable of being reduced by mechanical division to plane and smooth laminæ: such as the Topaz, especially the colourless variety; Euclase, Arragonite, Fluate of lime, and Carbonate of lead. Among those substances in which friction excites negative electricity, there are some which require only to be pressed, for the production of the same effect. An instance occurs in elastic bitumen, when it has been cut into a proper shape for the experiment. Mr. Becquerel has lately discovered that many other substances, such as cork, bark, hairs, paper, and wood, possess the property of producing electricity by compression.

(206.) Many substances, when reduced to powder, exhibit electricity, if they are made to fall upon an insulated metallic plate. This fact was first noticed by Mr. Bennet, after he had invented his gold-leaf electroscope. He found that powdered chalk, put into a pair of bellows, and blown upon the cap of the electroscope, communicates to the instrument positive electricity, when the pipe of the bellows is about six inches from the cap; but the same stream of powdered chalk electrifies it negatively at the distance of three feet. On being blown in a more copious stream from a pair of bellows without the pipe, the electricity is always negative; and the same effect takes place when the powder is let fall from another plate upon the cap of the instrument. This subject was pursued by Cavallo; but the most complete set of experiments relating to it is that of Singer, who employed in his researches the two following methods: first, that of sifting the powders on the cap of a delicate electrometer through a fine sieve, which was thoroughly cleaned after each operation; and secondly, that of bringing an insulated copper plate repeatedly in contact with extensive surfaces of the powders spread on a dry sheet of paper; the copper plate being brought in contact with the condenser after every repetition of the contact, until a sufficient charge was communicated.

(207.) The following substances, according to Singer, produce negative electricity when sifted on the cap of the electrometer: *viz.* copper, iron, zinc, tin, bismuth, antimony, nickel, black lead, lime, magnesia, barytes, strontites, alumine, silex, brown oxide of copper, white oxide of arsenic, red oxide of lead, litharge, white lead, red oxide of iron, acetate of copper, sulphate of copper, sulphate of soda, phosphate of soda, carbonate of soda, carbonate of ammonia, carbonate of potash, carbonate of lime, muriate of ammonia, common pearl-ashes, boracic acid, tartaric acid, cream of tartar, oxymuriate of potash, pure potash, pure soda, resin sulphur, sulphuret of lime, starch, orpiment.

(208.) The following substances produce positive electricity under the same circumstance: *viz.* wheat flour, oatmeal, lycopodium, quassia, powdered cardamom, charcoal, sulphate of potash, nitrate of potash, acetate of lead, oxide of tin.

(209.) The following catalogue exhibits the results of the experiments of contact with a copper plate; the different substances being arranged under the head of the electricity they really acquire, which is contrary to that of the copper plate. *Positive:* lime, barytes, strontites, magnesia, pure soda, pure potash, common pearl-ashes, carbonate of potash, carbonate of soda, tartaric acid. *Negative:* benzoic acid, boracic acid, oxalic acid, citric acid, silex, alumina, carbonate of ammonia, sulphur, resin. These experiments were several times repeated with uniform results.

(210.) The above mode of electrical excitation is probably merely a species of friction, differing only from the more ordinary instances by the mode of its application. But in other cases the electrical effects of contact are more distinctly exhibited, as when zinc filings are poured through holes in a plate of copper, upon the cap of an electrometer.

(211.) The following experiment, founded on one devised by Professor Lichtenberg of Gottingen, is an elegant illustration of the opposite electrical states of different powders. With the knob of a charged jar, trace on the surface of a smooth plate of glass, or of any resinous substance, various lines at pleasure; and then repeat the same operation in other parts with the knob of a jar charged with the opposite electricity. Let the surface thus prepared be gently dusted, by means of a powder-puff, with a mixture of powdered sulphur and red lead, previously triturated together in a mortar. By the contact and friction thus produced, the sulphur has been rendered negative, and the red lead positive; and each of the powders, when projected on the plate, will attach itself to the oppositely electrified lines, forming a series of red and yellow outlines. It is also observable, that the configurations assumed by these and other powders differ according to the species of electricity impressed upon the plate; positive electricity producing an appearance resembling feathers, and negative electricity an arrangement more like stars.

(212.) The most important circumstance in this inquiry, is the connection between electricity and the chemical properties of matter. It is observed by Sir H. Davy, that most of the substances that act distinctly upon each other electrically, are likewise such as act chemically, when their particles have freedom of motion: this is the case with the different metals, with sulphur and the metals, with acid and alkaline substances. Of two metals in contact, the one which has the greatest chemical attraction for oxygen acquires positive electricity, and the other the negative: so that if arranged in the order of their oxidability, as follows, zinc, iron, tin, lead, copper, silver, gold, platina, each will become positive when brought into contact with any that follow it in the series, and negative with any of those which precede it. In contacts of acids with bases, as of crystals of oxalic acid with dry quicklime, the former is negative, the latter positive. All acid crystals when touched by a plate of metal render it positive, the crystals themselves becoming negative.

(213.) Bodies that exhibit electrical effects by mutual contact, previous to their chemical action on each other, lose this power during combination. Thus if a polished plate of zinc be made to touch a surface of dry mercury, and quickly separated, it is found positively electrical, and the effect is increased by heat; but if it be so heated as to amalgamate, that is, unite chemically with the mercury, it no longer exhibits any signs of electricity. The case is analogous with copper and sulphur; and iron, when applied to mercury, produces more electricity than zinc, apparently from its being incapable, under ordinary circumstances, of forming a chemical combination with mercury.

(214.) On the other hand, there can be no question that electricity is occasionally, if not universally, elicited during chemical action. We have just seen that a dry acid becomes negative by contact with a metal, which is consequently thereby rendered positive. In this case no chemical combination had taken place. But Becquerel has shown that if the acid, instead of being in a dry crystalline form, be in a liquid state, and capable of acting chemically on the metal, the acid will become positive and the metal negative. The same conclusion may also be deduced from the experiments of Lavoisier and Laplace, on the action of dilute sulphuric acid on iron filings. That the oxidation of metals gives rise to electricity has been also shown by the experiments of Dr. Wollaston, from which it would appear that the electricity obtained in the common electrical machine is derived principally from this source. When he employed as the rubbing substance an amalgam of silver or of platina, which are metals very little subject to oxidation, he could obtain no electricity. An amalgam of tin, on the other hand, supplied a large quantity of electricity. Zinc acts still better than tin; but the best amalgam for this purpose is made with both tin and zinc, a mixture which oxidates more readily than either metal separately. As a further trial whether oxidation assists in the production of electricity, a small cylinder with its cushion and conductor was arranged in a vessel so contrived that the contained air could be changed at pleasure. After ascertaining the degree of excitement produced in atmospheric air, carbonic acid was substituted, but the excitement could not be renewed; while it was immediately reproduced on the readmission of common air. It must be acknowledged, however, that Sir H. Davy, in repeating these experiments, arrived at opposite results; for he states, that the machine acted equally well in hydrogen gas as in atmospheric air, and was even more active in carbonic acid gas, a circumstance which he attributes to the greater density of this gas.

(215.) Electricity is often developed by processes quite independent of chemical changes. This is evident from its production by the friction of two bodies of the same kind upon one another, as has been already noticed, (§ 35;) and also by the strong electricity which is manifested on the separation of the parts of the same body. Thus, if a piece of dry and warm wood be suddenly rent asunder, the two surfaces which have separated are found to be electrified, the one positively, the other negatively; and a flash of light is perceived if the experiment be made in the dark. The same phenomenon is observed when the plates of mica (Muscovy glass) are suddenly torn asunder; and even when a stick of sealing-wax is broken across; the two surfaces of fracture being in each case positive and negative respectively. Dr. Brewster discovered that the fracture of the unannealed glass tears, called *Prince Rupert's drops*, was attended with the evolution of electrical light, which pervaded the whole drop, so that its form was distinctly visible in the dark. The light appears even when the experiment is made under water.

(216.) There is every reason to presume that electricity is essentially concerned in the processes that are carried on in the living system both of animals and vegetables. In the animal economy more particularly, the operation of this agent is indicated in the processes of secretion, in the actions of the muscles and nerves, and probably, indeed, in all the vital functions. There are several kinds of fish, which are endowed with the power of accumulating large quantities of electricity, which they can discharge at pleasure through conducting bodies that come in contact with them, and thus communicate powerful shocks. This power is possessed in an eminent degree by the torpedo, which is a species of ray; but it is also met with in the Gymnotus electricus, the Silurus electricus, the Trichiurus indicus, and the Tetraodon electricus. But as this, as well as other subjects relating to animal electricity, involve considerations which properly belong to Galvanism, we must defer treating of them until this branch of electrical science is before us.

Chapter XV.

Electricity of the Atmosphere.

(217.) As the subject of atmospheric electricity is more especially a branch of the science of Meteorology, we shall content ourselves, in this place, with a very brief outline of the principal facts relating to it.

(218.) The atmosphere is very generally in an electrical state. This may be ascertained by employing a metallic rod elevated to some height above the ground, and communicating at its lower end, which should be insulated, with an electroscope. In order to collect the electricity of the higher regions of the air, a kite may be raised, in the string of which a slender metallic wire should be interwoven, so as to conduct the electricity. If the electroscope be sufficiently sensible it will usually indicate the prevalence of positive electricity in the atmosphere, the intensity of which increases according as the stratum examined is more elevated. In the ordinary state of the atmosphere its electricity is invariably found to be positive: and is stronger in winter than in summer; and during the day than the night. From the time of sunrise it increases for two or three hours, and then decreases towards the middle of the day, being generally weakest between noon and four o'clock. As the sun declines its intensity is again augmented, till about the time of sunset, after which it diminishes, and continues feeble during the night. In cloudy weather the electrical state is much more uncertain; and when there are several strata of clouds, moving in different directions, it is subject to great and rapid variations, changing sometimes from positive to negative, and back again, in the course of a few minutes. On the first appearance of fog, rain, snow, hail, or sleet, the electricity of the air is generally negative, and often highly so; but it afterwards undergoes frequent transitions to opposite states. On the approach of a thunder-storm these alternations of the electric condition of the air succeed one another with remarkable rapidity. Strong sparks are sent out, in great abundance, from the conductor; and it becomes dangerous to prosecute experiments with it in its insulated state.

(219.) The analogy between the electric spark, and more especially of the explosive discharge of the Leyden jar, with atmospheric lightning and thunder, is too obvious to have escaped notice, even in the early periods of electrical research. It had been observed by Dr. Wall and by Gray, and still more pointedly remarked by the Abbé Nollet. Dr. Franklin was so impressed with the many points of resemblance between lightning and electricity, that he was convinced of their identity, and determined to ascertain by direct experiment the truth of his bold conjecture. A spire which was erecting at Philadelphia he conceived might assist him in this inquiry; but, while waiting for its completion, the sight of a boy's kite, which had been raised for amusement, immediately suggested to him a more ready method of attaining his object. Having constructed a kite by stretching a large silk handkerchief over two sticks in the form of a cross, on the first appearance of an approaching storm, in June 1752, he went out into a field, accompanied by his son, to whom alone he had imparted his design. Having raised his kite, and attached a key to the lower end of the hempen string, he insulated it by fastening it to a post, by means of silk, and waited with intense anxiety for the result. A considerable time elapsed without the apparatus giving any sign of electricity, even although a dense cloud, apparently charged with lightning, had passed over the spot on which they stood. Franklin was just beginning to despair of success, when his attention was caught by the bristling up of some loose fibres on the hempen cord; he immediately presented his knuckle to the key, and received an electric spark. Overcome with the emotion inspired by this decisive evidence of the great discovery he had achieved, he heaved a deep sigh, and conscious of an immortal name, felt that he could have been content if that moment had been his last. The rain now fell in torrents, and wetting the string, rendered it conducting in its whole length; so that electric sparks were now collected from it in great abundance.

It should be noticed, however, that about a month before Franklin had made these successful trials, some philosophers, in particular Dalibard and De Lors, had obtained similar results in France, by following the plan recommended by Franklin. But the glory of the discovery is universally given to Franklin, as it was from his suggestions that the methods of attaining it were originally derived.

(220.) This important discovery was prosecuted with great ardour by philosophers in every part of Europe. The first experimenters incurred considerable risk in their attempts to draw down electricity from the clouds, as was soon proved by the fatal catastrophe, which, on the 6th of August, 1753, befel Professor Richman, of Petersburg, whose name has already been before us, (§ 123.) He had constructed an apparatus for observations on atmospherical electricity, and was attending a meeting of the Academy of Sciences, when the sound of distant thunder caught his ear. He immediately hastened home, taking with him his engraver, Sokolow, in order that he might delineate the appearances that should present themselves. While intent upon examining the electrometer, a large globe of fire flashed from the conducting rod, which was insulated, to the head of Richman, and passing through his body, instantly deprived him of life. A red spot was found on his forehead, where the electricity had entered, his shoe was burst open, and part of his clothes singed. His companion was struck down, and remained senseless for some time; the door-case of the room was split, and the door itself torn off its hinges.

(221.) The protection of buildings from the effects of lightning, is the most important practical application of the theory of electricity. We have only room for a few observations on the principles on which conductors for this purpose should be constructed. They should be formed of metallic rods, pointed at the upper extremity, and placed so as to project a few feet above the highest part of the building they are intended to secure; they should be continued without interruption till they descend into the ground, below the foundation of the house. Copper is preferable to iron as the material for their construction, being less liable to destruction by rust, or by fusion, and possessing also a greater conducting power. The size of the rods should be from half an inch to an inch in diameter, and the point should be gilt, or made of platina, that it may be more effectually preserved from corrosion. An important condition in the protecting conductor is, that no interruption should exist in its continuity from top to bottom: and advantage will result from connecting together by strips of metal all the leaden water pipes, or other considerable masses of metal in or about the building, so as to form one continuous system of conductors, for carrying the electricity by different channels to the ground. The lower end of the conductors should be carried down into the earth till it reaches either water, or at least a moist stratum.

For the protection of ships, chains made of a series of iron rods linked together, are, by their flexibility, most conveniently adapted. They should extend from the highest point of the mast some way into the sea, and the lower part should be removed to some distance from the side of the ship, by a wooden spar or outrigger.

The air of close rooms, vitiated by respiration, is found to be negatively electrified.

Chapter XVI.

Theoretical Views of the Nature of Electricity.

(222.) The preceding history of the phenomena relating to electricity, may prepare us for the discussion of some interesting inquiries concerning the real nature of this powerful and mysterious agent, and the theory of its operation.

The first question that presents itself is with respect to its materiality. Besides the well-known mechanical forces which belong to ordinary ponderable matter, the phenomena of nature exhibit to our view another class of powers, the presence of which, although sufficiently characterised by certain effects, is not attended with any appreciable change in the weight of the bodies with which they are connected. To this class belong heat, light, electricity, and magnetism: each of which, respectively, produces certain changes on material bodies, either of a mechanical or chemical nature, which it is natural to regard as the effects of motion communicated by the impulse of material agents, of so subtile and attenuated a kind, as to elude all detection when we apply to them the tests of gravity or inertia. If we admit heat and light to be material, analogy will lead us to ascribe the same character to electricity and to magnetism, notwithstanding their being imponderable.

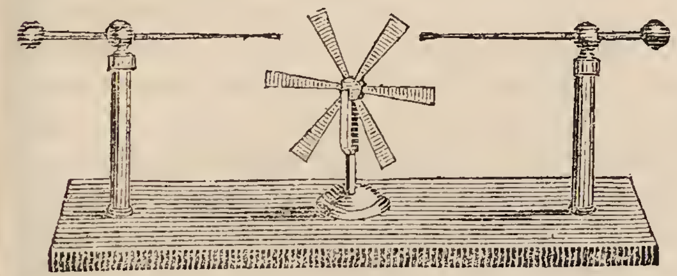
(223.) But the materiality of electricity has also been maintained on other grounds. The pungent sensation of the electric spark, the smart blow which accompanies the shock, the vivid line of light which marks its course, the varied sounds which attend its passage through the air, and the irresistible fury with which it bursts asunder the densest textures, all seem to denote the mechanical effects of sudden and powerful impulse; all seem to imply the rushing of a stream of fluid possessed of momentum adequate to produce these energetic motions. Can we refuse to ascribe the character of materiality to that which we not only see and hear, but feel also?

(224.) This argument has been endeavoured to be strengthened by a variety of experiments, from which the communication of impulse in a particular direction with respect to the species of electricity has been inferred. The stream of air, which proceeds from a pointed conductor when electricity is issuing from it, appears as if the air were carried forward along with the electric fluid. The direction of its motion is still more decidedly indicated by the different luminous appearances which accompany the escape of the fluid from, or its reception by, a pointed conductor. (See *fig.* 23.) We have already had occasion to notice the manner in which this curious fact appears to support the hypothesis of Franklin, implying the singleness of the electric fluid, (§ 98.)

(225.) The following experiment has also been adduced by Cavallo and by Singer, in support of the same opinion. Place on the table of the universal discharger a card bent lengthwise over a round ruler, so as to form a hollow cylindrical groove; or, what is still better, two straight sticks of sealing-wax, laid parallel to each other, so that the junction of their rounded edges may form a groove. In this groove place a pith-ball of about half an inch in diameter, and arrange the wires of the discharger with their points in the direction of the groove, and at four inches from each other, the ball being equally distant from each. On passing a small charge from one wire to the other, the ball will be driven from the positive to the negative wire, and this effect will be constant if the wires terminate in points; but if they are knobbed, the ball frequently vibrates between them, because the influence of the attracting surfaces upon the ball interferes with the regularity of the effect, and often renders the result equivocal.

(226.) The nature and place of the perforation effected in a card by the passage of an electric charge, of which we have already given an account (§ 159), appear to favour the same view of the subject. The following experiment, also, shews that the impulse is communicated most forcibly in the direction from the positive towards the negative conductor. A light float-wheel, the vanes of which are made of card paper, inserted in the circumference of a cork turning freely on a pin passed through its centre as an axle, will be put in motion by presenting to it an electrified point, apparently in consequence of the impulse of the stream of air which issues from the point. Whether the point be positively or negatively electrified, the direction of the motion, as well as of the stream of air, is always the same. But if the wheel be placed on an insulating stem, as in *fig.* 42, and introduced between the pointed wires of the universal discharger, which are to be placed as accurately as possible opposite to each other, and at the distance of an inch or more from the upper vanes; on connecting one of the wires with the positive, and the other with the negative conductor of an electrical machine, and exciting it, the wheel will move as if impelled by a stream from the positive to the negative wire. On reversing the connections, so that the electricity of each wire is changed, the motion of the wheel will likewise be reversed.

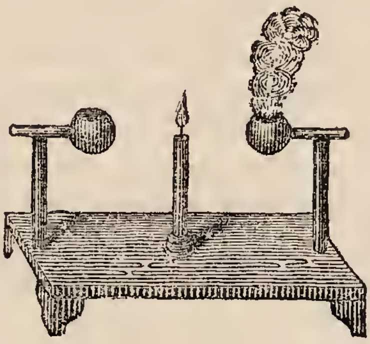
Fig. 42.



(227.) If a card be placed vertically, by inserting it in a small piece of cork that may form a base of about a quarter of an inch wide for it to stand upon, but so that it may be overthrown by the smallest impulse; and the pointed wires of the universal discharger be brought opposite to each other, and about a quarter of an inch below the upper edge of the card, which stands at an equal distance between them; on connecting the wires with a machine, or with an insulated jar, so as to effect an electric discharge between them, the card will be thrown down, and will constantly fall from the positive, and towards the negative wire.

(228.) The determination of a stream of electrified air in this direction is also rendered very sensible by the motions of smoke or vapour placed in the circuit of the electricity. Thus the flame of a taper placed between two oppositely electrified balls will constantly be blown from the positive to the negative side. *Fig*. 43 represents two hollow metallic balls, about three quarters of an inch in diameter, insulated on separate glass pillars, by which they are supported at a distance of two inches from each other: the upper part of each ball is hollowed into a cup, into which a small piece of phosphorus is to be put. A small candle has its flame situated mid-way between the balls, one of which is connected with the positive, and the other with the negative conductor of the machine. When the balls are electrified, the flame is agitated, and inclining towards the one which is negative, soon heats it sufficiently to set fire to the phosphorus it contains, whilst the positive ball remains perfectly cold, and its phosphorus unmelted. On reversing the connections of the balls with the machine, the phosphorus in the other ball will now be heated and will inflame.

Fig. 43.



(229.) However plausibly it may have been inferred, from a superficial view of these facts and experiments, that the electric fluid actually possesses momentum, and that it moves in a particular direction, a more rigid analysis of the phenomena will show that they in no degree warrant such an inference. All the mechanical effects that attend the transfer of electricity are ultimately resolvable into the sudden action of a repulsive power exerted among the particles of matter which are situated in the line of its course. They are only partiticular instances of the fundamental law of electric action, that bodies charged with the same kind of electricity repel one another. Thus the particles of air electrified by a pointed conductor are repelled by that conductor, and repel it also; and, moreover, repel one another: and the same effect takes place whether their electric state be of the positive or negative kind. Hence the stream of air which proceeds from any electrified point is very naturally accounted for. If the quantity of electricity which is transferred is considerable, it excites a more violent commotion among the particles which it influences in its passage. The intense energy of its repulsive action produces the most sudden and forcible expansion of that portion of the air which occupies this line; this air thus expanding must be impelled laterally against the surrounding particles, and must occasion their sudden compression. The evolution of heat and light is the necessary consequence of this violent compression; and the vibratory impulse being propagated in all directions is the source of the sound which attends the electric explosion. The sensation to which the passage of the electric shock through our bodies gives rise, is also, evidently, referable to an impression made on the nerves by the same repulsive action. In all this we can discern no positive proof of the operation of a material agent extraneous to the body itself and acting by mechanical impulse. The materiality of electricity, therefore, must still rest upon a similar foundation with that of heat, or of light.

(230.) If the electric power, or fluid, if we choose to consider it as such, does not act by its mechanical momentum, the arguments in favour of the motion of a single fluid from the positive to the negative body, derived from the appearances of the streams of electric light, (§ 97, 98,) the impulsion of a pith-ball, (§ 225,) the perforation of a card, (§ 159,) the rotation of a windmill, (§ 226,) and the determination of the flame of a taper (§ 228) in one constant direction, must fall to the ground, and can evidently be of no avail in deciding the great question, whether there be two electric fluids or only one. But, still, it is incumbent upon us to inquire upon what principle these remarkable differences in the phenomena of positive and of negative electricity can be accounted for, consistently with either hypothesis.

(231.) On an attentive examination of the phenomena they appear to be explicable on the supposition that the air, or medium through which the electricity passes, is, in the language of one theory, more disposed to admit of the passage of the vitreous than of the resinous electricity; or, to speak consistently with the Franklinean theory, that it is more disposed to receive the electric fluid from a conductor which is charged with it, than to part with it to an undercharged conductor which absorbs it. The consequences of this hypothesis are, that the vitreous electricity meets with less resistance in passing out from a body into the air, and is therefore carried forward more readily and more directly than the resinous electricity. The latter, in consequence of meeting with greater resistance to its exit, is more diffused in the surrounding space.

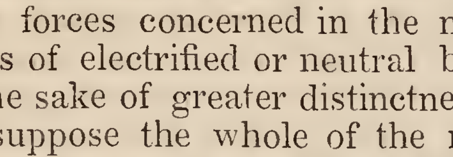
On the Franklinean theory the same effects will follow with reference to the propulsion of the electric fluid from the positive, and its absorption by the negative body.

(232.) That the peculiarity of the mechanical effects of the different species of electricity depends upon the properties of the air, which is the vehicle of its agency, and not upon any specific power in the agent itself, is shown by a modification of the experiment described in § 159, in which a varnished card, suspended between two conductors, was perforated at the point where it was touched by the negative wire. On repeating the same experiment under the receiver of an air-pump, Mr. Tremery found, that in proportion as the air is exhausted, the place where the card is perforated by the electric shock approaches nearer to the positive wire. When the pressure of the air is reduced to one-half, the hole is at the middle point between the two wires. At every discharge, a flash is seen to pass from each conductor to the place of perforation. The curious appearances presented by the edges of the perforations made in the leaves of a quire of paper, already detailed (§ 160, 161,) are not reconcileable with the supposition of a mechanical impulse acting only in one direction, but indicate the equal repulsive action of both kinds of electricity, when the disturbing influence of the air is withdrawn. It is a confirmation of this hypothesis, respecting the peculiar kind of obstruction which air opposes to the passage of electricity, that other substances have been discovered in which a similar property exists. Mr. Ermann, of Berlin, has found that the flame of alcohol is possessed of a greater conducting power with regard to positive, than to negative electricity. Alkaline soap, on the contrary, conducts negative electricity better than positive; and will, therefore, serve to insulate a feeble degree of the latter, at the same time that it permits the passage of the former.

(233.) It has always been urged as a strong objection to the theory of a single electric fluid, that it necessarily involves the condition of a mutual repulsion among the particles of ordinary matter. See § 52. Before attempting to combat this objection, it will be proper to enter into a somewhat fuller illustration of the position than we have already done; and for this purpose we shall avail ourselves of the assistance of a few diagrams, calculated to aid our conceptions of the forces concerned in the mutual actions of electrified or neutral bodies. For the sake of greater distinctness, we shall suppose the whole of the matter in the body, of which we are studying the actions, to be concentrated in a small space, and we shall represent this matter by a black square. In like manner, we shall suppose that the whole of the electric fluid contained in the same body is condensed into a small space, denoted by a white circle. The mutual actions of the matter or electric fluid in two adjacent bodies, are expressed by lines passing from the one to the other respectively; the attractions being distinguished by unbroken lines, and the repulsions by dotted lines.

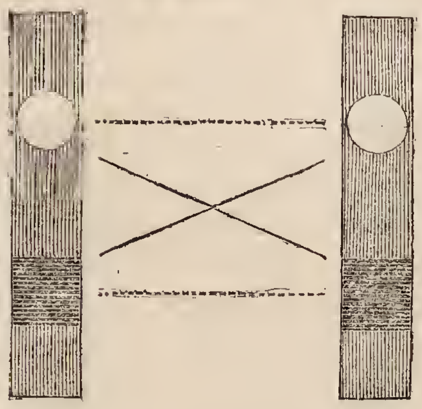
(234.) *Fig.* 44 represents a body B in a neutral state of electricity, by which is to be understood, that the quantity of fluid it contains exists in a proportion so exactly adjusted to the quantity of matter, as that its repulsion for a particle F of electricity at any distance, precisely balances the attraction of the matter for that same particle. While this equilibrium is preserved among the forces which would impel any electric fluid external to the body both from it and towards it, it is evident that the body will neither acquire nor lose electricity, but remain quiescent, whether it be insulated or not.

Fig. 44.



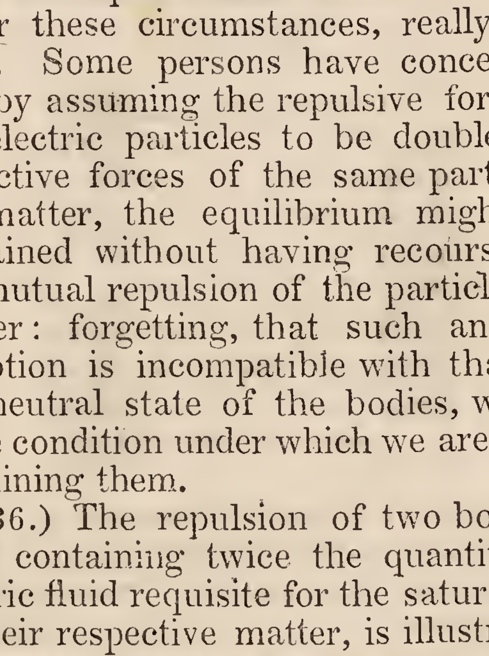
(235.) The state of the forces operating between two similar neutral bodies is shown in *fig.* 45. Here it follows *from the condition of neutrality, as above defined*, that the two attractive forces, denoted by the two whole lines, are *each of them* equal to the repulsive force between the two fluids, denoted by the upper dotted line. Actuated by these forces only, therefore, the two bodies would attract each other. The addition of a second repulsive force between the two portions of matter, as represented by the lower dotted line, is therefore necessary to account for the state of equilibrium which we find, under these circumstances, really obtains. Some persons have conceived, that by assuming the repulsive force of the electric particles to be double the attractive forces of the same particles for matter, the equilibrium might be explained without having recourse to the mutual repulsion of the particles of matter: forgetting, that such an assumption is incompatible with that of the neutral state of the bodies, which is the condition under which we are now examining them.

Fig. 45.



(236.) The repulsion of two bodies, each containing twice the quantity of electric fluid requisite for the saturation of their respective matter, is illustrated by *fig.* 46. All the forces represented

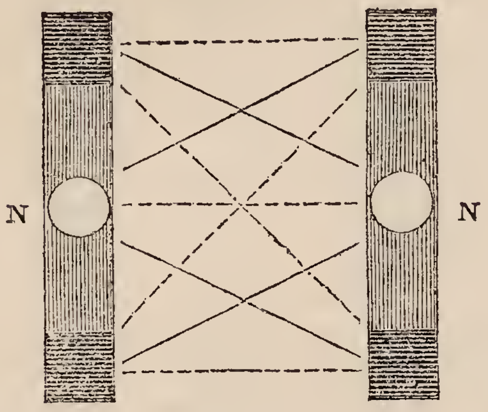
Fig. 46.



by the lines must, from the hypothesis, be regarded as equal in point of intensity: but the number of repulsive forces is as five, while that of the attractive forces is only as four: the former therefore will prevail.

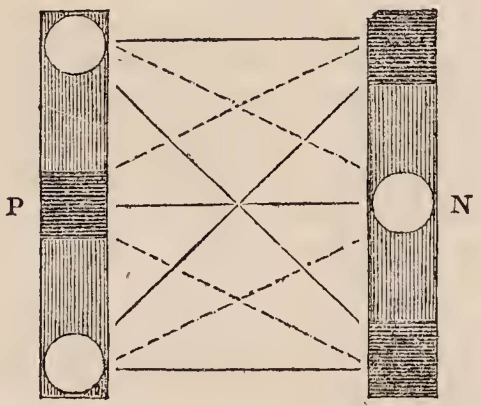
(237.) Precisely the same result will obtain in the case of two negatively electrified bodies, in which, as represented in *fig.* 47, the quantity of matter is twice as much as the fluid can saturate. In the former case it was the repulsion between the two portions of fluid which were in excess that destroyed the equilibrium; while in this case the same effect is produced by the mutual repulsion of the unsaturated portions of matter.

Fig. 47.



(238.) Lastly, we may collect from an examination of *fig.* 48, where a body positively electrified is supposed to be placed near one that is negatively electrified, that the ultimate effect will be determined by the attraction between the fluid in excess in the former, and the unsaturated matter in the latter: all the other attractions and repulsions exactly compensating each other.

Fig. 48.



(239.) It is a great, though a common error to imagine, that the condition assumed by Æpinus, namely, that the particles of matter, when devoid of electricity, repel one another, is in opposition to the law of universal gravitation established by the researches of Newton; for this law applies, in every instance to which inquiry has extended, to matter in its ordinary state, that is, combined with a certain proportion of electric fluid. By supposing, indeed, that the mutual repulsive action between the particles of matter is, by a very small quantity, less than that between the particles of the electric fluid, a small balance would be left in favour of the attraction of neutral bodies for one another, which might constitute the very force which operates under the name of gravitation: and thus both classes of phenomena may be included in the same law.

(240.) An objection has been urged by Biot against the hypothesis of a single fluid, on the ground that it implies an equal degree of attraction between the fluid and every species of matter, whereas in the case of other agents, such as heat and magnetism, the degree of their attraction is very different towards different kinds of matter. This objection does not apply to the hypothesis of the two fluids, for they are assumed as acting independently of any specific attractions for the bodies which contain them: hence their distribution in those bodies follows the same law, whatever be the specific nature of the materials of which the latter are composed.

(241.) We arrive, then, at the conclusion that there is no fact in electricity which cannot be explained on either of the two hypotheses: but to which side the balance of probabilities may incline, when the respective merits and demerits of each are taken into account, remains, perhaps, to be decided more by the taste than the judgment of the inquirer.

ERRATA IN PART I.

Page.	Col.	Line.	
6,	1,	3,	*from bottom, omit the line*—"Ice above—13° Fahrenheit."
7,	1,	10.	*omit*—"of six inches, that is."
9,	1,	36,	*for* "resinous," *read* "vitreous."
ib.	1,	37,	*for* "vitreous," *read* "resinous."
20,	1,	1,	*for* "silken," *read* "silk."
22,	1,	10,	*for* "spheroids," *read* "spheroid."
ib.	2,	10,	*from bottom, for* "smaller," *read* "greater."

(7)

ROGET, Peter Mark

MAGNETISM.

Chapter I.

General Facts and Principles.

(1.) The attractive power of the loadstone for iron was known in times of very remote antiquity, and has been, in all ages, a subject of curiosity and of wonder. It is a property which seems, at first sight, so unconnected with every other, as to form of itself a separate class among natural phenomena: and, although an immense mass of knowledge relating to Magnetism has been accumulated by the labours of successive generations, and embodied into a science of high rank and importance, yet the field it comprises is of comparatively limited extent. This arises from the great simplicity which characterises both the phenomena and the laws that govern them; a quality, however, which peculiarly invites a philosophic mind to undertake their investigation. A still more powerful motive to this inquiry will present itself when we reflect on the signal benefits mankind has derived from magnetism as applied to the purposes of navigation. The discovery of the compass, by the aid of which, the mariner, however distant from land, amidst cloudy skies, or in the darkest nights, is enabled, at all times, to steer his course with certainty, and traverse in all directions the wide expanses of ocean which separate the countries and continents of our globe, must unquestionably rank among the great discoveries that have essentially contributed to advance the civilization of the human race.

(2.) The term Magnetism expresses the peculiar property occasionally possessed by certain bodies, more especially by iron and some of its compounds, whereby, under certain circumstances, they mutually attract or repel one another, according to determinate laws.

(3.) This property was first noticed in a mineral substance called the *native magnet*, or the *loadstone*, which is an ore of iron, consisting chiefly of the two oxides of that metal, together with a small proportion of quartz and alumina. Its colour varies in different specimens, according to minute differences in the proportion of the oxides, and the nature of the other substances with which they may be found united: but it is usually of a dark grey hue, and has a dull metallic lustre. It is found in considerable masses in the iron mines of Sweden and Norway, and also in different parts of Arabia, China, Siam, and the Philippine Islands. Small loadstones have occasionally been met with among the iron ores of England.

(4.) There are several modes in which a piece of iron may be rendered magnetic, or converted into what is called *an artificial magnet*; and for all purposes of accurate experiment such a magnet is much to be preferred to a loadstone. The following is a simple and ready method of obtaining artificial magnets with a view to the investigation of the magnetic properties.

Let a straight bar of hard tempered steel, devoid of all perceptible magnetism, be held in a vertical position (or still better, in a position slightly inclined to the perpendicular, the lower end deviating to the north,) and struck several smart blows with a hammer; it will be found to have acquired, by this process, all the properties of a magnet.

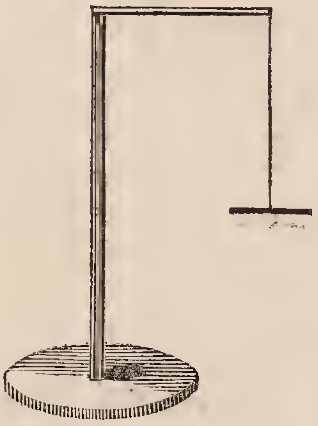
(5.) These properties are the four following:—viz. 1. Polarity. 2. Attraction of unmagnetic iron. 3. Attraction and repulsion of magnetic iron. 4. The power of inducing magnetism in other iron. These we shall now explain and illustrate.

§ 1. *Polarity.*

(6.) If a bar, which has been rendered magnetic, be supported in such a manner as to have entire freedom of motion in a horizontal plane, and be removed from the neighbourhood of all ferruginous bodies which might influence it, it will spontaneously turn round, and, after a few oscillations, will finally settle in a position directed nearly north and south. If it be disturbed from this situation and placed in any other direction, it will, as soon as it is again at liberty to move, resume its former position. The end of the bar which points to the north, is that which was lowermost at the time it acquired its magnetism by hammering: the end which, during that operation, had been the upper one, is consequently that which, when the magnet is free to move, directs itself to the south. The two ends of a magnet of this form are called its *poles:* the one which spontaneously turns to the north, being distinguished as the *north,* and the other as the *south pole:* and the tendency of the magnet to assume the above described position is called its *Polarity.* The straight line joining the two poles of a magnet is called its *axis.*

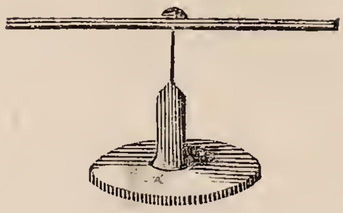
(7.) There are several ways of supporting a magnet so as to enable it to manifest its polarity. The readiest mode is to suspend it by a thread, fastened round it at the middle, so that it may be sufficiently balanced to preserve its horizontal position as it turns freely round its centre. It cannot, indeed, turn thus, without, at the same time, either twisting or untwisting the thread by which it hangs; and the reaction of the thread, the fibres of which tend to resume their original situation, or the force of torsion, as it is called, may prevent the magnet from assuming the precise position to which its polarity would have brought it. But by employing a very slender thread, and taking it of sufficient length, the force of torsion may be so much reduced as to be quite insensible in the experiments about to be described.

Fig. 1.



(8.) Another convenient mode of examining the horizontal movements of the magnetized bar is to poise it on its centre, hollowed into a cap, which is made to rest on a fine point fixed in a stand.

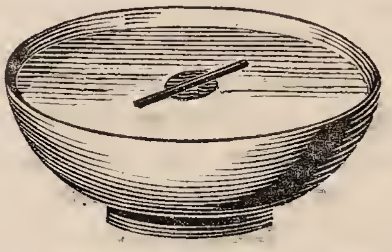
Fig. 2.



When thus fitted up, it acts like the needle of a mariner's compass; and in its principle is identical with that instrument.

(9.) We may sometimes find it more expedient to fix the magnet on a piece of cork, and thus make it float on water, in a basin. In this case, we must take care, however, that it be kept at a sufficient distance from the sides of the vessel to prevent its being affected by the capillary attractions of the water.

Fig. 3.

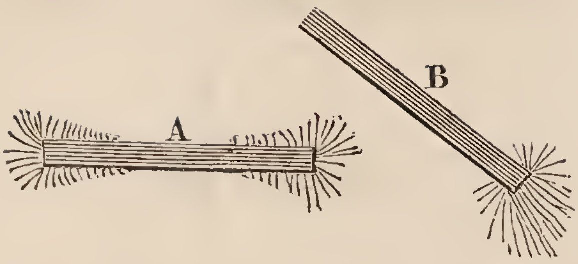


The same precaution must be used if the magnet be made to float on the surface of mercury, which is an excellent mode of giving it complete liberty of motion. But the vessel containing the mercury should be at least six inches in diameter, in order to guard against the effects of the curvature of the surface of the mercury near the sides. When the surface of the mercury is very clean and bright, which happens only when the metal is very pure, it allows of the ready motion of pieces of iron floating upon it. But it soon tarnishes, and the film of oxide which forms on the surface, becomes a great impediment to the freedom of motion of the floating body. The best way of rendering it clean, is to strain it through a funnel of paper, rolled up into a cone, having a small aperture at the point, of about the fortieth of an inch in diameter.

§ 2. *Attraction of Iron.*

(10.) If either pole of a magnet be brought near any small piece of soft unmagnetic iron, it will be found to attract it. Iron filings, for instance, are immediately collected together when a magnet is placed among them; and they adhere more especially to the poles (as shewn in *fig.* 4, A), from which,

Fig. 4.



when the magnet is lifted up, they remain suspended in thick clusters. (*Fig.* 4, B.) A small number of filings are also found adherent to the intermediate parts of the bar, but they are evidently attracted much more feebly than those at the ends; it may also be remarked that there is a part of the magnet, generally mid-way between the two ends, to which the filings have no tendency to adhere at all, and which appears therefore to have no power of attraction. Thus it appears that the attractive forces, whatever be their nature, reside chiefly at the poles.

(11.) It is an established law of nature, the knowledge of which we have derived by induction from a vast variety of phenomena occurring in every part of the material universe, that all *action* is attended by a corresponding *reaction*, equal in degree, but opposite in its kind, to the action itself. Mechanical philosophy, in all its departments, abounds with exemplifications of this fundamental principle; many of these, indeed, are matters of familiar observation. The stretched rope pulls back with equal force at both its ends; the compressed spring resists equally in two opposite directions; the exploding powder, at the same moment that it propels the ball, gives to the gun its recoil. In all the effects resulting from cohesion, from elasticity, from caloric, from animal force, from gravitation, whether actuating the minutest particles of matter, or the largest masses; whether exerted on the rolling waters of the ocean, or displayed on the grander scale of the planetary movements, the same universal law is rigidly observed. To every physical force there is opposed another and a similar force. No material agent can produce an effect upon another, without being at the same time subjected to an equal reaction from that other agent. An attracting body must of necessity be itself attracted, and a repelling body repelled. This perfect reciprocity of action takes place in all the agencies of electricity;—it exists also in those of magnetism.

(12.) If the two bodies exerting a mutual action upon one another, be very different in their size, the smaller of the two will necessarily exhibit the effects of this action more strongly than the larger, because, its mass being less, the same force will communicate to it a greater velocity of motion. We find, accordingly, that the small fragments of iron in the experiment just described, appear to fly towards the magnet, while their reciprocal action on the magnet itself is imperceptible. But this latter action may be rendered sensible by trying its effect on a magnet poised or suspended in any of the ways above mentioned; and we shall find that on presenting a piece of soft iron to either of the poles of the magnet, the latter is slowly attracted by the iron. The attraction, therefore, between the magnet and the iron is reciprocal.

Let us next see what influence magnets have upon one another.

§ 3. *Attraction and Repulsion of Magnetic Iron.*

(13.) For the purpose of examining the mutual action of two magnets, we may either present to the poised magnet another magnet held in the hand, or we may place two poised magnets in different positions with respect to each other. We shall find by sufficiently varying these positions, that when the poles of different magnets are brought near one another, they in some cases appear to be attracted towards each other, while in others they manifest a mutual repulsion. This, however, does not happen capriciously; for if we mark the poles according to the distinction already pointed out, we shall find that two north poles always repel each other:—that two south poles also repel each other:—but that the north pole of one magnet invariably attracts, and is of course attracted by the south pole of another magnet.

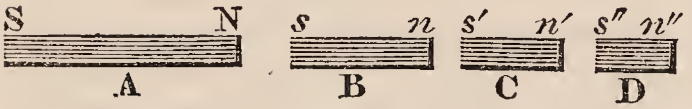
(14.) It thus appears that there are two species of magnetic powers, the northern and the southern, which in their mode of action are perfectly similar, but in their effects are directly opposite.

(15.) Such of our readers as have studied our Treatise on Electricity must here be struck with the pointed analogy which subsists between the phenomena of magnetic attraction and repulsion, and those of electricity. In both there exists the same character of double agencies of opposite kinds, capable, when separate, of acting with great energy, but being, when combined together, perfectly neutralized and exhibiting no sign of activity. As there were two electrical powers, the positive and the negative, or, as some prefer denominating them, the vitreous and the resinous, so there are two magnetic powers, distinguished as the northern and the southern polarities; or, as some choose to designate them, the austral and the boreal. The parallel is most exact. Both sets of phenomena are governed by the same characteristic law, which may be expressed by the following concise and general formula, namely, —between like powers there is repulsion; between unlike, attraction.

§ 4. *Induction.*

(16.) The communication of magnetic properties to iron or steel by the mere approach of the poles of a magnet, is also analogous in its principal circumstances to electric induction.

Fig. 5.



If the north pole, N, of a magnet A (*fig.* 5), be brought near to the end *s* of an unmagnetized bar of iron, B, that end will immediately acquire the properties of a south pole, while the opposite, or distant end, *n*, will at the same time be converted into a north pole. If, instead of the north pole, N, the south pole, S, had been presented to the bar, the changes effected in B would have been just the reverse; the adjacent end would have acquired the northern, and the distant end the southern polarity.

(17.) Thus we may observe that each pole of a magnet induces the opposite kind of polarity in that end of the iron which is nearest to it, and the same kind on the remotest end; just as happens in the induction of electricity, in which the positive state induces the negative, and the negative the positive state, in those parts of an insulated conductor which are nearest to the electrified body; and a similar state of electricity in the distant end.

(18.) That the iron, while it remains in the vicinity of the magnet, possesses the magnetic properties, may be shown by a variety of experiments.

First, it attracts other iron. If we take, for instance, a key (*fig.* 6), and hold it horizontally near one of the poles of a strong magnet, also lying in a horizontal position, but not touching the key; and if we then apply another light piece of iron, such as a small nail, to the other end of the key, the nail will hang from the key, and will continue to do so while we slowly withdraw the magnet horizontally from the key. When the magnet has been moved beyond a certain distance, the nail will drop from the key, because the magnetism induced on the key becomes, at that distance, too weak to support the weight of the nail. That this is the real cause of its falling off may be proved by taking a still lighter fragment of iron, such as a piece of very slender wire, and applying it to the key. The magnetism of the key will still be sufficiently strong to support the wire, though it could not support the nail: and it will continue to support the wire, even when the magnet is yet further removed; at length, however, when the distance is still greater, the wire, in its turn, drops off.

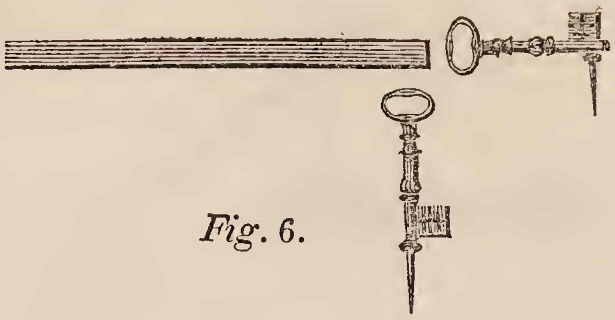


Fig. 6.

The same effects may be observed if the nail be placed in contact with the near end of the key; but they are generally less distinct, on account of the direct influence which the magnet exerts on the nail, and which interferes in some degree with the action of the key, so that the results become complicated.

(19.) The same series of phenomena take place when the key is held above or below the pole of the magnet, or on either side of it. The key will hold the nail or wire suspended from either end, as long as the magnet is near enough to exert sufficient influence on the key.

(20.) If the key be laid upon a piece of paper on a table, and several small bits of wire, or iron filings, be strewed round one end of the key, which we suppose to be devoid of all magnetic properties, no adhesion will be perceptible between the fragments of iron and the key. Let us now approach the pole of a magnet to the other end of the key; we immediately observe the filings and lighter pieces of iron spontaneously, and of one accord, move towards the key, and adhere to it just as if the key had itself become a magnet. They also collect and cohere together, as if animated by a common sympathy. When this has taken place, let us suddenly remove the magnet: that moment all these effects cease at once, the key returns to its natural or unmagnetic state; the bits of iron which had attached themselves to it immediately fall off, and show no tendency either to cohere among themselves, or to adhere to the key.

(21.) Secondly, the vicinity of a magnet to a piece of iron gives it the property of attracting and repelling the respective poles of another magnet, in the same way as a magnet would have done. The truth of this proposition may easily be proved by placing a small compass needle poised as in *fig.* 2, in various situations relative to the ends of the key or any other piece of iron of a lengthened shape, while in the vicinity of the magnet. It will be seen by this examination that the piece of iron has acquired by induction two poles, the qualities of which will be discovered by their attractions or repulsions of the poles of the compass needle, as they are respectively presented to each; and it will be found that these two poles are disposed in the manner specified above.

(22.) Thirdly, the iron, which has become magnetic by induction, has at the same time acquired the power of inducing a similar state of magnetism on the iron in its neighbourhood. Thus, while the bar B, *fig.* 5, is rendered magnetic by the influence of the magnet A, it exerts itself a similar power on another bar C, rendering it also magnetic. The bar C, in its turn, will act in like manner upon another bar, D, and so on. In this way the influence of the magnet A may be made to extend along a series of iron bars or pieces of any other shape, each acquiring magnetism by the inductive power of the preceding piece; and in its turn inducing magnetism on the next.

(23.) But this is not all. The piece of iron which has been rendered magnetic by the vicinity of a magnet, not only acts upon the other iron that is near it, but also reacts upon the magnet from which its power is derived, and increases the intensity of its magnetism. The power of a magnet is, in fact, augmented by the exertion of its inductive influence on a piece of iron in its neighbourhood. A simple experiment is sufficient to prove this fact.

Let a piece of iron be suspended from one of the poles of a straight magnet; and let the weight which this magnet will carry be ascertained by attaching to the iron a scale, capable of holding the weights necessary for this trial, and which may be gradually increased till the piece of iron drops off from the magnet. Repeat this experiment, having previously placed a bar of iron in contact with the other pole of the magnet, and it will be found that the magnet will now support a much greater weight; showing the increase of power it has derived from the presence of the bar of iron which has been applied to the other pole, and the induced magnetism of which, although solely derived from the magnet, reacts, by a kind of secondary induction, upon that magnet. We have already had occasion, in the Treatise on Electricity, to notice the same kind of reaction in the case of electric induction. The increased intensity which a magnet acquires by induction often leads to the permanent acquisition of power by the magnet. Hence we may understand the reason why a magnet that is employed for magnetizing a neutral bar of steel, by means of its inductive power, becomes itself stronger by the operation.

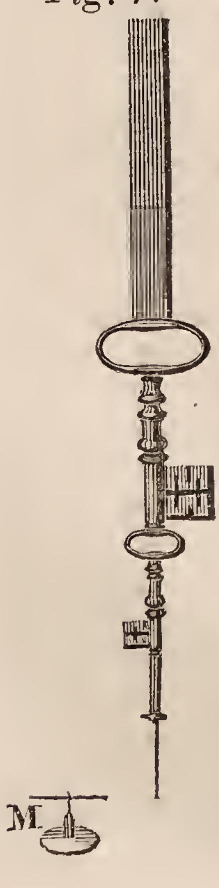
(24.) It is a necessary consequence of the law of magnetic induction that it is accompanied by attraction: for the polarity of the adjacent end of the piece of iron on which the magnetism is induced, is always of the opposite kind to that of the pole of the piece which induces it: according to the fundamental law of magnetism, therefore, a mutual attraction must take place between them. The remote end of the piece on which the magnetism has been induced is indeed repelled, because its polarity is similar to that of the inducing pole: but it is evident that the attractive action of the adjacent and dissimilar poles will always be stronger than the repulsive action of the more distant poles; and will therefore always prevail.

(25.) This remark leads us to a very important step in the generalization of the magnetic phenomena. We have hitherto spoken of the attractive power of magnets for iron as one of the primary facts in the science: but we now see that it is merely a necessary result of a more general law, namely, that of induction, together with the law of action of the two polarities upon each other:— or, in other words, that it is itself comprehended in these more general facts. A magnet attracts a piece of unmagnetic iron, not from any inherent disposition to attract it in that state, but in consequence of its inductive influence, which converts it, for the time, into a second magnet, having its poles so disposed with relation to the first magnet, that the adjacent parts have always opposite polarities; and attraction, therefore, takes place between them. Thus the pieces A and B, *fig*. 5, attract each other, simply because the induced magnetism of the end *s* is of the opposite kind to that of the pole N. In like manner B and C attract each other, because the polarities of the adjacent poles *n* and *s′* being of a dissimilar kind, their mutual action is attraction. With respect to this ultimate effect, the inductive influence of either pole is exactly alike, and leads to the same result.

(26.) We may now understand the reason why, when a magnet is placed in a heap of iron filings, and then lifted up, the filings attach themselves in clusters to the poles, arranging themselves in lines, and adhering together by a force of attraction which extends from each individual particle to those which precede and follow it. They form, indeed, by their mere juxtaposition, under the influence of the large magnet, a series of minute magnets, of which the poles are similarly situated in each, and being alternately north and south, the adjacent ends attract one another.

(27.) This disposition of the poles may be verified by making an experiment of the same kind on a larger scale, suspending from the end of a strong magnet a piece of iron, such as a key (*fig*. 7), from the lower end of which a smaller key may be made to hang in consequence of its induced magnetism. To this may be appended a still smaller piece of iron, such as a nail; and we may thus proceed, adding piece after piece, till the lower one will exert only sufficient attraction to sustain a very small weight of iron, such as a small needle. The polarities of the lower ends of each piece, if examined previously to each additional piece being appended to it, will be found to be constantly of the same kind as that of the lower end of the magnet from which the whole is suspended. This may be ascertained by its attracting or repelling the poles of a small magnetic needle balanced on a point, and supported on a stand, as shown at M, *fig*. 7.

Fig. 7.



(28.) The knowledge of the general fact, that magnetic induction always tends to produce attraction between the adjacent parts of the bodies which act upon each other, enables us to explain many phenomena, which might otherwise appear to be at variance with the simple laws of attraction and repulsion already stated. Thus we find that the dissimilar poles of two magnets attract each other with a force which is greater than the repulsion exerted between the similar poles of the same magnets; and this happens because, in the former case, the tendency of induction is to strengthen the magnetic power of the adjacent dissimilar poles; but, in the latter case, where the poles are similar, each pole tends, by its inductive influence, to weaken the magnetism of the other. This is exemplified in a still more striking manner, when a weak magnet is brought near to a much more powerful one: in which case we find that, although when they are at a moderate distance from each other, either pole of the weaker magnet is repelled by the similar pole of the strong one; yet, if it be brought very near, and especially if it be made to touch the latter, it is attracted, and will even adhere with some force to the strong magnet. This effect evidently results from the powerful inductive influence of the strong magnet, which, for the time, destroys the feeble polarity of the weak magnet at the part immediately adjacent, and impresses upon it a polarity of an opposite kind: whence attraction follows as a necessary consequence of contrary polarities.

(29.) The intensity of magnetic power developed by induction in an iron bar, was found by Mr. W. S. Harris to be inversely as the distance of the inducing pole from the adjacent end of the bar on which it acts. It would also appear, from the experiments of the same gentleman[*], that the intensity of the magnetism induced on the remote end of the bar, is, with the same inductive power acting on the nearer end, inversely as the length of the bar.

Magnetic induction is not confined in its operation to any particular direction; thus a bar of iron will be rendered magnetic if placed at right angles, or at any other inclination to the axis of the magnet, or line joining its two poles, as in the situations represented in *fig*. 8. The ends, *s*, *s*, of the bars B, C, adjoining the north pole N of the magnet A, will still become south poles by induction, and the ends *n*, *n*, north poles. Under these circumstances, if the inclination be less than a right angle, as in the case of C, the opposite pole of the magnet *s* begins to exert an inductive influence on the other end of the bar *n*, which concurs with that of the pole N in rendering *n* a north and *s* a south pole. The most favourable position for the bars receiving the full inductive influence of both poles is that of parallelism with the magnet A, as shown in *fig*. 9.

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[*] Transactions of the Royal Society of Edinburgh for 1829.

Fig. 8.

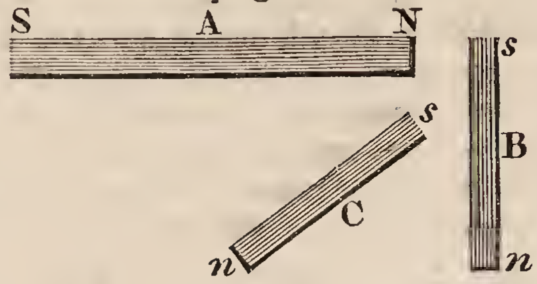
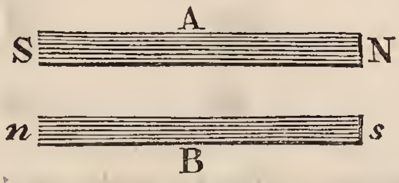
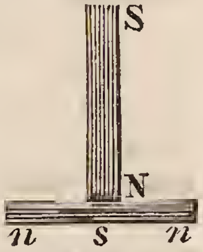


Fig. 9.



(30.) Complicated effects result from bringing the magnet in contact with other parts than the ends of a piece of iron. Thus, if the north pole of a magnet be placed in the middle of an iron bar, as in *fig*. 10, both extremities of the bar are rendered north poles, while the middle is a south pole.

Fig. 10.



(31.) If the north pole of a magnet be placed on the centre of a round iron plate (*fig*. 11), so that its axis may be perpendicular to it, the plate will have a south pole in its centre, and every part of its circumference will have the properties of a weak north pole. If the plate have the form of a star (*fig*. 12), each of the points will have a stronger northern polarity than in the last case. Analogous effects may be observed in pieces of iron of an irregular shape when acted upon by a magnet; the part immediately adjoining to the north pole of the magnet acquires the properties of a south pole, and all the remote protuberances have a feeble northern polarity.

Fig. 11.

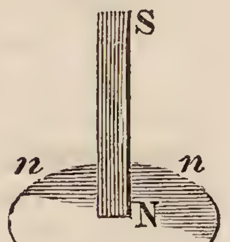
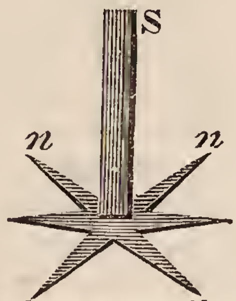


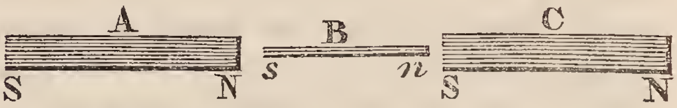
Fig. 12.



§ 5. *Complex Induction.*

(32.) When two magnets are placed so as to exert an inductive influence on the same bar, it will depend on their relative position, whether they shall conspire to produce the same polarities in the ends of the bar, or whether they shall oppose each other, and produce contrary polarities. If the bar B, *fig*. 13, lie between the two magnets A and C, and be in the same line with them, and if N, the north pole of A, be adjacent to S, the south pole of C, the intermediate bar B will receive a magnetism of the same kind from the inductive power of both the magnets, its south pole *s* being adjacent to N, and its north pole *n* to S. Its magnetic power will, therefore, be considerably greater from the united influence of the two magnets, than it would have been from the influence of one only.

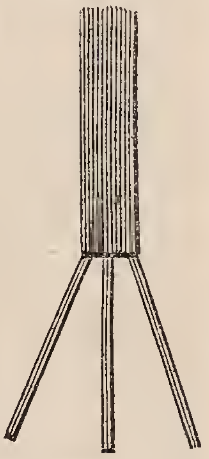
Fig. 13.



(33.) It may here be observed that the order in which the poles of these pieces A, B, and C, succeed one another, is exactly the same as that which obtains in the successive induction of magnetism along a series of iron bars, as in *fig*. 5. Now the same consequences follow from this arrangement in the one case as in the other. We have just seen that when C is a bar already rendered magnetic, the magnetism of the iron bar B, which the magnet A had induced upon it, is increased by the presence of C. In like manner, we find that the magnetism of the bar B, in the case above referred to, *fig*. 5, is increased by the presence of another piece of iron C, placed at its end, although that piece, previously to its being so placed, was entirely free from magnetism. This increase is owing to the piece C having become magnetic by its position with respect to B, which had itself been rendered magnetic by the vicinity of the magnet A. C having thus become a temporary magnet by the action of B, reacts upon B, and, exerting a new inductive influence, tends to increase its magnetism in the same manner as if it had been a permanent magnet. Thus it is that in a series of iron bars held together by induced magnetism, each piece tends to increase the strength of the preceding piece, and the whole coheres together with greater force than if no such reaction took place. This circumstance affords a further explanation of the strong cohesion we observe among the particles of iron filings, which hang in long threads from the poles of a magnet.

(34.) A closer attention to the consequences which flow from magnetic induction, will also enable us to explain another remarkable fact, which the adhesion of the strings of iron filings presents. It is that each separate filament, although composed of parts that attract each other in the direction of their length, yet shew a tendency to keep distinct from the neighbouring filaments, and even appear to repel one another. In order to understand this, let us consider the condition of several slender iron bars placed side by side, and adhering to the north pole of a magnet, as shewn in *fig*. 14. The inductive power of the magnet, as we have seen, will render each of the ends in contact with that pole, a south pole, while all the remote ends will be north poles. Hence, the bars will all have their similar poles near each other, and this will happen at both their extremities, and they will accordingly repel one another. As long as they adhere to the magnets by one end, this repulsion will be prevented by that adhesion from shewing itself; but at the other ends, which are at liberty to move, it will be strongly manifested, and the bars will be observed to separate or diverge from one another. Now this is very nearly the condition of the filaments composed of particles of iron. The polarities of those parts of each which are in contact, are neutralized and become scarcely sensible; but those of the extremities, being uncompensated, exert their full power, and produce the observed repulsion of the filaments.

Fig. 14.



(35.) This effect of induction is exceedingly well illustrated by the following experiment of Cavallo: let two short pieces of iron wire, *fig*. 15, be each fastened to a thread, the threads being joined at their other ends and formed into a loop, by which they are to be suspended from a hook or pin, so as to have full liberty to move. On bringing the pole of a magnet, the south pole for instance, at a certain distance below the wires, it will occasion them to recede from each other, as shewn in *fig*. 16, indicating the repulsion which takes place between the adjacent ends of the wires, in consequence of their being similarly affected by the inductive power of the magnet; the lower ends of both being rendered north poles, and the upper ends south poles. This divergency of the wires will continue to increase until the magnet has approached to a certain limit. But if the magnet is brought nearer than this limit, its own attractive force becomes so strong as to overpower the repulsion that exists between the lower ends of the wire; and therefore brings them nearer to each other, as shewn in *fig*. 17; while the repulsion of the upper ends *s*, *s*, still continues to manifest itself, by keeping them remote from one another. On removing the magnet entirely, the wires immediately collapse, their magnetism being only of a transitory nature. But if the same experiment be made with sewing needles, instead of soft iron wires, the needles will often continue to repel each other after the removal of the magnet, having acquired some degree of permanent magnetism by the circumstances in which they have been placed.

Fig. 15.

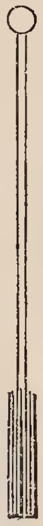


Fig. 16.

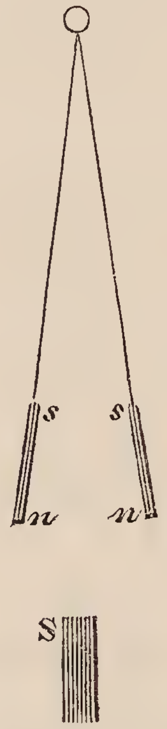
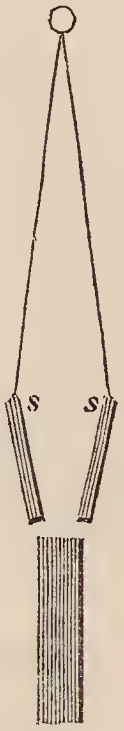


Fig. 17.



(36.) If four wires be suspended in a manner similar to those in the last experiment, each by its separate thread, the induction of a similar magnetism upon all of them will produce a mutual repulsion among them, and they will of course all diverge from one another. But if the wires be made of steel, so as to retain whatever magnetism may be communicated to them, and a northern polarity be given to the lower ends of two of the wires, but a southern polarity to the lower ends of the other two wires, when each of these pairs is kept apart, the wires will repel each other; but if both pairs are brought together, all the four wires will unite and adhere together. The reason is that those wires which have opposite polarities attracting each other, unite to form a pair in which the polarities are balanced, and the repulsion each had before exerted towards those which were similar in the other pair, is now entirely neutralized. The same thing will happen, however numerous are the pairs of wires that are dissimilarly magnetized.

(37.) In order that a bar of iron may receive the combined inductive influence of two magnets, it is not necessary that they should all be situated in the same line as in the example already given. The same effect will result if the bar be at right angles to the two magnets, as in the following figure (18), provided the two ends be immediately acted upon respectively by the poles of opposite denominations of the magnets. The attraction of a bar in this situation is much increased by the conspiring inductive influence of the two magnets; and the force exerted is more than double of that by which it would have adhered to either of the magnets when singly employed. This may be verified by attaching the scale of a balance to the bar (*fig.* 19,) and adding weights till its adhesion to the magnets is overcome: these weights will be found to exceed the sum of the weights which the two magnets would have supported, by means of the adhesion of the same iron bar, if they had been applied separately.

Fig. 18.

(38.) While such is the effect of the application of the dissimilar poles of two magnets to the ends of a bar of iron, namely, that of conspiring to induce the same kind of magnetism, it is likewise evident that an effect of an opposite kind must result when the dissimilar poles of the magnets are both applied to the same end of a bar. These poles, being of different kinds, will produce contrary effects; their inductive influence will oppose, instead of assisting, each other, and the magnetism induced on the bar will be only that resulting from the difference, instead of the sum of their intensities. If the bar be of some length, and if the magnets be of equal strength, and applied close to each other, their actions upon the remoter parts of the bar will be so nearly equal, that they will almost entirely neutralize each other, and no sensible degree of magnetism will be excited. Thus if while a key is supported by a magnet as in *fig.* 20, we gradually bring down upon it a second magnet, with its lower pole of the opposite kind to the lower pole of the first magnet, it will tend to induce in the key a polarity of an opposite kind to that which it has received from the first magnet. In as far as it exerts that influence it diminishes this magnetism, and consequently weakens the attraction. Another cause also operates in diminishing the attraction. The polarity induced upon the adhering end of the key is of the contrary kind to that of the pole of the first magnet; it is therefore of the same kind with that of the second magnet which is brought near it, and which, therefore, as far as the key retains its induced magnetism, must repel it. Accordingly, it happens that when the second magnet, if sufficiently powerful, is brought within a certain distance from the upper end of the key, it destroys its power of adhering to the magnet, and the key drops off.

Fig. 19.

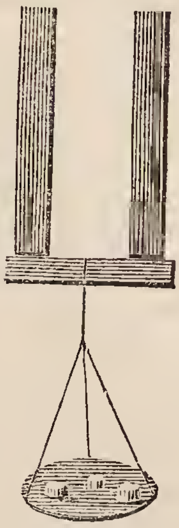
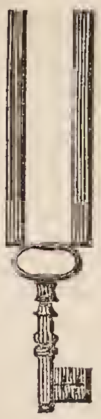


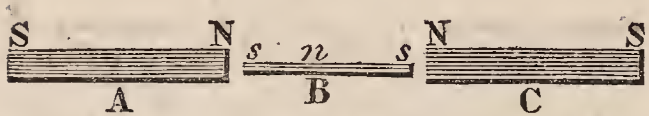
Fig. 20.



A similar counteraction of magnetic induction will take place, when the other pole of the second magnet, that is the pole of the same denomination as that of the first magnet to which the key adheres, is applied to the lower end of the key. The first action of the lower magnet, as it approaches the key under these circumstances, is to repel it; but on being brought still nearer, its inductive influence becomes so great as to reverse the poles of the key, which is now attracted by the pole which before repelled it: it then generally drops off and adheres to the lower magnet.

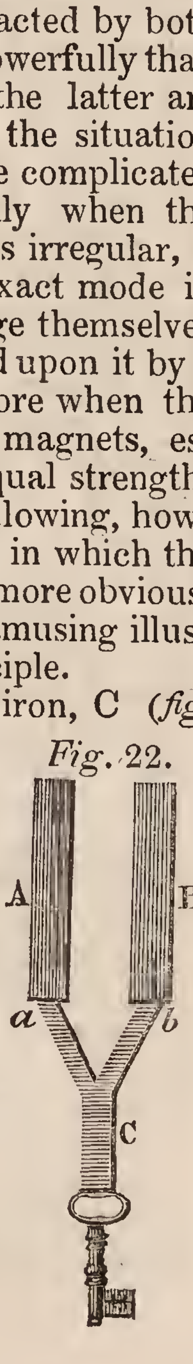
(39.) The effect of applying similar poles to the two ends of a bar of iron is generally that of inducing the opposite polarity on both ends of the bar, and the same polarity at the middle. Thus, *fig.* 21, the north poles N, N, of the two magnets A and C being applied lengthwise to the ends of an intermediate bar B, will render it a magnet with three poles, those at the end being south poles, and the middle being a north pole. In this case the bar will be attracted by both the magnets, though less powerfully than when the acting poles of the latter are of opposite kinds, as in the situation shewn by *fig.* 13. In more complicated cases, and more especially when the form of the piece of iron is irregular, it is difficult to predict the exact mode in which the poles will arrange themselves when magnetism is induced upon it by a single magnet, and still more when the operation of two or more magnets, especially if they be of unequal strength, is to be estimated. The following, however, is one of those cases in which the process that takes place is more obvious, and which furnishes an amusing illustration of the general principle.

Fig. 21.



(40.) Take a piece of iron, C (*fig.* 22), formed into the shape of a fork, or of the letter Y, and suspend it by one of the branches of the fork to the north pole of a magnet A; its lower end will immediately acquire a northern polarity, and will attract another small piece of iron, such as a key, which may therefore easily be supported by it. While the key is thus hanging from its lower end, apply to the other branch of the fork the south pole of another magnet B, the key will instantly drop off. The reason is that the magnet B tends to induce upon the remote or lower end of the fork, a contrary polarity to that which is induced upon it by A, and thus destroys its power of attracting. The fork will have a south pole at *a*, a north pole at *b*, while its lower end will be neutral. If, on the contrary, the north pole of the magnet B had also been applied to the branch *b* of the fork, its influence would have conspired with that of A in inducing a northern polarity at C, and the key would have been more strongly attracted.

Fig. 22.



§ 6. *Different Qualities of Iron and Steel with regard to Magnetic Susceptibility and Retentiveness.*

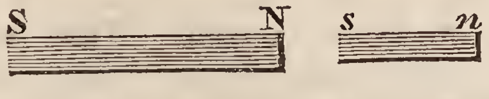
(41.) All the effects we have hitherto described, as attendant on the induction of magnetism on iron, are of a temporary nature, depending altogether on the influence excited by the neighbouring poles of a magnet; for we find that, the moment the magnet is removed, all these effects cease, and the iron returns to its original state of neutrality, and loses all its magnetic properties. But the case is different when steel is made the subject of experiment. Magnetism, it is true, may be induced on steel; but the induction proceeds very slowly, and is, at first, much more feeble than it is with iron. On the other hand, steel does not, like iron, lose what it has acquired; for, on the removal of the magnet which gave it the magnetic properties, it retains these properties permanently: it has, in fact, become itself a real magnet.

(42.) This remarkable difference existing between iron and steel in their respective susceptibility to receive, and capacity to retain magnetism, must, no doubt, arise from some peculiar arrangement of their particles, the exact nature of which is at present entirely unknown.

It is, however, exceedingly analogous to the difference in the qualities of electrics and non-electrics with regard to the power of conducting electricity; and the magnetic phenomena depending upon it admit of being explained on an hypothesis very similar to that by which we are enabled to account for the electrical phenomena which correspond to them. The two magnetic polarities may be conceived to reside constantly in all iron or steel, and in the natural or neutral condition of these bodies, may be regarded as in a state of equilibrium, and as equally distributed throughout the whole mass. But this state of equilibrium in a bar is disturbed by the influence of a magnetic pole in the vicinity, which exerts an inductive influence. This new force which comes into play, tends to transfer one kind of polarity to one end of the bar, and the opposite polarity to the other end. In iron, these changes are readily effected, on account of the facility which its peculiar texture affords for the transmission of these agencies in both directions. No sooner is the cause which has produced these changes, and maintains the separation of the polarities, removed, than all the effects cease; for there exists no obstacle to the return of the two polarities to their original situations: they revert, therefore, to their former state of equal distribution, and the condition of neutrality is restored in every part. But it is not so with steel; the constitution of which is such as to interpose impediments to the transfer of the polarities from one part to another. It requires a certain time before the obstruction, whatever be its nature, can be overcome; and before the new state of distribution, which induction tends to establish, can be completed; nor can the changes themselves ever be accomplished to the same extent as they are in bodies which present no such obstacles. It is also a necessary consequence of the resistance which the texture of steel presents to any changes taking place in its magnetic condition, that these conditions, when once induced, tend to remain in a great degree fixed. Hence we see the reason why steel bars admit of being rendered permanently magnetic, while the magnetism induced upon iron is only temporary. A steel bar, which has as great a degree of magnetic power as it is capable of retaining, is said to be *saturated* with magnetism.

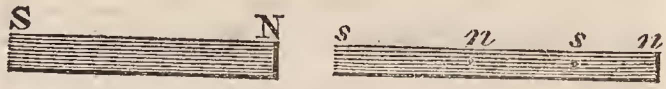
(43.) In order to obtain an exact knowledge of the progress of magnetic induction in steel, we should place a bar of this material very near to the pole of a strong magnet, and in the same line with it, and provide ourselves with a very small and delicate compass needle, poised on its centre, as in *fig*. 2, by which the polarity of each part of the bar may be examined in succession. It will be evident that an inductive action commences immediately on the magnet's being presented to the bar; for the latter is attracted and adheres very strongly to the magnet from the very first.

Fig. 23.



The end next the magnet has, therefore, acquired a polarity of a contrary nature to that of the magnet. For the sake of greater clearness of illustration, we shall suppose the actual pole of the latter to be a north pole. The near end of the steel bar is at once converted into a south pole; but if we examine the remote end we do not find it so immediately converted into a north pole. A sensible time is required for effecting the change in the latter; and it will be found, upon a more careful investigation, that the different parts of the bar from south to north, acquire, in succession, this northern polarity, which at last settles in the extremity. If the bar be of considerable length, it often happens that the northern polarity never reaches thus far, but stops at a nearer point; and in that case, we generally find a weaker south pole appearing at some greater distance; and this pole also travels slowly onwards till it attains its furthest limit. This is often succeeded by another north pole, and even a greater number of alternations will sometimes take place; each successive pole, however, becoming weaker and more diffused in proportion as they are more numerous and more distant. The points where the polarities thus change from the one kind to the other have been called *consecutive points*. It is evident that alternations of this kind must very much disturb the regularity of the magnetic actions of bars in which they exist, and complicate the resulting phenomena.

Fig. 24.



It would appear, from a variety of facts hereafter to be detailed, that a certain time is in all cases required for the complete operation of magnetic induction.

(44.) There are certain circumstances and modes of treatment which tend to quicken the progress of this induction. The first of these is concussion. Whatever excites a tremulous or vibratory motion among the particles of the steel, promotes the transmission of the magnetic polarities, and favours the induction of magnetism. Striking on the bar with a hammer is found to produce this effect in a remarkable degree; and the more so if it occasion a ringing sound in the steel, which is an indication that its particles are very generally thrown into vibratory motion. But any other cause producing agitation among the particles assists in the induction of magnetism.

(45.) The transmission of an electric discharge through a steel bar under the influence of a magnet, is sufficient to produce permanent magnetism. That the electricity acts here only by its mechanical operation, is proved by the effect being the same, whatever be the direction in which it is transmitted; that is, whether the positive stream of electricity be made to pass from right to left, or from left to right, along any part of the steel bar; or whether it be passed longitudinally or transversely through it. This mechanical operation of electricity is, however, to be carefully distinguished from an influence of a totally different description, which it is capable of exerting in producing magnetism, and the operation of which will be the subject of a distinct treatise hereafter to be published.

(46.) Heat also appears to act by removing the obstructions to the transmission of magnetism which exist in steel, in its ordinary state, and by thus reducing it nearly to the condition of soft iron. Accordingly, if a steel bar be heated, and placed in circumstances favourable to magnetic induction, if it be placed, for example, in the immediate vicinity of a magnet, and then suddenly cooled, it will be found, on its removal from the magnet, to have become strongly and permanently magnetic. The greatest degree of magnetism is produced by heating the steel to redness, and, while it is under the influence of a strong magnet, quenching it suddenly with cold water.

(47.) It will readily be understood that since the magnetism of a steel bar remains permanent, only because the peculiar texture of the steel presents an insuperable obstacle to its resuming its natural state of uniform distribution, all the causes which diminish this obstructing force, will give occasion to the escape of portions of this imprisoned magnetism, and will make the bar approach nearer to a neutral condition. In other words, its magnetism will be impaired and weakened, by the very same causes which favoured its acquisition when under the inductive process. It is accordingly found that any mechanical concussion, or any rough usage, has a tendency to destroy the power of a steel magnet. Dr. Gilbert, who was one of the earliest discoverers in this science, found that a magnet which he had impregnated very strongly was very much impaired by a single fall on the floor: and it has been observed since his time, that a magnet is more injured by falling on a stone pavement, or receiving blows which cause it to sound or ring, than by being struck with any soft or yielding substance.

(48.) In like manner the application of heat to a magnet is invariably attended by a dissipation of its magnetic power. It is even sensibly affected by the heat of boiling water; and a red heat totally destroys its magnetism. It has been observed by Mr. Canton, that if the temperature of the magnet has been raised only to that of boiling water, although it loses much of its power during the operation, yet that a great part of it is again recovered on its becoming cool. But after it has been heated to redness, no part of its magnetism is recovered on cooling.

(49.) The precise nature of the influence which heat has upon magnetism is far from being clearly understood; and there appears to be much discordance in the accounts given by different authors on this subject. This appears to have arisen in a great measure from a want of attention to the circumstance that the operation of heat is of two kinds: for while, on the one hand, it facilitates the induction of magnetism, on the other it weakens magnetic action. In those cases where the effects depend upon the readiness with which a piece of iron receives magnetism by induction, heat will favour this process; thus, soft iron is more disposed to be attracted by a magnet when hot than when cold, provided the heat be not excessive. But in as far as relates to permanent magnetism, the action of heat is to impair or destroy it; so that steel, when heated, is less capable of retaining its power than it is when cold. This happens in consequence of its being brought nearer to the condition of soft iron, by the separation of its particles. By raising the temperature sufficiently high, to a red heat for instance, the whole of its permanent magnetism is at once destroyed, though it will still be susceptible, while in that state, of receiving temporary magnetism by induction, and therefore of being attracted by another magnet.

(50.) The direct tendency of heat to diminish magnetic power must also be taken into account in our estimate of the preceding phenomena. It not only promotes the destruction of permanent magnetism, but diminishes likewise the effects of that which is of a temporary nature. The degree in which heat possesses this direct influence, can be estimated only under circumstances in which no permanent change has been produced in the magnetism of a bar subjected to the change of temperature; that is, provided we find on the return of the bar to its former temperature, that it has retained all the power it had before the experiment. The limit beyond which no proper distinction can be accurately drawn between the effects of this twofold operation of heat, appears, according to the experiments of Mr. Christie, to be below 100° of Fahrenheit. From this temperature downwards the power of a magnet increases as it becomes colder; and this augmentation proceeds as far as the lowest temperature that has been tried.

(51.) The following are the results of an extensive series of experiments upon this subject made by Mr. Christie*. Commencing with a temperature of −3° of Fahrenheit, up to one of 127°, the intensity of magnetic power decreased as the temperature of the magnets increased. From an experiment he made at the Royal Institution, in conjunction with Mr. Faraday, in which a small magnet, enveloped in lint, well moistened with sulphuret of carbon, was placed on the edges of a basin containing sulphuric acid, under the receiver of an air-pump, he found that the intensity of the magnet increased to the lowest point to which the temperature could be reduced, and that the intensity decreased on the admission of air into the receiver, and consequent increase of temperature. This, he observes, is in direct contradiction to the notion which has been entertained of intense cold destroying the magnetism of the needle. Captain Middleton had announced† his having frequently observed that a compass appeared to be deprived of all magnetic power from cold, while he was navigating among the ice in Hudson's Bay; but recovered its power when brought into the cabin and warmed by the fire: and that this repeatedly occurred. There can be no doubt that this must have been owing to some other cause than the one he assigned.

With a certain increment of temperature, the decrement of intensity is not constant at all temperatures, but increases as the temperature increases. From a temperature of about 80° the intensity decreases very rapidly as the temperature increases, and beyond the temperature of 100°, a portion of the power of the magnet is permanently destroyed.

(52.) The effects produced on unmagnetized iron, by changes of temperature, were observed by Mr. Christie to be directly the reverse of those produced on a magnet; an increase of temperature causing an increase in the magnetic power of the iron, the limits between which, he observed to be 50° and 100°. This is in perfect conformity with the views we have above explained, of the nature of the operation of heat with regard to magnetism.

(53.) Although the direct tendency of heat to diminish magnetic power may, in a red hot bar, be not sufficient to prevent its receiving induced magnetism, yet when the temperature is still further raised, even this capability is destroyed; and accordingly we find that, at a white heat, iron appears to be totally insusceptible of any magnetic action. There are still, however, some curious anomalies occurring in the magnetic action of iron at these very high temperatures, of which further investigation alone can furnish the explanation.

(54.) In the account we have now given of the properties of iron and of steel, with regard to their capabilities of acquiring and of retaining magnetism, we have all along referred to those of pure metallic iron in its softest and most ductile state, and to those of steel which has been brought to its greatest degree of hardness by immersion in cold water after being heated; for it is in these two states that they exhibit the strongest contrast in these respects. We often, however, meet with this metal in states possessing intermediate degrees of the above qualities; that is, acquiring magnetism with less facility than soft iron, and retaining less of it than hard steel. It may be laid down as a general propo-

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[*] Philosophical Transactions for 1825, p. 62.

[†] Philosophical Trans. for 1738, vol. xl. p. 310.

sition, liable however to some exceptions, that the power of retaining magnetism in any specimens of iron or steel, is in proportion to its hardness.

(55.) But some of the combinations of iron with other substances affect its capacity for magnetism, independently of the hardness of the compound. A slight degree of oxidation pervading the mass of iron appears to increase its power of retaining magnetism; but a greater degree renders it totally insusceptible of being affected by the magnet, or of possessing any magnetic properties whatsoever. Combinations with phosphorus, with arsenic, or with tin, were found by Mr. Gay Lussac to produce compounds somewhat resembling those of carburet of iron or steel in their capability of retaining magnetism. Every thing depends, however, upon the proportions in which these several substances are united with the iron; for if they exceed a certain quantity, they totally incapacitate the compound from acquiring any magnetic properties.

(56.) It is only the finest and purest soft iron, free from all knots and veins, that returns to the state of perfect neutrality after it is removed from all extraneous magnetic influence. Iron is seldom found in this perfectly pure state; but even the purest iron may be rendered capable of permanently retaining magnetism, if it has been twisted or hammered violently. The slight superficial oxidation it undergoes by the action of the atmosphere, will also make it susceptible of some degree of fixed magnetism. But in its common state, iron may, on the whole, be regarded as incapable of any long retention of the magnetism which it may have received by induction.

(57.) It would appear from the experiments of Mr. Scoresby*, that the texture of all iron, even the most malleable, presents a certain degree of resistance to the transmission of magnetic power; for if a bar of iron be placed in circumstances favourable to its acquiring magnetism by induction, it does not acquire it in the degree of intensity of which it would be capable, were there no such internal obstructions to the transmission. If, under these circumstances, it be subjected to percussion, which, as we have seen, favours the transfer of magnetism in obedience to the attractive and repulsive forces that act upon it, it is found to acquire a much higher intensity of magnetic power than it would have received without such percussion. Nor is the whole of this power lost on the removal of the inducting cause; a part is retained by the iron, the internal structure of which appears to have undergone some alteration by the percussion.

As connected with this subject, we may notice the following curious observation of the same experimentalist*. Bars which had been strongly magnetized, and had their magnetisms destroyed or neutralized, either by hammering, heating, or by the simultaneous contact of the two poles of another magnet placed transversely, were always found by him to have a much greater facility for receiving polarity in the same direction as before, than in the contrary direction. Hence, it generally happened in his experiments, that one blow with the original north end downward, produced as much effect as two or three blows did with the original south end downwards. He also observed, that the polarity of *pokers*, generally supposed to be permanent, and considerable in intensity, was rather transient and weak; for in no instance did he meet with a poker the magnetism of which he could not destroy by a blow or two with a hammer on the point; and in general, two blows, even when the poker was held in the hand, and not rested upon any thing, were sufficient to invert the poles.

(58.) Soft steel is not much more retentive of magnetism than iron in its ordinary state. It is only when hardened that its magnetic powers become in any degree sensible. Dr. Robison states, that when steel is tempered to that degree which fits it for watch springs, it may acquire a strong magnetism, which it exhibits immediately on the removal of the magnet. But it dissipates very rapidly; and in a very few minutes it is reduced to less than one half of the intensity it manifested while in contact with the magnet, and to less than two-thirds of what it was immediately on removal from it. It continues to dissipate for some days, though the bar be kept with care; but the dissipation diminishes fast, and it retains at least one-third of its greatest power for any length of time, unless carelessly kept or injudiciously treated.

(59.) Steel tempered for cutting-tools

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* Transactions of the Royal Society of Edinburgh, vol. ix. p. 252.

* Philosophical Transactions for 1822, p. 251.

(the same author observes), such as chisels, punches, and drills for metal, acquires magnetism still more slowly by induction, and receives less of it while in contact with the magnet; but it is less disposed to lose it, and finally retains a larger portion of what it had acquired. Steel made as hard as possible is still longer in acquiring all the magnetism which simple juxtaposition can give to it. It acquires less than the former; but ultimately retains a much greater proportion. The loadstone, or native ore of iron, resembles very hard steel in these respects; that is, in the time necessary for its greatest impregnation, and in the durability of the acquired magnetism.

(60.) We have seen that iron, or any of its compounds, when free from magnetism, is attracted by a magnet only in consequence of the induction of magnetism upon it by the magnet which attracts it. It follows, therefore, that the degree of susceptibility to induction may be accurately measured by the attraction which results from this property. With this view Mr. Barlow made a series of experiments to ascertain the relative attraction which different species of iron and steel had for the magnet; and obtained the following specific results, the relative magnetic power of each substance being expressed by numbers*.

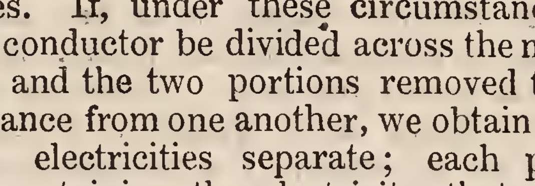
Malleable iron	100
Soft cast steel......	74
Soft blistered steel ..	67
Soft shear steel.....	66
Hard blistered steel..	53
Soft shear steel.....	53
Hard cast steel.....	49
Cast-iron..........	48

§ 7. *Fracture.*

(61.) We have hitherto been able to trace a very close analogy between the phenomena of magnetism and electricity, as far at least as relates to the law of action, and the influence of induction; but in pursuing it beyond this point it fails us entirely. Electricity, whether positive or negative, is not only capable of being excited by induction, but may be actually transferred from one body to another; but the transference of the magnetic polarities is a phenomenon which was never, in a single instance, known to take place. A body may, without difficulty, be rendered positively or negatively electrified; that is, it may be charged with a redundance of one or other of the two kinds of electricity; and the influence or agency, call it by what name we please, that has been gained by one body, is the same as that which has been lost by the other. It is not so with magnetism. There is never any transfer of properties, but only the excitation of those which were already inherent in the body operated upon. We always find in the same magnet, that the intensities of the two polarities, although each may occupy different portions of it, or be concentrated in some points, and diffused over others, yet still on the whole exactly compensate each other. We never can obtain a portion of iron or steel endowed wholly with either the northern or the southern polarity. Each appears to be strictly confined within the boundary of the surface of the body which contains it.

(62.) When a conductor of electricity, of an oblong shape, is placed near an electrified body, but not sufficiently near to receive any part of its electricity, it becomes electric by induction, the two ends of the body having opposite electricities. If, under these circumstances, the conductor be divided across the middle, and the two portions removed to a distance from one another, we obtain the two electricities separate; each portion retaining the electricity that had been induced upon it. The condition of a magnet appears to be exactly analogous to this in reference to the distribution of magnetic power; for the northern polarity appears to be collected in one half of its length, and the southern polarity in the other; and each of these agencies seems, indeed, to be almost entirely concentrated in the very extremities of the bar. What, then, ought to happen conformably with this analogy, were we to break a magnet (A, *fig.* 25,) across its middle? Might we not expect by this means to obtain the two polarities separate, each still contained in the same portions where they had before resided?

Fig. 25.



(63.) The result of this experiment is exceedingly curious, and what certainly no previous reasoning could have led us to anticipate. Each portion, B C, of the fractured magnet is at once converted into a magnet, perfect in itself; that is, each respectively has a north pole at one end, and a south pole at the other. That end of the magnet which, previously to the fracture, was a north pole N, continues to be a north pole, while the other end of that fragment *s*, that is, the broken end, becomes a south pole. The converse is true of that fragment B which originally contained the south pole of the magnet. It thus appears that the two fractured surfaces *n* and *s*, are now converted, the one into a north, and the other into a south pole, although that part had, in the original magnet, been apparently in a neutral state.

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* Philosophical Transactions for 1822, p. 117.

(64.) Similar consequences ensue from the subdivision of one of these fragments into any number of portions, however great; each lesser fragment constituting in itself a complete magnet furnished with its two poles.

(65.) It is observed by Æpinus, who made many experiments on the effects of the fracture of magnets, and the observation has been confirmed by others, that the neutral point in each fragment of the broken magnet is at first much nearer to the place of their former union than to their other ends. He states that in the space of a quarter of an hour after the separation, the neutral points advance nearer to the middle of each, and continue to do so, by small steps, for some hours, and sometimes days, and finally become stationary at the centre.

When a magnet is split according to its length, the two portions will have sometimes contrary, and sometimes the same poles as they had when they formed one piece. When one portion is much thinner than the other, the slender fragment generally has its poles reversed*.

Chapter II.

Laws of Magnetic Forces.

§ 1. *Relation of Intensity to Distance.*

(66.) It would be inconsistent with the elementary views to which we are at present confining ourselves, to engage in the investigation of the mathematical law which regulates the variations of intensity of the magnetic forces, both attractive and repulsive, at different distances. We shall here only observe that this law has been made the subject of diligent and careful inquiry by some of the most eminent philosophers of modern times; and shall content ourselves with merely stating the final result of their labours. It has been ascertained most satisfactorily that the same law of variation obtains in magnetic attractions and repulsions with relation to proximity, as in the electrical: namely, that the intensity of the force by which magnetic polarities act upon each other is inversely as the square of their distance. In this respect, therefore, they agree, not only with the electrical forces, but also with that of gravitation; and it would appear, indeed, to be a property common to all forces which emanate in every direction from a central agent.

(67.) The variations of the intensities of magnetic attractions and repulsions exerted between any two poles depend solely upon the distances at which they are placed; and are in no degree affected or interfered with by the interposition of other bodies which are not themselves magnetic. Numerous experiments have been made with a view of discovering whether there exists any substance which can modify or intercept the action of magnets when placed between them and the body acted upon; but the result has been uniformly the same: namely, that the intervening bodies, of whatever kind they were, provided they were not susceptible of magnetism, occasioned no difference in the observed effects.

This subject, however, involves a question, hereafter to be discussed, as to the magnetic susceptibilities of substances which are not of a ferruginous nature.

§ 2. *Mutual Action of Two Magnets.*

(68.) The general law of magnetic force with relation to distance being once established, it becomes interesting to follow its consequences and applications under a variety of circumstances. These consequences are always, even in the simplest cases, more complicated than electrical arrangements; because in magnets the two polarities are always conjoined, and their influence is never perfectly isolated. In studying the mutual actions between two magnets, or even between one magnet and the smallest conceivable piece of iron, we have always four polarities in activity, the two residing in one body, and the two residing in the other; these polarities are not strictly confined to particular points in the magnet: for although much concentrated at the two ends, they exist with less intensity in other parts of the magnet.

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* Derham, Philosophical Transactions, vol. xxiv. p. 2138.

(69.) Let us, however, suppose, for the sake of simplification, that the magnetic forces emanate solely from the two poles at the extremities of the magnet M, *fig*. 26, with its axis placed horizontally, while a smaller magnet B suspended on a point, or in other words, the needle of a mariner's compass, and which we shall therefore designate as *the needle*, is presented to it in the vicinity of its north pole, N, and with its centre in a line with the axis of the magnet. The north pole of the magnet attracts the south pole of the needle, and tends to turn it in the direction indicated by the arrow at *s*. It also repels the north pole of the needle, turning it in the direction indicated by the arrow at *n*. These two actions, it will be seen, both conspire to give the needle a rotatory motion in the same direction with regard to its centre, and to bring it into the position represented in the next figure, (27,) in which the south pole of the needle is turned directly towards the north pole of the magnet.

Fig. 26.

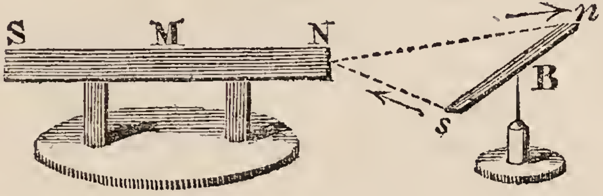
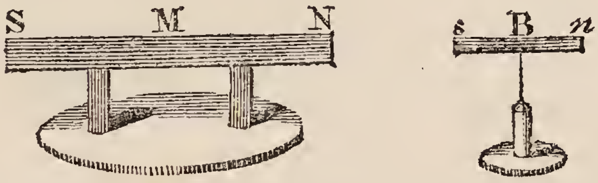


Fig. 27.



The influence of the south pole, S, of the magnet operates in a manner exactly contrary to that of its north pole; but being at a greater distance, its intensity is less; and all that it can effect is to subtract somewhat from the forces with which the needle would have been impelled, if the north pole of the magnet had acted alone. The general result as to the rotatory motion, is therefore determined by the predominance of the actions of the north pole of the magnet, and remains as before stated.

(70.) The tendency in one magnet to assume a particular position with relation to another magnet, is termed its *directive force**. It results, as we have seen, from the conjoined influence of two forces, the one acting on the north, and the other on the south pole, and is therefore equal to the sum of these forces.

(71.) If we now consider what tendency the needle has to approach to, or recede from the magnet, we shall find the same forces, which in the former instance conspired together, now opposing each other. It is, in the first place, evident that while the needle is in the position shewn in *fig*. 26, that is, at right angles to the magnet, the attraction of the adjoining north pole of the magnet for the south pole of the needle, is balanced by its repulsion for its north pole; and the needle, although strongly urged by these forces to turn round its centre, has no tendency, on the whole, to recede from, or approach the magnet. When, however, it arrives at the position shewn in *fig*. 27, its south pole *s* being nearer to N than its north pole *n*, the attractive action is more powerful than the repulsion, and the needle is, consequently, now impelled towards the magnet. But the force which thus impels it results from the difference only of two contrary forces, the one attractive, the other repulsive.

(72.) Hence we may conclude that the directive force, which consists of the sum of two forces, is in all cases considerably greater than the attractive force exerted upon the whole needle; this latter force being only equal to the difference between the same forces. The ratio between the directive and attractive forces will be increased, either by diminishing the length of the needle, or increasing that of the magnet. Hence the polarity of a small needle may be considerable, while its attraction is quite insensible.

(73.) Let us next transfer the needle to the situation shown in *figs*. 28 and 29, in which its centre is in a line drawn from the centre of the magnet, and at right angles to its axis; the needle being supposed, as in the former case, to be so balanced as to turn freely in a plane which passes through the centre of the needle, and both poles of the magnet. Let it be placed in the position indicated in *fig*. 28, with one of its poles directed towards the middle of the magnet. The directive force is here compounded of four forces: the attractions of N for *s* and of S for *n*; and

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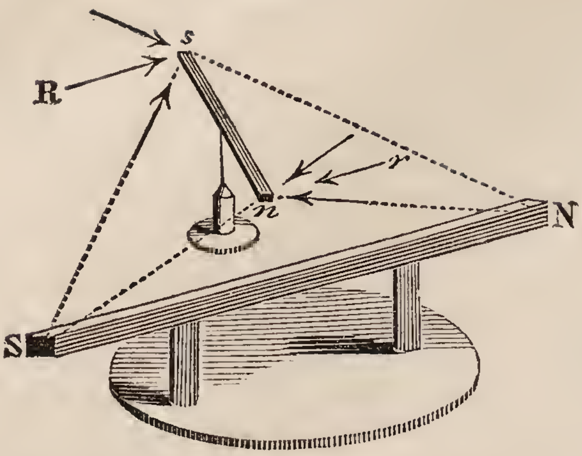
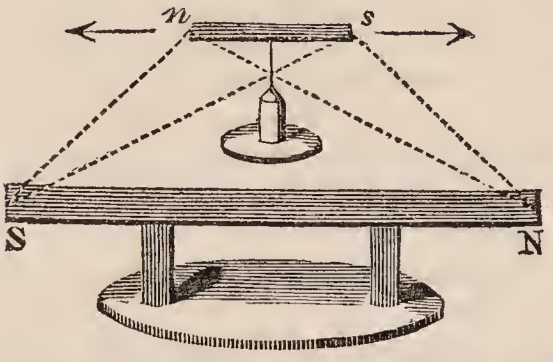
* Dr. Gilbert expressed it by the term *verticity*.

Fig. 28.



the repulsions of N for *n*, and of S for *s*. They respectively impel the poles *n s* of the needle in the directions denoted by the small arrows parallel to the lines in which these forces act. Those which act upon the remote pole of the needle *s*, compose a resultant having the direction of the upper horizontal arrow R, at right angles to the length of the needle, which is also the radius of its revolution. Those forces which act upon the opposite pole *n*, compose another resultant force in the opposite direction, expressed by the lower horizontal arrow *r*. Now these two resultant forces having opposite directions, and acting at the opposite ends of the needle which turns upon its centre, conspire in producing a rotation in the same direction with relation to that centre; and will tend to bring the needle into the position shown in *fig.* 29, in

Fig. 29.



which its direction is parallel to that of the magnet, but in which its poles are reversed when compared with those of the magnet; that is, the north pole of the needle being on the side of the south pole of the magnet, and its south pole on the side of the north pole of the magnet. This relative situation has been called by some authors *the subcontrary position.*

(74.) Here also it may be remarked, that in consequence of the greater proximity of the poles of the different denominations, compared with that of the poles of the same name, the sum of the attractive forces exceeds that of the repulsive; the former will therefore prevail, and the needle will have a tendency to move towards the magnet in the direction of the line connecting their centres.

(75.) A similar process of reasoning, derived from the same principles, will enable us to determine the resultants of the forces which act upon the needle when its centre is situated in different directions relatively to the axis of the magnet: and consequently what will be its movements, and what its final position of equilibrium. In oblique positions, indeed, the process of investigation becomes more complicated, for it is necessary to take into consideration the different intensities of each of the four forces concerned, with reference not only to the respective distances of the poles of the needle from those of the magnet, but also to their respective directions in the plane of rotation.

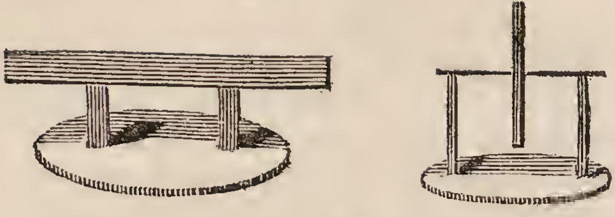
(76.) If the plane of rotation, to which the movements of the needle is limited, be one which does not pass through the poles of the magnet, the complication of the problem becomes still greater. There are, however, three general results at which we may arrive, which tend very much to simplify the resolution of questions relating to this subject.

(77.) The first is, that if we suppose the needle to be at perfect liberty to move on its centre in all directions, the position of equilibrium at which it will arrive by the conjoint action of all the forces which impel it, will always be situated in the plane which includes the poles of the magnet and the centre of the needle. This plane may be called, for the sake of distinctness, *the magnetic plane;* and the position assumed by the needle in this plane may be called its *magnetic* position.

(78.) Secondly, when the movements of the needle are limited to any particular plane, its position of equilibrium is that which makes the nearest approach to the magnetic position that the case will admit of. It will therefore be situated in a plane passing through the magnetic position, and at right angles to the plane of revolution.

(79.) Thirdly, if the plane of revolution be perpendicular to the magnetic position, the needle will be in a state of equilibrium with regard to the forces exerted upon it by the magnet, in all positions. Such a plane may be called *the plane of neutrality*. An example of this is shown in *fig.* 30, where the needle,

Fig. 30.



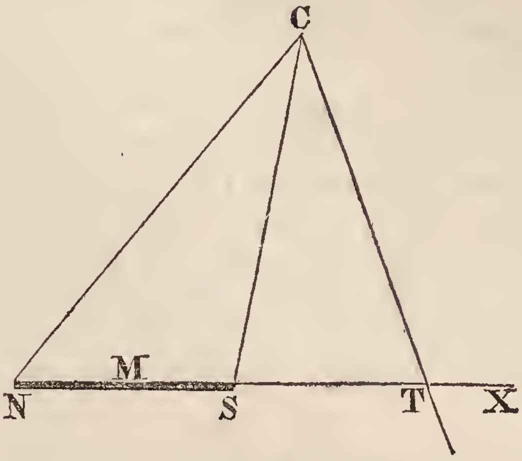
turning on a horizontal axis in a line with the magnet, is limited in its motion to a vertical plane perpendicular to its position of equilibrium. In this case it will have no tendency to assume any one position in preference to another, in this plane.

§ 3. *Magnetic Curves.*

(80.) In order still further to generalize our views, let us conceive the needle to be exceedingly short when compared with the length and distance of the magnet; and we shall then arrive at still more simple conclusions with regard to its positions of equilibrium in the magnetic plane. The two poles of the needle may, with regard to the action of the magnet, be considered as coincident; the intensities of the actions of any one of the poles of the magnet upon them are so nearly equal that their differences may be regarded as infinitely small. The attraction of the magnet for this minute needle, an attraction which, as we have seen, depends upon their difference, must accordingly be inappreciable. But the directive force, on the contrary, depending on the sum of these actions, must be very effective; and it is to the operations of the latter of these forces only that we need direct our inquiries.

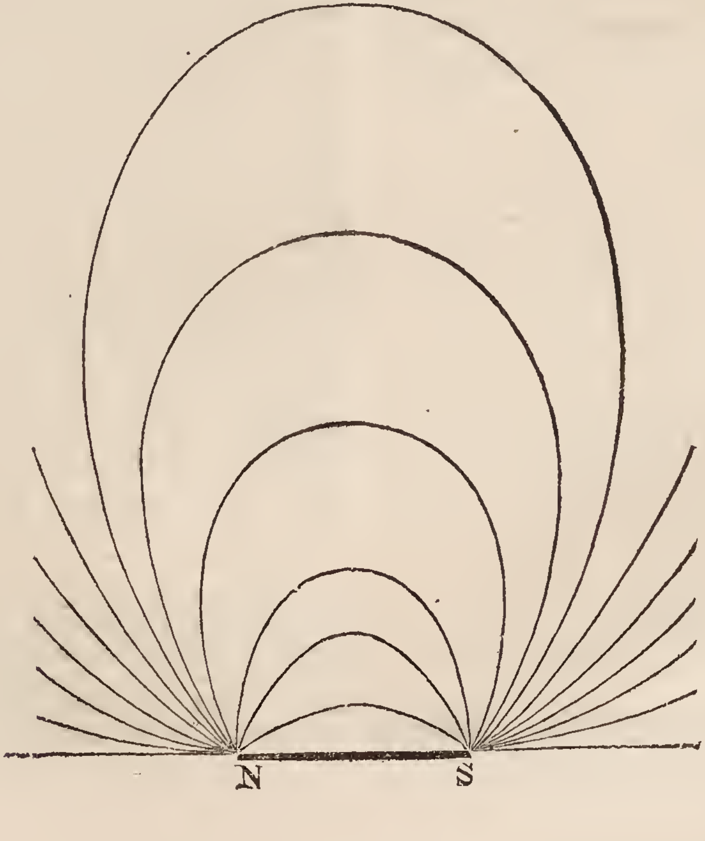
(81.) The problem to be solved is this: given the position of the magnet M, (*fig.* 31,) and of its two poles N and S, and also the place of the centre C of the needle, which is supposed to be at liberty to revolve only in the magnetic plane, to find the direction C T, at which the rotatory force resulting from the action of the north pole N of the magnet on the two poles of the needle in the direction C N, exactly balances that resulting from the action of the south pole S of the magnet on these poles, in the direction C S, each force having an intensity reciprocally proportional to the squares of these respective lines.

Fig. 31.



It may be mathematically demonstrated * that if such be the law of the magnetic forces, the direction of the needle is that of the tangent of a peculiar curve of an oval shape, which has been denominated the *magnetic curve.* Every magnet having two poles N and S (*fig.* 32) has a system of mag-

Fig. 32.



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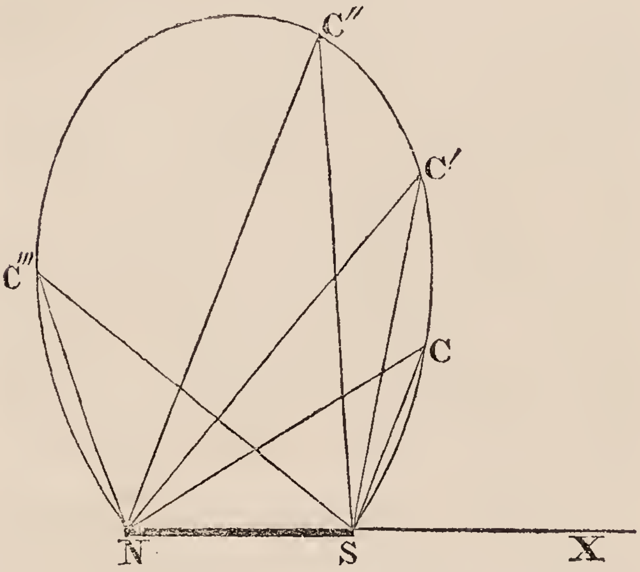
* Demonstrations of this and of the other fundamental properties of the magnetic curves are given in the Journal of the Royal Institution, for Feb. 1831, by the author of this Treatise.

netic curves related to the line joining these poles, and which may be called its axis. The general form and disposition of these curves, according to their different distances from the magnet, is shown in the figure.

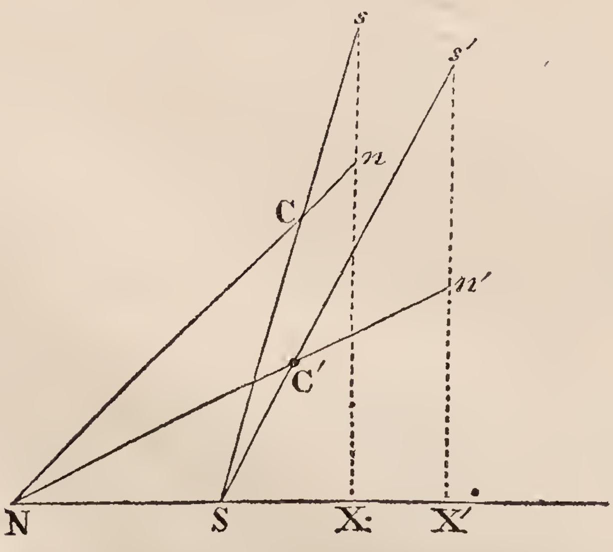
(82.) The magnetic curves have the following remarkable property; namely, that the difference of the cosines of the angles, which lines, drawn from any point in the curve to the two poles, make with the axis, taken on the same side, is constant. Thus, in the curve S C C′ C″ N, *fig.* 33, the sum of the cosines of the angles C N X and C S X, is equal to the sum of the cosines of the angles C′ N X and C′ S X. When, however, the angle C″ S X exceeds a right angle, its cosine being negative, it will be the sum (instead of the difference) of the cosines of the polar angles C″ N S, C″ S N, that is constant. When the angle C‴ N X is also obtuse, both the cosines being negative, it is again their difference that is constant.

Fig. 33.



(83.) If two radii of equal length, N *n*, S *s*, *fig.* 34, be made to revolve in the same direction round their respective centres N and S, while their other extremities, *n* and *s*, are kept continually in such a relative position as that a line drawn through them shall always be perpendicular to the axis N X, then the line, constituted by the successive points of intersection C, C′ of the radii, will be a magnetic curve *.

Fig. 34.



(84.) The most expeditious method of delineating a great number of magnetic curves related to the same base, in order to obtain a general view of the entire system of these curves, is to describe from each pole, N, S (*fig.* 35), as a centre, the equal circles or semi-circles, A A, B B, with as large a radius as the paper will conveniently admit of; and, dividing the axis, produced till it meets both circles, into any number of equal parts, to mark off, on the circumferences of both the circles, the points where they are cut by perpendiculars from these points of division; then, drawing radii from the centre of each circle to the divisions of its respective circumference, the mutual intersections of these radii will form different series of points indicating the course of the magnetic curves which pass through them. In the present case these curves are composed of a succession of diagonals of the lozenge-shaped interstices formed by the intersecting radii, as is shown in the upper half of *fig.* 35.

(85.) The forms and disposition of these curves are elegantly illustrated by the lines in which iron-filings arrange themselves when acted upon by a powerful magnet. In order to exhibit them, we need only place a sheet of paper or pasteboard immediately over a straight magnetic bar laid flat upon a table, and scatter lightly some very fine iron-filings over the pasteboard; which is best done by shaking them through a gauze bag. If we then tap gently upon the paper, so as to throw them into a slight agitation, they will arrange themselves with great regularity in lines, which exactly follow the course of the magnetic curves, extending from one pole of the magnet to the other. These minute fragments of iron, being rendered magnetic by induction, have their dissimilar poles fronting each other, and therefore attract one another, and adhere together

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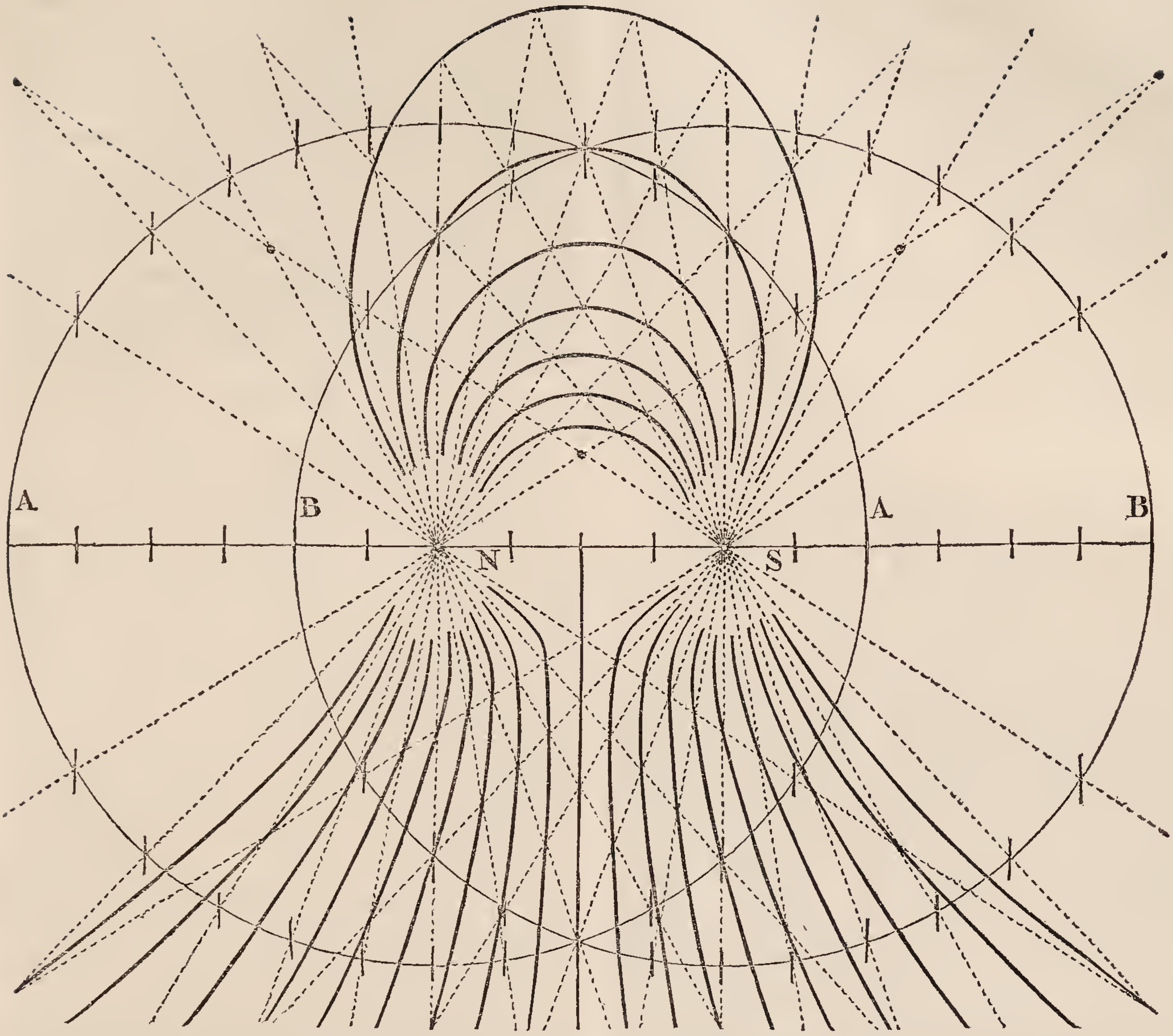
* The author of this Treatise has constructed a system of rulers by which magnetic curves may be mechanically delineated, founded on the principle stated in the text. The description of this instrument is contained in the paper above referred to in the Journal of the Royal Institution.

Fig. 35.



in the direction of their polarities, which is that of the tangent to the magnetic curve: thus affording a beautiful ocular exemplification of the mathematical properties of these curves.

(86.) By continuing to tap upon the paper, the filings arrange themselves still more visibly into separate lines; but here a curious, and perhaps unlooked for phenomenon presents itself. The lines gradually move and recede from the magnet, appearing as if they were repelled, instead of attracted, as theory would lead us to expect. This arises from the circumstance that each particle of iron, or cluster of particles, is thrown up into the air by the shaking of the paper, and, while unsupported, immediately turns on its centre, and acquires a position more or less oblique to the plane of the paper. This is shown in *fig.* 36, in which M represents a section of the magnet, P P a section of the paper, and *f f* the position of the filaments of iron thrown up into the air. The end of each filament nearest to the magnet is thus turned a little downwards, and the filament falls upon the paper at a point a little more distant than that which it before occupied; and thus, step by step, it moves further and further from the magnet, till it reaches the edge of the paper and falls off.

(87.) When the magnet, instead of being beneath the paper, is held above it, the effect is just the reverse. In this latter case, the lower ends of the filaments having a tendency to turn towards the magnet, the filings gradually collect under it, when made to dance by the vibrations of the paper, instead of falling outwards as they did before. This will be rendered apparent by *fig.* 37, where the letters indicate the same objects as in the preceding figure.

Fig. 36.

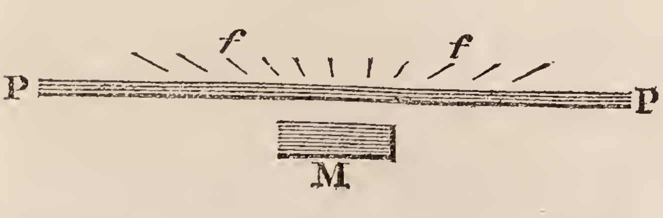
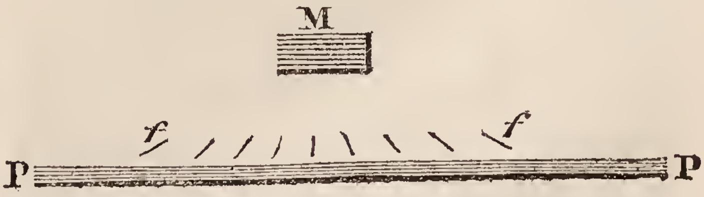
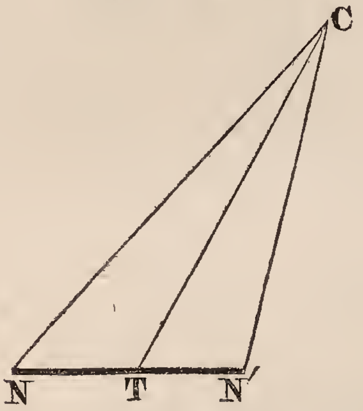


Fig. 37.



(88.) Magnetic curves of a different kind are constituted by the balanced actions of two poles of the same denomination placed near to each other. When, for instance, a second north pole N′ (*fig.* 38) is substituted, instead of the south pole S, both poles will act in a similar manner, and in directions not very different.

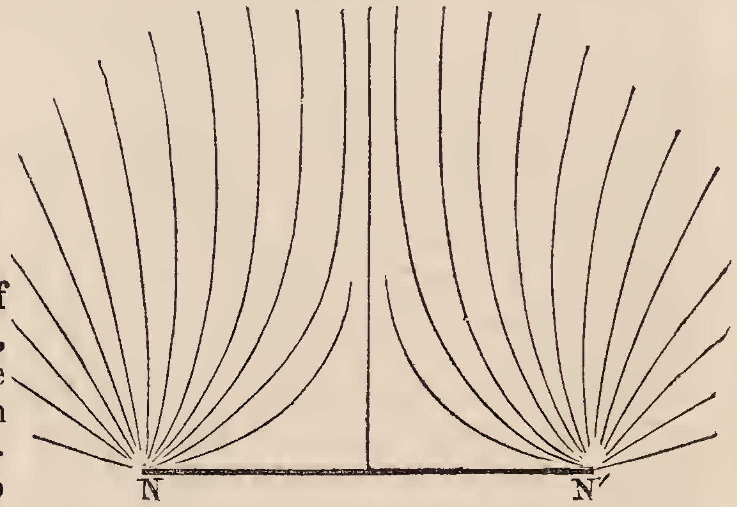
Fig. 38.



In order to render the conditions of this case as simple as those of the last, we must suppose that the action of the south poles belonging to the two north poles N, N′, whose action we are examining, is, from their remoteness, too feeble to influence the results. In the former case, where the actions of the two poles were of a contrary kind, the resultant of their joint action, or the line C T (*fig.* 31) passed in a direction intermediate between N C prolonged and C S, and therefore cut the axis N X at some point in the prolongation of N S. But in the present case, the two magnetic poles being of the same kind, their action is similar, and their resultant is a force of which the direction is intermediate to the lines C N and C S; and this line produced must cut the axis somewhere between N and N′. In consequence of this change of position, which produces a change in the sign of the cosine of the angle C S T, which is now C N′ T, the relation of the cosines of the polar angles is as follows; namely, that the sum (and not, as before, the difference) of the cosines of the angles which lines, drawn from any point in the curve to the two poles, make with the axis, taken on the same side, is constant. This applies to the case in which the angle formed by C N′ with the produced axis is acute, and its cosine positive. When it is obtuse (or C N′ N acute), the cosine becoming negative, it is their difference which is constant.

(89.) The intersections of the radii, drawn according to the method above described, § 82, will also point out the course of those curves which belong to the case where the acting poles are similar. For this purpose they must be taken in a different order of arrangement, and followed in the lines of the other diagonals of the lozenge-shaped intervals between the intersecting radii; that is, of the diagonals which cross those constituting the curves in the former case, as is shown in the lower half of *fig.* 34. These *divergent* curves, as they have been called in contradistinction to the former or *convergent* ones, are delineated in *fig.* 39; and may, in like manner, be exhibited by the arrangements of iron-filings round two similar poles.

Fig. 39.



(90.) When the actions of the four poles of two magnets are taken into account, the magnetic curves expressive of the direction of a needle influenced by them, become, of course, much more complicated.

Chapter III.

Terrestrial Magnetism.

§ 1. *Variation of the Compass.*

(91.) It has been already stated (§ 6), that if a magnetic bar be poised on its centre so as to move freely in a horizontal plane, and if no ferruginous body be sufficiently near to affect it sensibly, it will assume, when left at liberty, a direction nearly north and south. When disturbed from this situation, it returns, after several oscillations, to the same position. On this property is founded the mariner's compass, which is of such essential use in navigation. In moving horizontally towards the position which it thus tends to assume, the needle of the compass is said to *traverse*.

(92.) It is found that in this country, as well as throughout Europe, the north pole of the compass deviates a certain number of degrees to the westward of the exact northern direction. This deviation from the true geographical meridian has been called the magnetic *declination;* but it is more usually known by the term *Variation of the Compass.* The vertical plane which passes through the direction of the horizontal needle at any particular place is termed the *magnetic meridian* of that place, in contradistinction to the *geographical* or *true meridian*, which is a vertical plane passing through the poles of the earth.

(93.) There are but few places on the earth where the compass points directly to the poles; that is, where it exhibits no variation. As far as observation has extended, these places are situated in a line which encompasses the globe, and is called *the line of no variation.* In many of its portions it appears to form part of a great circle of the sphere, but in others it deviates much from regularity, presenting many flexures in its course. It may be considered as commencing from a point which may be designated as the principal arctic magnetic pole of the earth, and the exact situation of which is not yet perfectly ascertained, although the late voyages of discovery in these regions have enabled us to form a tolerable approximation to the precise spot, which appears to be a point somewhere to the westward of Baffin's Bay. After crossing the United States of North America it passes along a tract of the Atlantic, a little to the eastward of the windward West India Islands, till it touches the north-eastern point of the South American continent. Thence it stretches across the Southern Atlantic towards the south pole, where navigators are unable to follow it. It re-appears in the eastern hemisphere to the south of Van Dieman's Land, and passing across the western part of the Australian continent, is again found in the Indian Archipelago. Here, according to Biot, it divides into two branches, one of which crosses the Indian Sea and enters Asia at Cape Comorin; it then traverses Hindostan and Persia, and passing through the western part of Siberia stretches over to Lapland and the Northern Sea. The second branch pursuing a more directly northern course, traverses China and Chinese Tartary, and makes its exit from Asia in the eastern division of Siberia, where we again lose it in the Arctic seas. Between these there must exist an intermediate line of no variation in some part of the continent of Asia; but the observations we possess regarding it are, as yet, too imperfect to admit of any attempt to trace it correctly.

(94.) If we consider these Asiatic lines of no variation as composing a single band, we may then consider the globe as divided by this and the corresponding American line into two hemispheres. In that hemisphere which comprehends Europe, Africa, and the western parts of Asia, together with the greater portion of the Atlantic, the variation is to the west. In the opposite hemisphere, which comprises nearly the whole of the American continents, both North and South, and the entire Pacific Ocean, together with a certain portion of Eastern Asia, the variation is to the east.

§ 2. *Dip of the Magnetic Needle.*

(95.) But in order to arrive at a knowledge of the real influence which the earth exerts on a magnetic needle, it is not sufficient to ascertain the position it assumes when its movements are confined to a horizontal plane, as it is in the mariner's compass of the ordinary construction: we must place it in such circumstances as will allow it to move freely in a vertical plane also. But to effect this in an unexceptionable manner is extremely difficult. The great obstacle with which we have to contend is the force of gravity, which by acting in one direction, interferes with the operation of the force of terrestrial magnetism, which acts in a different and in an oblique direction.

(96.) The readiest mode of removing the influence of gravity, is to affix a steel needle to a cork, or other buoyant substance, and to immerse it in water, adjusting the specific gravity of the two bodies, so that they may remain suspended in the middle of the fluid without any tendency either to float or to sink; taking care at the same time that the centre of gravity of the whole coincides with the centre of its figure, so that, when the needle is unmagnetic, and united to the cork, the two together, placed in any position in the fluid, shall have no tendency to take any other position. If the needle be now rendered magnetic, and replaced as before, it is found to assume a position nearly vertical, that is, making an angle with the plumb line of about 20 degrees, the north pole of the needle being turned about 25 degrees to the westward of the true north. Its deviation from the plane of the meridian is equal to the variation of the horizontal needle. Its inclination to the horizontal plane, or 70°, is called the dip. But this method, though well fitted for illustrating the general fact, and the principle on which it depends, is not adapted for accurate measurement. For this purpose we must have recourse to other contrivances.

(97.) The magnetic force may, by the ordinary dynamic method of the resolution of forces, be resolved into two forces, the one acting vertically, the other horizontally. The latter of these forces, namely, the horizontal force, is the only one with the action of which gravitation does not interfere; and accordingly, the mariner's compass indicates by its motions, the effects of this part of the terrestrial magnetic force, and this only. In order to ascertain the vertical force, we must proceed in a different manner. The needle must be furnished with an axis, at right angles to its length, and adjusted very carefully, so that it may pass as exactly as possible through its centre of gravity. This, of course, can only be done when the needle is wholly free from magnetism, and secured, in the manner hereafter to be pointed out, from all magnetic influence which the earth might exert upon it. The axes should be supported horizontally in such a manner as to allow the needle complete freedom of motion in a vertical plane. The needle being thus balanced, will have no tendency to incline to one side rather than to another, and will remain at rest in any position in which it may happen to be left, as long as no extraneous force is applied to it. When this has been accomplished, the needle is to be magnetized, by the methods hereafter to be described, as strongly as possible, and it is then to be replaced on its supports, which are to be turned so that the plane in which the needle is allowed to move, may coincide with that of the magnetic meridian. It will be found that, in this situation, the end of the needle to which a northern polarity has been imparted, will preponderate, or *dip*, as it is called, and after a certain number of oscillations, will settle at a determinate point. The line which its axis assumes under these circumstances, is termed the *magnetical direction*, or *position*. The dip of the needle was first observed by Norman.

(98.) The inclination of the needle, or *dip*, like the variation, differs in different parts of the globe. The latest accurate observation of the dip in London, of which we have any record, is that of Captain Sabine, who ascertained it, in August, 1828, to be 69° 47'.* As a general rule, to which, however, there are many exceptions, the dip diminishes as we approach the equator, and increases as we recede from it on either side. Towards the polar regions it is very great, and as we come near to the poles, it approaches to a right angle. At the magnetic poles themselves, the dipping needle would, of course, be exactly perpendicular to the horizon. Those places on the earth where the needle is perfectly horizontal, that is, where there is no dip, are in a line that encircles the globe, and is termed the *magnetic equator*.

(99.) As the magnetic poles are not situated exactly at the poles of the earth's rotation, but at some little distance from them; so, the magnetic equator does not coincide with that of the earth; though it does not in any part deviate widely from it. In a general way we may consider it as a great circle of the globe inclined to the terrestrial equator at an angle of about 12 degrees; its intersections with it being situated at the longitudes 113° 14' west, and 66° 46' east from the meridian of Greenwich. Such, at least, is the result given by all the observations made for an extent of more than one half of its circuit, in the Atlantic and Indian Oceans, and that part of the Pacific which is nearest to the South American continent, as appears from a table of these observations given by Biot*. But a remarkable anomaly is met with when we trace the course of the magnetic equator across the Pacific Ocean. This line is found in the southern hemisphere in the American continent, and joins the equator as before-mentioned, at a longitude of about 113°; but still further to the westward, at longitude 156° 30' it is again met with at a distance from the equator and to the south of it. In the Sea of China at 116° east longitude, it is found to the north of the equator, which it must therefore have crossed at some intermediate point; and it is again inflected towards the south, so as to traverse the equator at the eastern node already mentioned.

It appears, therefore, from these observations, that there are at least three points in the terrestrial equator where the magnetic equator coincides with it; and the probability is, that there are four: because, if the latter curve passes to the northern side of the equator at its western coincidence, it must again cross it before it can arrive at the southern situation in which it has been met with in longitude $156\frac{1}{2}°$. These inflexions will, therefore, assume a figure, with relation to the terrestrial equator, somewhat like that represented in *fig.* 40, where the dotted line *m m m*, is the magnetic, and the continuous line, *e e*, the terrestrial equator.

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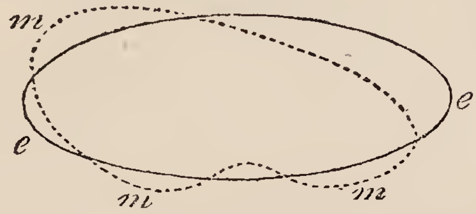
* Philosophical Transactions for 1829. Since the above was written, we are informed that the dip has been ascertained by Capt. Segelcke to be 69° 38' at Woolwich, in Nov. 1830.

* Traité de Physique, tome III. p. 130.

Fig. 40.



§ 3. *Variations in the Intensity of Terrestrial Magnetism.*

(100.) Besides the variation and the dip, which together constitute the magnetic position, and which differ much in different situations, there is also a third circumstance highly deserving our attention in connexion with this subject, namely, the intensity of the force which directs the needle towards this position. Extensive observations of the relative intensities of the magnetic force of the earth in different parts of its surface, are of greater value in enabling us to understand the general system of terrestrial magnetism than those in the dip or variation. We know that this force varies greatly in different latitudes; but our information with regard to the exact amount of this variation is exceedingly scanty, both from its importance not having been felt, and the consequent omission of the proper observations with regard to it, and also from the greater difficulty there is in conducting the experiments which are required to ascertain it.

(101.) The best mode of estimating the comparative intensities of the magnetic action in the same needle in two different places, is to count the number of oscillations which it makes in a given time, a minute for example, on its being disturbed from its position of equilibrium, while it is resuming that position. The movements of the needle being regulated by the same dynamical laws which govern the oscillations of the pendulum, it is a necessary consequence of those laws, that the intensity of the force producing the oscillations, is proportional to the square of the number of oscillations performed in a given time. Mr. Graham appears to have been the first who devised this method of measuring the magnetic intensities.

(102.) The first accurate observations of this kind were those made by Humboldt, and by De Rossel: who have completely established the general fact, that the intensity of the force of terrestrial magnetism increases as we recede from the equator, where it is weakest, till we approach the poles: at the magnetic poles themselves, it is probably greater than at any other spot. We have every reason to expect that great light will be thrown on this department of the science from the labours of Professor Hansteen of Christiana, who is now travelling at the expense of the King of Sweden, and with the permission of the Emperor of Russia, for the purpose of observing the magnetic dip, variation, and intensity, over the whole of the North of Europe and of Asia. He has especially directed his attention to trace the course of the lines of equal intensity, or *isodynamic lines* as they have been called: that is, the lines connecting those places where a needle freely suspended in the magnetic direction, and drawn a certain number of degrees from this position, makes the same number of vibrations round the point of rest in an equal time.

§ 4. *Hypothesis of the Magnetism of the Earth.*

(103.) From a consideration of the general facts that have now been stated with respect to the influence of terrestrial magnetism, it will be sufficiently evident that the earth acts upon magnetised bodies in the same way as if it were itself a magnet; or rather as if it contained within itself a powerful magnet lying in a position nearly coinciding with its axis of rotation. This hypothesis was originally proposed by Dr. Gilbert in his work entitled "Physiologia nova de Magnete, et de Tellure magno magnete," published in the year 1600; and Kepler ranks this hypothesis among the greatest discoveries in the annals of science.

(104.) In order to make this hypothesis agree with facts, we must assume that that pole of the terrestrial magnet which is situated in the northern regions of the earth, attracts the north pole of the compass needle, and consequently that it has the same properties as the south pole of an ordinary magnet. The opposite pole of the earth, or that situated in the antarctic regions, has the contrary properties, for it attracts the south pole of the compass; and therefore corresponds in its properties to the north pole of a common magnet.

(105.) It may be necessary to remark that this circumstance of the south pole of the terrestrial magnet being situated near the north pole of the earth, and *vice versa*, has occasionally created a confusion of terms. Some authors have taken a fancy to reverse the names we have hitherto given to the magnetic polarities: assuming that it is more correct to set out by calling that property which distinguishes the pole of the terrestrial magnet situated in the northern regions, the northern polarity: and consequently to give the name of the south pole to that pole of the compass, or ordinary magnet, which is attracted towards it, and which of course has the opposite polarity. For the same reason they would call the antarctic pole of the magnet of the earth, *the south pole*, and that end of the needle which is turned towards it, *the north pole*. Mr. Savery * endeavoured to avoid this confusion of terms by using the word *end*, in contradistinction to that of *pole;* and this phraseology is adopted by Mr. Christie in his papers in the Philosophical Transactions, as appears from the following passage: "To prevent any ambiguity, I must here state, that by the *south pole* of a magnet, I understand always the end which, when the magnet is freely suspended, points towards the north pole of the earth; so that the *north end* is the *south pole*, and the *south end* the *north pole* of a magnetic needle †." It matters little which set of terms are used, provided they are clearly defined, and all persons agree to abide by these definitions. But whereever a diversity of practice exists, it is then best to adhere to that which most generally prevails: in the present case the authorities in favour of the nomenclature we have adopted are much the most numerous.

(106.) Some have attempted to avoid the confusion which the changes just mentioned would lead to, by the introduction of the terms *Boreal* and *Austral* instead of *north* and *south:* the former set of terms having reference to the natural magnetism of the earth, the latter to that of the needle, or artificial magnet: that is, they would express what we have all along called the *northern polarity*, by the term *Austral polarity;* and the *southern polarity*, they would translate by the expression *Boreal polarity*. We should not have dwelt upon this comparatively unimportant topic, were it not that this change of language is sanctioned and adopted by Biot and most of the Continental writers on magnetism.

(107.) Assuming it, then, as an hypothesis, that the earth contains in its axis, or near it, a powerful magnet, let us note the consequences which follow from it, and compare them with the facts. We shall begin with those that relate to the inductive power of the earth's magnet: a power which will be exerted in the direction which a magnetised needle, at perfect liberty to move, would assume in consequence of the action of terrestrial magnetism; that is, in the direction of the magnetic position. In this part of the globe this position is, as we have seen, not very far from the perpendicular to the horizon. A bar of unmagnetised iron placed in the vertical position, or near it, ought therefore to become magnetic from the influence of the earth, and merely in consequence of its position. Its lower end should exhibit the properties of a north pole, and its upper end those of a south pole. All this agrees perfectly with experience. An iron bar held nearly upright will be found, at its upper end, to attract the north pole of a compass needle, and repel the south pole: it is, therefore, itself a south pole. Its lower end, on the contrary, will attract the south, and repel the north pole of the compass: and has therefore a northern polarity. That these properties of the ends of the bar depend altogether on the position of the bar itself, is proved by reversing its position; when the two ends will be found to have exchanged polarities merely by their change of situation: the upper end being always a south, and the lower end a north pole. On the other hand, if the bar be placed in a position at right angles to the magnetic position, (for example, horizontally, and with the ends directed to the east and west,) it will not exhibit any characteristic magnetism.

(108.) The magnetism which a soft bar of iron derives from its position with relation to the earth, is, as we have just seen, of a transitory kind; immediately lost on turning the bar so that it makes a right angle with the magnetic position; and again acquired, but with contrary poles, when its position is reversed. But this is not the case with harder bars, for, by remaining for a considerable time in a vertical position, they are found to acquire a sensible and permanent magnetism. This is generally the case with the stationary iron bars belonging to a building, and even with pokers and other fire-irons which have long been kept in an upright position. This circumstance will also readily account for the permanent magnetism of that class of iron ores to which the loadstone belongs. Indeed it is perhaps not going too far to assert with Professor Robison that all the magnetism which we observe, whether in nature or art, is either the immediate or the remote effect of the magnetism of the earth.

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* Phil. Trans. for 1730, p. 295.

† Ibid. for 1823, p. 344.

(109.) All the phenomena which we have already described as the consequences of induced magnetism proceeding from ordinary magnets, are exemplified also in the case of that derived from the magnetism of the earth. It is most readily induced, but soonest lost, in the softest kinds of iron and steel; it is slowly acquired, but more permanently retained in hard-tempered steel. Percussion promotes the change, of whatever kind it may be, which the position of the bar relatively to the earth has a tendency to produce. Hence we see the reason why the steel bar described in § 6 became permanently magnetical by being struck, while in a vertical position, with a hammer. Mr. Scoresby found that even a bar of soft iron, held in any position, except in the plane of the magnetic equator, may be rendered magnetical by a blow with a hammer or other hard substance; and both ends seem to acquire, by this treatment, an equal degree of magnetism.

On the other hand, an iron bar, possessing permanent polarity, when placed any where in a direction at right angles to the magnetic position, and struck several times, has its magnetism always much weakened, and may even be deprived of the whole of its magnetism by a single blow. This affords, indeed, an excellent method of depriving iron of its magnetism. Rough treatment of any kind, such as filing or scouring the surface of iron, and more especially bending or twisting it, when in the magnetical position, tends to impart to it the magnetism corresponding with that position; or to destroy its previous magnetism, if it be subjected to the same treatment in a position at right angles to this.

Iron heated to redness, and quenched in water, in a vertical position, was found by Mr. Scoresby to become magnetic; the upper end acquiring the southern, and the lower end the northern polarity. Hot iron, according to the same experimentalist, receives more magnetism of position than the same when cold. An iron bar is rendered magnetical by passing an electrical discharge through its axis, provided it be in a position favourable to induction by the earth: and the polarity it acquires corresponds with the effects of this induction. Electricity appears to act, in this instance, merely by its mechanical agency, and independently of a peculiar influence of another kind which it possesses, and which will be the subject of future inquiry.

(110.) Let us now examine how far Dr. Gilbert's hypothesis corresponds with the actual phenomena of the variations of magnetic position in different parts of the globe. For this purpose, it will be necessary to revert to what was explained in a former chapter regarding the positions which a small needle assumes when under the influence of a strong magnet in its vicinity, and variously situated with respect to it. These positions, we have seen, are tangents to a magnetic curve passing through the two poles of the great magnet, and through the centre of the needle. The direction of the tangent, which is the same as that of the dipping-needle, together with that of a vertical line, or one perpendicular to the horizon, will determine the plane of the magnetic meridian, for it is the plane which includes both these lines. The compass-needle, which turns in an horizontal plane only, will arrive at its position of equilibrium when it is situated in the plane of the magnetic meridian, because it then makes the nearest approach of which it is susceptible, to the position of the dipping-needle, which is that towards which the magnetic influence of the earth tends constantly to bring it.

(111.) In those parts of the globe where the dip is very small, the horizontal needle is capable of taking a position very nearly approaching to that of the dipping-needle: hence the terrestrial magnetism is exerted in bringing it to this position with very little loss of its force. This happens in the equatorial regions of the earth. In high latitudes, on the contrary, where the dip is great, the forces which actuate the horizontal needle, act more obliquely, and therefore to great disadvantage: hence the compass-needle is more feebly impelled; the point of rest is less decidedly marked, and the compass traverses slowly. The absolute intensity of the terrestrial force is, indeed, greater in the latter case than in the former; but the increase is not sufficient to compensate for the greater obliquity of its action. If we could place ourselves exactly over the north or south magnetic pole of the earth, the dipping-needle would take a vertical position, and the horizontal compass would no longer be sensible to the influence of terrestrial magnetism, but would remain at rest in any position in which it might happen to be placed.

(112.) All these consequences of the hypothesis which ascribes terrestrial magnetism to the influence of a magnetic power in the central regions of the earth, and of which the direction nearly coincides with its axis of rotation, may be experimentally illustrated by placing a strong magnet in the centre of an artificial globe. The points on the surface which are opposite to the poles of the magnet, are to be marked as the terrestrial magnetic poles. A great circle being traced equidistant from these poles, will be the magnetic equator, dividing the globe into the northern and southern magnetic hemispheres. Great circles passing through the poles, and crossing the equator at right angles, will be magnetic meridians; of which the one which also passes through the poles of the earth's rotation will be the lines of no variation. Smaller circles parallel to the magnetic equator, will indicate situations when the dip is the same in all. The lines of equal variation will be curves of particular forms less easily determinable. The accordance of fact with theory may now be verified by placing in different situations, on the surface of a globe so prepared, a small needle suspended as freely as possible by a fine thread, which holds it balanced as nearly as possible at its centre of gravity, and observing the positions it assumes in each situation.

(113.) But when we come to compare the regular lines thus traced from theory, on the supposition of a single central magnet, with the lines which observation points out as those indicating the actual variations of the magnetism of the earth, we meet with very remarkable discordances. Many have been the attempts made to explain the irregularities and anomalies in the course of the magnetic lines by suppositions of various kinds. There is reason for believing that the northern and the southern magnetic poles do not occupy points on the globe diametrically opposite to each other, which would be the case if the magnetic influence emanated from the centre of the earth. It has been supposed, in consequence, that the terrestrial magnet, or centre of magnetic force, was eccentric. But this supposition alone will not suffice; for there are various indications of the influence of more than one pole in each hemisphere of the earth; and the probability is that these poles are of very unequal intensities. Other irregularities exist which appear to owe their existence to the influence of causes entirely local and of limited extent, such as might be supposed to be derived from large masses of iron situated at different depths beneath the surface of the earth.

(114.) The observations best calculated to decide the important question of the existence of secondary magnetic poles, appear to be those of the variations of magnetic intensity, from which we derive the knowledge of the isodynamic lines already adverted to (§ 102); for these lines will necessarily arrange themselves in regular order around the point or points in each hemisphere when the intensity is greatest, that is around each respective pole. If these poles were single, and placed opposite to each other in the globe, one in the northern and the other in the southern hemisphere, the lines of equal intensity would form parallel circles, analogous to those of geographic latitude. Captain Sabine remarks * that the observations on this subject, made previously to those of Professor Hansteen, appeared to corroborate such an hypothesis; for, although they extended widely over the magnetic parallels in the northern hemisphere, namely, from the least almost to the greatest intensity, yet they were confined, in respect to longitude, to a space little more than a quarter of a hemisphere; and to that quarter which is immediately opposite to the countries visited by Professor Hansteen. Within the space that had been thus examined, the isodynamic curves appeared to arrange themselves with comparative insignificant deviations, in parallel circles around a point situated in the north-eastern part of Hudson's Bay, and, as nearly as could be judged, about the intersection of the sixtieth degree of geographical latitude

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* Quarterly Journal of Science. Sept. 1829, p. 3.

with the meridian of 80° west of Greenwich.

But M. Hansteen was led by a more careful consideration of the slight apparent deviations which had been noticed, and of the general disposition on the globe of the lines of dip and variation, to infer the existence of a second point of principal magnetic action in the northern hemisphere. This fact may now, indeed, be regarded as fully established by his recent observations; the isodynamic curves being found to arrange themselves systematically round two poles, the one in Hudson's Bay and the other in Siberia; and to be governed in the courses which they follow, partly by their distances respectively from those points, and partly by a disparity in the absolute attractive force at the points themselves: the maximum intensity in Siberia appearing to be weaker than that in Hudson's Bay, and existing at a point situated in longitude 102° east of Greenwich, which is as nearly as can be judged, 180° from the present position of the corresponding point in Hudson's Bay, and in a latitude somewhat to the north of 60°, but which, it is to be hoped, will soon be more particularly determined.

§ 5. *Progressive Changes of Variation and of Dip.*

(115.) The most singular and unaccountable circumstance relative to terrestrial magnetism remains yet to be noticed; namely, that it does not remain constantly the same in the same place, but undergoes a slow and progressive change. The variation of the compass is itself variable, not merely in different regions of the globe, but at different periods of time. Thus, the needle in London, in the beginning of the seventeenth century, was inclined a few degrees to the eastward of the true north. In 1659 or 1660, it pointed exactly north; or in other words, the variation was reduced to zero; and, of course, London was at that time one of the points of the line of no variation. After this, the variation became westerly, and has continued so to the present time. The line of no variation, therefore, has been progressively, but slowly moving in a westerly direction, and has now passed over to North America.

Similar changes have taken place at Paris; but the line of no variation appears to have passed over that city rather later than it did over London: for it was not till the year 1664 that the magnetic coincided with the true meridian. In 1814, it was 22° 34′ west. In October, 1829, the variation at Paris was ascertained, by M. Arago, to be 22° 12′ 5″ west*.

At London, the westerly variation continued to increase till the year 1818, when it amounted to 24° 30.′ This appears to have been its maximum; for since that time it has somewhat diminished, and is at present about 24°.

It appears, from the table given by Mr. Gilpin †, that the annual change in the variation has diminished, in each successive period, since the beginning of the last century. In the preceding century, that is from 1622 to 1692, the annual change was about 10′; from 1723 to 1773, it was about 8′; from 1787 to 1795, about 5′; from that time to 1802, only 1′.2: in 1818 it was reduced to zero.

(116.) The dip has also undergone corresponding changes, though less considerable ones than the variation. In 1680, the dip in London was 73° 30′; in 1723 it was 74° 42′: since which time it has been observed to diminish progressively, though, as it would seem, not quite regularly.

			Authorities and Localities.
In	1773 it was	72° 19′	Dr. Heberden.
	1786 ,,	72 8	Gilpin, Royal Society's Rooms Phil. Trans. for 1806, p. 419.
	1805 ,,	70 21	
	1818 ,,	70 34	Capt. Kater, Regent's Park.
	1821 ,,	70 3	Captain Sabine, Chiswick.
	1828 ,,	69 47	Ditto.
	1830 ,,	69 38	Capt. Segelcke, Woolwich.‡

On the continent of Europe the dip has undergone a similar diminution of late years. The dip at the observatory at Paris, in the year 1814, was 68° 36′, according to the determination of M. Bouvard. In June, 1829, it was ascertained by M. Arago to be 67° 41′.3.

(117.) Captain Sabine, by comparing the present dip with that observed for the last fifty years, concludes that the mean annual diminution is about 3′. Mr. Barlow finds that these observations accord much more nearly with what would take place on the supposition of a uniform motion of revolution in the magnetic pole round the pole of the earth. From the most authentic observations on the dip and variation of the needle in London, he calculates that the longitude of the northern extremity of the magnetic polar axis which it obeys was, in 1818, 67° 41′ west, and its latitude 75° 2′ north. If we suppose that the motion of this pole has been uniform since the year 1660, when, from the disappearance of variation, its longitude must have been zero, and that it has preserved the same distance from the terrestrial pole, its annual motion of revolution must have been about 25′. 4. It would, therefore, require eight hundred and fifty years to make an entire revolution of 360°. Computing from these data, it would follow that the variation ought to reach its maximum when the longitude of the magnetic pole is 70° 23′ west. It would have arrived at this situation about the year 1823; about which time, as it would appear, the variation was stationary, having attained its real maximum, and having, since that period, actually retroceded.

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* Annuaire pour l'An 1830.

† Phil. Trans. for 1806, p. 395.

‡ For the information relative to the last of these determinations, we are indebted to the kindness of Professor Barlow, of Woolwich, who states that Captain Segelcke, of the Norwegian Navy, and a friend of Professor Hansteen, employed in this determination of the dip, the same needle which the latter had with him in his recent tour in Siberia.

(118.) On calculating what the dip should be in 1823, according to Mr. Barlow's hypothesis, a very near agreement with actual observation is found to take place. It follows, however, from this hypothesis, that the dip has not an uniform decrease, but that it is changing much more rapidly at this present time than it has ever done since magnetical observations have been made. Its decrease, during the five years preceding 1824, has been nearly half a degree, and it ought to have diminished to an equal extent during the following five years. Mr. Barlow* has computed that in

	The Variation should be	and the Dip
1828	24° 29′	69° 43′
1833	24 26	69 21

The near accordance of these results with what has actually been observed, is considered by him as a strong confirmation of the truth of his hypothesis.

(119.) It would appear, then, both from observation and from theory, that the dip is at present changing more rapidly than the variation; and the theory leads to the expectation that it will continue to decrease together with the dip, for about two hundred and fifty-five years, at the end of which period, that is in 2085, the longitude of the magnetic pole will be 180°; the variation will then be nothing, and the dip only 56°, which will be its minimum; they will then both increase together for the next two hundred and sixty years, when the needle will have its greatest easterly variation, and will then again return towards the north, the variation decreasing, but the dip still increasing, for one hundred and sixty-five years longer, namely, till about the year 2510, when the magnetic pole will be again in the meridian of London; the variation will then be zero, and the dip will amount to 77° 43′. It is to be observed, however, that Mr. Barlow advances this merely as an hypothesis, the truth of which remains to be determined by future experience.

(120.) A curious hypothesis was advanced by Dr. Halley, and supported with some ingenuity, in order to explain the progressive changes that take place in the variation of the compass. He supposes the globe we inhabit to be a mere external shell, enclosing, towards its centre, a detached magnetic nucleus, of a spherical shape, which revolves with the external shell on a similar axis, with nearly the same velocity. He supposes both these spheres to be magnets, having each two poles; but the poles of the one not exactly corresponding in situation with the poles of the other. The difference of the periods of rotation of the two spheres, he conceives to be exceedingly small, yet sufficient to become sensible after the lapse of years, and to occasion a change in the relative situation of the two sets of magnetic poles; and hence would arise changes in the direction of their resulting actions, and corresponding changes in the variation of the magnetic needle. However ingeniously this hypothesis may have been framed, it was too bold and fanciful to have been ever generally adopted. Its author, indeed, has the candour to acknowledge that it is beset with numerous difficulties, which further experience, extended over a long period of time, can alone enable us to remove. He concludes his paper in the Philosophical Transactions in which he has developed his theory, with the following sentence: 'But whether these magnetical poles move altogether with one motion, or with several—whether equally or unequally—whether circular or libratory; if circular, about what centre; if libratory, after what manner; are secrets as yet utterly unknown to mankind, and are reserved for the industry of future ages*.'

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* See his Essay on Magnetic Attractions, 2nd Edition, p. 218.

§ 6. *Diurnal Changes of Variation and Intensity.*

(121.) Independently of the changes already noticed, the position of the magnetic needle is liable to certain slight variations, according to the time of the day, and also according to the season of the year. The daily change in the variation was discovered in 1724 by Mr. George Graham, and has been confirmed by many subsequent observers. This change, however, is exceedingly minute, and requires the most careful observation, and the most delicate instruments to render it sensible, even in the horizontal needle; and it is still more difficult of detection in the dipping needle, which does not admit of the same degree of delicacy of suspension.

(122.) Professor Barlow, to whom the science of magnetism is so much indebted for its more recent improvements, has devised a mode of rendering these diurnal oscillations much more perceptible, by diminishing the ordinary directive power of the needle, through the influence of one or two magnets, placed in such positions with respect to the needle as to counteract, and thereby neutralize, as it were, the terrestrial action. The effect of the ordinary action being thus removed, he was led to expect that the extraordinary cause, whatever it might be, which produced the daily variation, would exhibit its effects much more perceptibly; and thus not only the amount of the changes it produces, but also the period of their taking place, and of the maximum of their operation, might be ascertained with great precision. These expectations have been amply realized by the success of his own experimental researches, and also by those of Mr. Christie, which are detailed in several papers in the 'Philosophical Transactions †.'

(123.) The general result of the experiments of the latter of these observers was, that the deviation of the horizontal needle from the mean position was easterly during the forenoon, and was of greatest amount at about eight o'clock, thence returning quickly to its mean position, which it attained between nine and ten o'clock, after which it became westerly; at first increasing rapidly, so as to reach its maximum at about one o'clock in the afternoon, and then slowly receding during the rest of the day, and arriving at its mean position by about ten o'clock at night. The state of the weather, and more particularly that of the temperature, had considerable influence on the nature and extent of the changes.

(124.) Mr. Christie remarks that the changes which are observed to take place cannot be explained by a change in the directions alone of the terrestrial forces, but that their characters agree, as nearly as we can possibly expect, with the effects that would take place from an increase of intensity at the time that the direction deviates towards the west.

(125.) The occurrence of diurnal changes of intensity at Christiana in Norway have been ascertained by M. Hansteen; the same conclusion being deducible from his observations of the vibrations of a needle very delicately suspended; and also from those of Mr. Christie, made with a different apparatus, and by a totally different method.

(126.) M. Hansteen found that the minimum intensity occurs about half past ten o'clock in the morning, that is, about two hours after the westerly deviation has commenced, and the maximum intensity at half-past seven in the evening, that is, about the same time after the return towards the east. Mr. Christie * found that the terrestrial magnetic intensity is the least between ten and eleven o'clock in the morning; the time, nearly, he observes, when the sun is on the magnetic meridian; that it increases from this time until nine and ten o'clock in the evening; after which it decreases, and continues decreasing, during the morning, until it attains its minimum already stated.

(127.) The general dependence of these variations of magnetic position on diurnal changes of temperature is sufficiently apparent from the results hitherto obtained. But the prosecution of the inquiry involves considerations of another kind, connected with a subject we have not yet touched upon, namely, electro-magnetic and thermo-magnetic phenomena.

(128.) The mean diurnal changes of variation were found by Mr. Canton to differ at different seasons of the year, being greatest in June and least in December[*], and he has given the results of his observations in a tabular form. Mr. Gilpin investigated this subject at a later period, and gives also tables of the diurnal changes of variations in each month, which he found to be very different in different years. The following table contains the results of these different observations[*].

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* Phil. Trans. for 1683, p. 220.

† For 1823, 1825, and 1827.

* Philosophical Transactions for 1825, p. 51.

Mean Diurnal Changes of Variation.

Months.	Canton in 1759.	Gilpin in 1787.	1793.
January	7′ 8″	10′.2	4′.3
February	8 58	10 .4	4 .6
March	11 17	15 .	8 .5
April	12 26	17 .4	11 .7
May	13 0	18 .9	10 .4
June	13 21	19 .6	12 .6
July	13 14	19 .6	12 .5
August	12 19	19 .4	12 .1
September	11 43	15 .5	9 .8
October	10 36	14 .3	7 .0
November	8 9	11.1	3 .8
December	6 58	8.3	3 .8

CHAPTER IV.

Theories of Magnetism.

§ 1. *Mechanical Theories.*

(129.) IN the general view we have now given of the present state of our knowledge with regard to magnetism, we have strictly confined ourselves to the statement of facts, unmixed with hypothetical speculations as to the nature of the powers from which they proceed. We have solely endeavoured to generalize the facts, as far as their nature and extent would warrant. The result has been their reduction to a small number, such as the mutual attractions and repulsions of magnetic iron according to certain laws,—the induction of these properties on other iron,—the differences in the capacities of receiving and of retaining these properties, existing in different kinds of ferruginous bodies,—and the magnetic influence of the globe of the earth.

(130.) But the human mind is so constituted as to refuse being restrained within the boundaries of a rigid inductive philosophy. Incited by an irresistible desire of exploring the secrets of Nature, it scruples not as to the means of forcing her to disclose them; and borne on the wings of imagination and conjecture, presses forwards with an eagerness which often betrays it into courses widely deviating from the truth. Yet good is often found to result from these erratic excursions of our faculties: they infuse fresh interest into the pursuit of knowledge; they inspire with the hope of success; they invigorate those powers which must be exerted to attain it. The spark which kindles a train of light is sometimes struck out in the conflict of discordant speculation; and amidst a multitude of attempts, some effort, more happy than the rest, elicits an important discovery. No great or comprehensive fact in science was ever established, without being preceded by a bold though sagacious conjecture. Hypothesis of some kind or other is invariably the precursor of truth.

(131.) Magnetism, ever since it occupied the attention of philosophers, has been a fertile soil for hypothesis. That a shapeless and unorganized lump of metal should have the power of drawing towards itself another equally rude and unfashioned piece lying at a distance, or of forcing it to move away, as if both were alive, and animated by some principle of active sympathy; and that all this should take place, whatever may be the number or kind of the intervening bodies, nay even in the apparent absence of any connecting medium, are phenomena of too remarkable a nature not to excite in us a lively curiosity to learn their cause. No wonder that the ancients, who had but imperfect notions of the real objects of philosophy, were impressed with a vague notion of their connexion with immaterial agency. Mag-

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[*] Phil. Trans. for 1759, p. 398.

[*] Phil. Trans. for 1806, pp. 416, 417.

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MALKIN, Sir Benjamin Heath

ASTRONOMY.

Introductory Observations.

In treating of any science which is grounded upon physical facts and appearances, two courses are generally open. We may begin with a statement of the results observed, and by gradual investigation extricate from them the principles on which they depend: or else, if these principles have been ascertained, we may begin by stating them, and may deduce from them the consequences which would follow on the supposition of their truth; and finally, by comparing these consequences with the appearances presented by Nature, and finding them to correspond, we may satisfy ourselves of the truth of those principles which we originally assumed. The former is necessarily the course of discovery; the latter is often the most concise and convenient method of instruction after the discovery has been made. In some cases there is little practical distinction between the two methods; for instance, the fundamental principle of Hydrostatics is the equal pressure of fluids in all directions; and the fact that they do press so is one of the first and most obvious results of observation and experiment; and from the time that it is ascertained, the experimental and hypothetical mode of discussing the subject may very nearly coincide. In other sciences, on the contrary, the first effect of observation is to lead us to conclusions very distant from the fundamental principles which we finally adopt, or even at variance with them. In these cases the simpler and shorter mode of instruction will generally be the second which we have mentioned. Among these sciences Astronomy is eminently distinguished; for almost all the immediate results of observation are contrary to its true principles, and it is not without much labour and reasoning that the truth can be extricated from the mass of error in which it is involved. Astronomy has been a favourite study from the earliest periods: it is only within the last two hundred years that its true principles have been at all generally received; it is only from the time of Newton that they have been adequately explained.

In the present treatise, nevertheless, the results of observation will not be explained from principles assumed in the first instance, but the principles of astronomy will be deduced, as far as they can be so without complicated mathematical investigation, from observation. There are several reasons which seem to render this the most desirable course of proceeding, although adopted at the sacrifice of much conciseness, and of any very logical precision of arrangement.

The present treatise is principally addressed to a class of readers not habitually accustomed to severe reasoning, and is intended for those who know nothing of astronomy when they enter upon its perusal; and to them the course which we have preferred will probably be at once more interesting and more intelligible than the other. The general appearances of the heavens, the succession of day and night, the apparent courses of the heavenly bodies, are objects of interest and curiosity to all, however ignorant of the laws which regulate them, or the consequences which may be deduced from them. And they are not only interesting, but to a certain extent familiar; sufficiently so to perplex the reader of statements apparently at variance with them, and to deprive him of the greatest satisfaction that the student of a new science can feel, the power of at once comparing his deductions with facts, and convincing himself experimentally of the soundness of his reasonings by the accuracy of the results to which they lead him. A mind habituated to close reasoning upon merely hypothetical truth may be satisfied without such confirmation; yet it is agreeable to all, and to those which have not been thus exercised it is almost necessary; but, on the hypothetical system, it is the last point they arrive at. We have therefore preferred that course, which begins by stating and classifying appearances of general interest, and by explaining them in a manner consistent with popular observation; especially as we thus very early arrive at conclusions of high importance, which depend on those appearances, and follow from them as necessary consequences, in whatever manner they are themselves finally explained.

This however is not the only object sought in the adoption of the proposed course. One of the most important and laborious exercises of the mind consists in extracting from complicated results the simple principles on which they depend, in developing truth from the mass of confusion which often conceals it. There is no department of science which furnishes so long and so curiously connected a train of this kind of investigation as Astronomy: none in which the process is more curious, or the results more satisfactory. It has also this additional advantage, that it is capable of being made intelligible in its general outlines, almost without reference to mathematical investigation; and with none which requires more than a very limited portion of mathematical acquirement. It consequently seems to furnish to those, whose attention has not been directed to the severer studies, some opportunity of applying that peculiar discipline to the mind which those studies best furnish, and which moral and metaphysical inquiries, from the more vague and qualified character of the elements on which they depend, almost entirely fail in affording.

In speaking of the truths of Astronomy as admitting of explanation, almost without recourse to mathematical investigation, we only mean that the fundamental observations and principles admit of being stated intelligibly, and that much reasoning may be grounded upon them without introducing it. Of course however a science which is conversant entirely with mathematical relations can only be established on mathematical principles. Those therefore who are unacquainted with the elements of mathematics, cannot follow the whole process of the deduction; but they may trace at least its general course, assuming the correctness of those investigations which they are unable to prosecute, but taking their results as the elements of a new train of reasoning, which will, in many cases, be perfectly intelligible to every one who is acquainted with the common relations of number and quantity. With a view to facilitate such a course, all the more complicated mathematical investigations (of which, however, none run into difficulty, for the fuller development of the subject is reserved for a treatise of a purely scientific character) have, as far as possible, been removed from the text into notes*, and certain portions of the text itself, of too much importance to the general progress of the deduction to be thus removed, are included between brackets, in cases where they have appeared likely to offer difficulty to any class of readers. In these instances care has been taken, wherever the result deduced is of importance to the future course of the reasoning, to state it, if possible, in terms intelligible to those who are unable to pursue its investigation, and thus to enable them to resume the thread of the general argument. With all these assistances, however, it is impossible to remove some degree of difficulty and abstruseness even from the most popular parts of the treatise: a long process of investigation cannot be followed without thought and diligence, and the results of reasoning from their very nature can only be comprehended by some effort of reason and attention.

It may be necessary to remark, that while we profess to deduce our principles from the first and fundamental observations, we do not pretend to follow the historical course of discovery. For our purpose the class of objects which present the most simple phenomena requires the first attention: in the historical course of investigation, the most remarkable appearances were probably those which first attracted attention, and thus furnished the groundwork of theory. All that we profess to do, is to point out those observations from which we may most readily and conclusively deduce the principles of which we are in search: and in so doing we shall often call attention at an early period to facts and appearances, easily ascertained when the attention of an observer is directed to them, but which probably excited little attention till the progress of science showed their importance.

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* There are some notes also, which have been inserted in that shape, not on account of their difficulty, but as bearing only an incidental relation to the text. That there may be no confusion between the two classes of notes, the notes which present any mathematical difficulty will, as well as the similar passages of the text, be included in brackets.

Chapter I.

Section 1.—*First Observations on the Stars—Division into Constellations—Daily rotation of the Heavens.*

To an observer situated anywhere upon the earth, the heavens offer the appearance of a vast concave vault, in the centre of which he is himself placed, and which is bounded by a plane extending to the sky; this plane is called *the horizon*, and itself appears circular. In this vault we perceive several different sorts of bodies: the sun visible by day only (for its sinking below the horizon terminates the day, when that word is used in opposition to night); the moon visible either by day or by night; and the stars (among which, for the present, we include the planets and comets) visible generally by night only. All these bodies appear to the eye to be situated in the vault itself: for we have no means, by mere ocular observation at a single place, of estimating their respective distances, and we consequently refer all alike to one imaginary surface in which they appear to be placed, and which, as we find no apparent difference in their distances, we imagine to be spherical. A slight degree of observation will shew that all these objects partake generally of one common motion. The sun and moon, from their great apparent size, and the remarkable changes of their situation and appearance, more forcibly attract our attention than any others; but the stars, on account of their number and the opportunities which they in consequence afford for comparative observations, as well as from other circumstances which will hereafter be explained, are the preferable objects for the first and fundamental observations.

A very moderate degree of attention is enough to assure us that the stars, with the exception of a few called *planets**, from a Greek word signifying *wanderers*, always hold very nearly the same positions with respect to each other. If we consider a few stars to form a group, we may observe this group night after night, and its shape and appearance will be always the same. There are not anywhere in the heavens different groups of stars of considerable extent, so resembling each other that an observer can be in any danger of mistaking one group for the other. And as the groups cannot be mistaken, the individual stars composing them may thus be certainly recognized, however any single stars in each may resemble each other in magnitude, colour, and brightness. Being thus able to recognize a star which we have once observed, we may prosecute our observations upon it night after night, and year after year. For the immediate observations of an individual no more than this is requisite; but when he wants to register their results, or to inform others of their nature, it is evident that he can no longer be satisfied with this mere power of identifying to his own satisfaction the particular star which he observes at different times, but he must have some means of distinguishing between the different stars which he has himself observed, and of announcing to others which it is, among all the heavenly bodies, to which he has especially applied his attention. For this purpose we again have recourse to those groups of stars, by which we originally distinguished each particular star from every other. These groups, when divided for the convenience of reference, are called *constellations*, (*i. e. collections of stars,*) a name which is also applied to those portions of the heavens which they respectively occupy; and the whole surface of the heavens has been long divided in this manner. The divisions are arbitrary in themselves, and often, perhaps, ill chosen; but as the only real use of them is for the convenience of reference, the one important object is to have a single received standard; and it would consequently be very undesirable to alter them even for the purpose of making what would originally have been a simpler and more distinct division. The surface of the heavens being thus divided into constellations, consisting each of a moderate number of stars, the stars in each are catalogued, and are arranged nearly in the order of their apparent brightness; the brightest stars being designated by the earlier letters of the Greek alphabet, the less bright by the later letters of the same; and if there be more stars in the constellation than there are letters in that alphabet, other alphabets, generally the small Italic and large Roman alphabets, are used to denote the stars next in importance, and if all the stars in the constellations are not thus distinguished, the least considerable of all are catalogued by numbers. These stars being registered on maps of stars[*], or on celestial globes, or their places being defined in the manner we shall hereafter explain, become known bodies; and any astronomer making observations, on any particular star, may communicate them to any other, who will at once know the star in question, and be able to compare those results with his own observations and conclusions. Besides this mode of distinction, some of the brighter and more remarkable stars have been distinguished by particular names: and a vague classification has been made of stars, according to their lustre, into what are called different *magnitudes;* the brightest which appear the largest to the naked eye, although none have any ascertainable diameter when viewed through a telescope, being called of the first magnitude, the next class of the second, and so on. The stars of the fifth magnitude are barely discernible by the unassisted eye; and beyond these there are stars of the sixth, seventh, eighth, and even lower magnitudes, which we discover by means of telescopes. The number of stars of each magnitude increases as their brilliancy diminishes: thus in the Astronomical Society's catalogue of 2881 stars, there are only 21 stars above the second magnitude, (18 of which are called of the first magnitude, and 3 between the first and second magnitude), about 50 of the second magnitude and between the second and third, about 180 of the third magnitude, and between the third and fourth; and so on. As an instance of these different modes of characterizing stars, we may take the principal stars of the constellation Orion. The brightest star, α of Orion, is also of the first magnitude, and is sometimes called Betelgeuse, though this name is nearly obsolete; the next, β of Orion, is also of the first magnitude, and is called Rigel[†]; then there are four of nearly equal brilliancy, γ, δ, ε, ζ, of Orion, all of the second magnitude, and none having any received name; then follow stars of the third magnitude, then of the fourth, and so in succession[‡].

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* The comets may be considered as a class of planets.

Let us now examine what are the appearances presented to an observer in the course of a single bright night. He sees a vast variety of stars distributed about the heavens, and all, at first sight, seeming to be at rest. A very short course of observation, however, proves that this appearance of rest is fallacious. The stars all continue at the same apparent distances from each other, but their positions vary with respect to the horizon and the observer. On one side of the heavens they are seen gradually to rise higher above the horizon, and some stars appear above it, which at first were not seen at all; on the other, they sink towards the horizon, and some which were originally visible, sink below it and disappear. Confining our observations to a particular star, we shall see it rise above the horizon at a particular spot, gradually increase in elevation till it reaches its highest point, when it is said to *culminate*, and then sink by like degrees, and finally fall below the horizon at another spot which may also be ascertained. The same observation may be made with other stars, and extended to any number. It is not even necessary that the observations of different stars should be made at the same time, or that the observations of the rising and setting of any star be made on the same night; for the same star is invariably found to rise and set in the same spots, although at different periods of the night. Now each arch of the horizon intercepted between the points of rising and setting of any star may be *bisected*, or divided into two equal parts; and the points of bisection will themselves be at the extremities of a diameter of the circle. It is found from observation, that this diameter is the same for every star; and the points of the horizon through which it passes are called *the North and South points*. The side of the heavens where the stars rise is the *East side*, that where they set, *the West;* the East and West points being those equi-distant on each side from the North and South points. If through the North and South points a plane pass perpendicular to the horizon, the intersection of this plane with the celestial hemisphere will be a semi-circle; and this semi-circle is called the *meridian of the observer*, or

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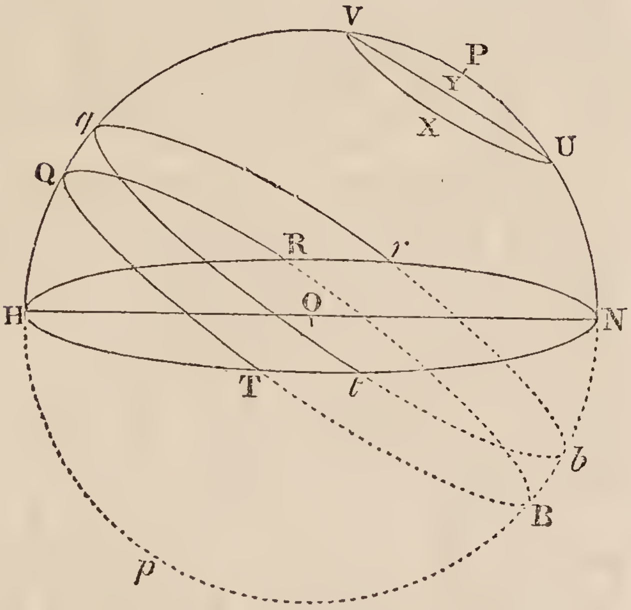
[*] A map of the stars (comprised in six sheets) is published under the Superintendence of the Society for the Diffusion of Useful Knowledge, in which these constellations are represented.

[†] These, and many of the names of the stars, were given by the Arabians, who, during the eighth century, and for a long period after, cultivated astronomy more diligently than any other people.

[‡] In the map referred to, Orion will be found nearly in the centre of Plate 2. This constellation may be best seen in the heavens for the purpose of comparing the original with the representation, about twelve o'clock at night in the middle of December, ten in the middle of January, and eight in the middle of February.

of the place of observation. A figure will explain these positions, although it is difficult sufficiently to represent the various dimensions of the sphere on a plane surface; and the reader should therefore, if possible, examine a globe with reference to these points.

Fig. 1.



Let H R N T (*fig.* 1.) represent the horizon of an observer; O, the centre of that circle and of the hemisphere H Q *q* N, being his position. Let T be the spot where a star first rises, T Q R its path above the horizon, and R the point where it sets; and let the arch T H R of the horizon, intercepted in one direction between the points T, R, be bisected in the point H, and the arch T N R intercepted in the other direction, be bisected in the point N. It is clear that H and N are at the two extremities of a diameter, for HR = HT, and RN = NT, and therefore, H R N (the sum of H R and R N) = H T N (the sum of H T and T N); or H R N, H T N are each of them semi-circles. Now, suppose *t q r* to be the path of another star above the horizon, *t* its point of rising, *r* its point of setting; it will be found that the arch *t* H = the arch H *r*, and *t* N = N *r*; that is to say, the points N and H in this case also bisect the arches of the horizon intercepted between the points of rising and setting. The same will be found to be true of every star (with some apparent exceptions to be presently noticed), and the points N and H are consequently fixed points, and independent of the particular stars by observation of which they were ascertained. Let H Q *q* N be the intersection with the celestial sphere of the plane passing through N H and perpendicular to the horizon; this is the meridian of the observer at O, and as the points N and H are fixed, the vertical plane drawn through them must be fixed also, and this meridian therefore is a fixed line. It will also be found by observation, that the points Q, *q*, where the paths of the different stars meet this meridian, are the most elevated points of those paths respectively, and consequently that a star which rises and sets attains its greatest height above the horizon, or *culminates*, when it is upon the meridian; and we may add, that the paths, T Q R, *t q r*, of the stars observed, appear to be parallel to each other, and that each path is divided by the meridian into parts T Q, Q R, or *t q*, *q r*, apparently equal and similar; that is to say, T Q equal and similar to Q R, and *t q* to *q r*.

In one part of the heavens the appearances are a little different. If we look to the Northern part we shall see many stars which never sink below the horizon. These stars pass the meridian twice, once at the lowest, and once at the highest point of their course. In moving from the lowest to the highest point, their course is entirely to the East of the meridian; in passing from the highest to the lowest point, it is entirely to the West of that line. These parts also in this case, as well as in that already mentioned, are similar and equal; and if the course of one of those stars be represented in the figure by the line U X V Y, every point in that line will be found to be equidistant from a certain point P, situated in the sphere, and U X V Y will be a circle, and the point P its pole. Besides this, it will be found that the path of every one of these stars which never set is a circle described at a given distance from the same point P, which itself appears stationary; and the position of this point being ascertained, it will further be found, that each of the paths T Q R, *t q r*, described by stars which alternately rise above the horizon and fall below it, and which appear, as we have already mentioned, to be parallel to each other, are themselves also portions of circles, every point of which is equidistant from the same point P, and which therefore are parallel to each other, and to the path U X V Y, described by a star which never sinks below the horizon. The point P therefore is the pole of all the parallel circles described by the stars; and if we suppose the sphere to be completed, as it is by the dotted lines in the figure, and *p* to be the point directly opposite to P, *p* will be the other pole of the same circles; or there are two fixed points, one P found by observation, the other *p* deduced from it, to which the motion of the stars may be equally referred. O being the centre of the sphere, will be a point in the line which joins P, *p*. The stars which are seen to describe their whole circle round the pole are called *circumpolar* stars. We shall also find, that in the time in which any particular star describes a third or fourth part, or any given portion of its circle, all others describe the same portion of theirs; and consequently they all continue in the same positions with respect to each other, though their places vary with respect to the horizon and the observer.

Having thus far ascertained the appearances which the stars present, let us see if we can thence deduce any conclusions respecting the occasion of them. For this purpose let us suppose the hemisphere, H Q P N, to be made a complete sphere, as is done by the dotted lines in the figure. Let us also suppose that the whole sphere has a motion of rotation round a line joining P, *p*, which is called *the axis;* but that H R N T, the horizon, or the line in which the plane bounding the visible hemisphere meets the heavens, continues fixed: and let us see what would be the appearances presented in such a case. Let us take the case of a star upon the horizon at T. It is clear that, as by the rotation of the sphere it was transferred from T, it would appear to move in a line of which every point was equidistant from P, for every point in that line would be determined by the actual distance of the star from P. Its apparent path therefore would be a portion of a circle, every point of which is equidistant from P; and, in point of fact, we have already seen that it is so. In the same manner, if there be a star which never falls below the horizon, and whose distance from P is P V, its apparent path would be a circle, of which every point is at the same distance P V from P, or it would be represented by a circle, U X V Y, which we have already seen to represent the apparent path of a star which never sets *. Each of these circles, thus described by the motion of different stars, having every point in it equidistant from the same point P, they all would, as before, be parallel circles. Again, as each of them is described in consequence of the same general motion of rotation of the whole sphere, each would be described in the same time; namely, the time of that rotation; and in the same manner, in any portion of that time, each star would describe the same portion of its own circle; namely, the same portion of that circle which the sphere describes of a complete revolution. All these are the appearances which we have already seen that the heavens, in fact, present.

The appearances presented by the motions of the stars may then be accounted for on the supposition that the sphere of the heavens revolves round an axis joining P, *p*. They cannot be explained however on this supposition, except by supposing that the sphere goes through a complete revolution. The motion of a star which is seen to rise and set, as that whose path is T Q R, might be explained by imagining the sphere to make only a part of a revolution; and the magnitude of that part would depend on the proportion which the visible path T Q R bore to the whole circle of which it formed a part; but the stars which never set, as that whose path is U X V Y, are seen to describe the whole circle, and their motion therefore can only be thus explained on the supposition of a complete revolution of the heavens on the axis P*p*. If however this be the case, the motions of the stars which sink below the horizon must also be continued below it, or they will describe below it the remaining parts (those represented by the dotted lines) of the circles T Q R B, *t q r b*. Let us see if we have any means of discovering, by observation or reasoning, whether they do so.

The first remark that occurs on this question, is that the supposition that they describe below the horizon the remainder of the circle, of which they are seen to describe part above it, at once accounts for one circumstance that seems to admit of no other explanation. We trace the path of a star from its rising at T, to its setting at R, and then lose sight of it; but on the next night we again see it appear at the same point, T. We know therefore that the star is in some way transferred from the point R, where it sets, to the point T, where it rises; and the most probable way in which we can suppose this transference effected, is the continuation below the horizon of the same motion which it had when above it, or the description of that circle, R B T, which it would describe on the supposition that the whole heavens revolve round the observer.

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* These positions may be thus illustrated:—If you take a top, or any body which you can spin with great steadiness and accuracy, and place a spot of ink upon it, and then spin it with great velocity, so that the spot returns to the same place in less time than is necessary for the eye to lose the impression last made by it (See Treatise on Optics, p. 44), the effect produced will be that of a line encompassing the top, and forming a circle upon it, of which every point will be equidistant from the extremity of the axis upon which the top spins. The apparent track of the point is of course the same, whether its motion be quick or slow; but by the rapidity of its motion we gain the advantage of actually seeing the whole track at once.

If however we take the case of a star rising just at the time when the stars begin to appear in the evening, and setting as day breaks on the following morning, it is evident that its path below the horizon, if it be described at all, must be described by day; or that the same motion of revolution continues by day, which we seem to have ascertained to exist by night. Does observation then confirm, or disprove this conclusion? The sun and moon are visible by day, but their motions, although they generally confirm it, are of a more complicated nature, and we therefore do not wish to draw our inferences on this point from them; and the stars are not visible to the unassisted eye when the sun is above the horizon. The telescope however, in the hands of a skilful observer, for only such a one can make the observations necessary for this purpose, removes this difficulty; with it he can, even when the day is brightest, ascertain the positions from time to time, and consequently the motions, of many of the brighter stars; and the result of these observations is, that the stars are ascertained to describe in the day-time the same courses which they are easily seen to trace in the night: and we consequently come to the conclusion that their motions may be accurately comprehended and explained, on the supposition that the whole heavens revolve about an axis, passing through the position of the observer, and carry the particular stars with them in their revolution.

If this be so, and the meridian of the place, H Q P N, be continued, as by the dotted line N B *p* H, below the horizon, so as to complete the circle, this lower part of the circle will again intersect the circles T Q R B, *t q r b*, in the opposite points, B, *b*, to those, Q, *q*, where the upper part of it met them above the horizon; and as Q, *q*, were the points most elevated above the horizon, B, *b*, will be those most depressed below it; or in other words, every heavenly body, which sinks below the horizon of a particular place, will be most depressed below it, when it passes the meridian of that place below the horizon, and of course below the pole. We have already seen a corresponding result with respect to circumpolar stars, when they cross the meridian below the pole though above the horizon.

As yet we have only considered the conclusions which an observer, confined to a single point on the earth's surface, would arrive at on this subject. We will now proceed to examine how they will be affected by a comparison with the results of other observations, made at a different place. The account which we have given of the observations made at one place, applies with equal correctness to all; that is to say, an observer situated anywhere upon the earth, finds that the apparent paths of the stars are circles, or portions of circles, each having every point in it equidistant from two fixed points, one in the observed heavens, and one in the other part of the sphere, supposed to be completed, and each bisected by a line passing through the visible fixed point, and dividing the visible heavens into two equal portions. In each case therefore, this line is what we have termed the meridian of the place of observation; and every place therefore has a meridian, passing through a fixed and immoveable point in the heavens. The position of this point may be ascertained by observation at each particular place, and it is found to be the same at all; the other extremity of the axis also is the same in each place.

We come therefore to this conclusion, that the axis P *p* round which the revolution of the heavens takes place is a fixed and determined line, not depending on the situation of the observer: and this is one circumstance necessary to the establishment of our theory, that the apparent motions of the stars may be attributed to the revolution of the heavens round a fixed axis; for if observations made at each place gave a different axis, they would be inconsistent with such a supposition. The points, P, *p*, are only imaginary points, being those where the axis P *p* meets the imaginary sphere of the heavens; they are however important to be known, and go by the name of the *poles of the heavens*. They are points, as we have already seen, in the meridian of every place, and therefore they no where appear either in the East or West side of the heavens; if however we conceive the heavens divided by a vertical plane, passing through the East and West point at any place*, the points P and *p* will always be on opposite sides of this plane; that is to say, the one on the North side of it, the other on the South, and the same point P is always on the same side of the plane. If therefore (in *fig.* 1.) P represent the pole, which to an observer at O is on the Northern side of the heavens, P is always on the Northern side, and is called on this account the *North Pole* of the heavens, and in like manner *p* is the *South Pole*†. There are however two points on the earth, (the poles of the earth) where the points P, *p*, are the one directly over the head, the other directly under the feet of the observer; here therefore there is no North or South point, and we shall hereafter see that the phenomena from which we deduced our definition of these points,—namely, the rising and setting of stars, do not take place at these situations. We have already seen that P, *p*, are points in the meridian of every place; all these meridians therefore intersect each other at the two poles. If *p*, the South Pole, be above the horizon, P, the North Pole, will of course be below it.

One circumstance may here require explanation before we proceed farther. We have already seen that the centre of the heavenly sphere is a point in the axis P *p*, and that this centre appears to be the situation of the observer; and we have also said that the results of observation are the same, wherever on the earth's surface he be placed. If two observers be at situations, the one one thousand miles East of the other, the situation of both cannot be in the line P *p*; but if the one is in it, the other must be nearly one thousand miles out of it: yet they both appear to be in it. We know from very simple reasoning, or we may easily satisfy ourselves by trial, that a small change of position in the observer does not affect the apparent position of a very distant object. Thus, if there be two trees, or two spires distant ten miles from each other, and two men stand half-way between them, the one precisely in the line joining them, and the other a yard on one side of it, each will alike feel that, to all common observation, he is exactly in the line which unites them. The angle between the two directions in this case, would be considerably less than half a minute*, and would not be observable except by instruments of some delicacy. In the same manner, if the distance to the points P, *p*, be excessively great in proportion to the distance between the situations of different observers, each observer will seem to be in the same position with respect to the points P, *p*, and the line joining them. There is therefore nothing absurd or contradictory in the apparent coincidence of each situation with the line P *p*, if we only suppose the points P, *p*, so remote from the earth, that any line drawn on its surface is too small to be estimated in comparison with that distance; and we get therefore a notion of the vast distance of those points, instead of a difficulty affecting the notion of such a revolution, as we have supposed to take place. If however every point on the earth's sur-

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[*] The circle made by the intersection of such a plane with the sphere of the heavens is called the *prime vertical*.

[†] A well-known star, α of the Little Bear, called also *Polaris* or the Pole Star, is at the distance of only one degree and fifty minutes (see the next note) from the North Pole P. Its motion, like that of all other stars, is in a circle, every point of which is equidistant from P. As P therefore is a fixed point, and the Polestar always very near it, the observation of the Polestar furnishes a very easy method of finding very nearly the fixed North Pole of the heavens. The meridian being a vertical circle, and passing through the pole to the horizon, the points where it intersects the horizon are those points of the horizon respectively nearest and most distant from the pole, and thus the North point of the horizon is that nearest to, and the South point of the horizon that most distant from, the North Pole.

[*] Every circle is conceived to be divided into 360 parts called degrees, each degree into 60 minutes, each minute into 60 seconds, each second into 60 thirds, and so on. One degree is written 1°; one minute 1′, one second 1″; and so on; though the divisions beyond seconds are quite as frequently expressed in decimal parts of a second. Thus nine degrees, fifteen minutes, twelve seconds and twenty-four thirds, are written 9° 15′ 12″ 24‴, or, as 24 thirds are $\frac{24}{60}$, or $\frac{4}{10}$ of a second, they are also written 9° 15′ 12″. 4. A degree therefore is the 360th part of a circle, a minute the 21600th part of it; half a minute, the quantity mentioned in the text, the 43200th part of a circle. The degrees, minutes, &c., of one circle, will of course occupy more space than those of another, exactly in the same proportion as the one circle is longer than the other, for they are in each the same proportional part of the whole circle. They correspond to the same angle in every case, but differ in linear magnitude, as the circles on which they are measured differ.

face be apparently in the line P p, so must its centre also, which lies in the midst between these points. The axis P p therefore may be considered to pass apparently through the centre of the earth; and we shall hereafter see the strongest physical reasons for believing that it actually does so.

There are several reasons which convince us, on very slight examination, that the earth itself is, speaking loosely, of a globular figure. They are collected and explained in the first pages of the "Treatise on Mathematical Geography;" and it is therefore unnecessary here to go into any account of them*. But it is important here to point out how we are enabled by this globular figure of the earth to verify the conclusions we have already formed respecting the motions of the stars. The earth being convex at all points, the horizons of different places, which are always planes touching the earth at those points, will be inclined to each other at all different angles; and the height of the pole above the horizon will vary in consequence. The height of the pole above the horizon measures what is called *the latitude* of the place. We shall hereafter explain why: it is sufficient now to state, that whenever we speak of the latitude of a place, as having a particular value, for instance 50°, we mean that the pole is there at the height of 50° above the horizon. We have already seen that there are certain stars which never sink below the horizon at a particular place, and which at that place are called *circumpolar* stars. If we take another situation, so that the height of the pole above the horizon be greater there than at the former place, some of those stars which before rose and set, will now (if the supposition, that they continue to describe below the horizon the remainder of the circle which they were seen to describe in part above it, is a true one) have their whole course above the horizon: in other words, they also will be circumpolar stars at the second place of observation; and we may there ascertain, by complete observation, whether they do actually describe the whole circle, as we have supposed they would: and in every case it is found that they do so. Again, the earth being globular, there are points on its surface from which an observer will actually see the opposite part of the heavens from that presented at any given places; and the question, whether the same motions be continued below the horizon which we observe above it, may in this way also be brought to the test of complete trial, and the truth of the doctrine will be established, if we find that in every part of the earth the same appearances of circular motion are observed in the paths of the stars above the several horizons of each observer. And this we find to be universally the case. In every way therefore the original conclusions, which we draw from observations at a single place, are confirmed by a comparison of those made at several.

We shall hereafter see that there is another and simpler mode of accounting for these appearances, by ascribing a motion of rotation to the earth itself. The appearances themselves however are exactly those which would result from the rotation of the heavens; and our only proof that the other theory will account for them, will be by shewing that it must necessarily produce the same appearances as the actual rotation of the heavens would do. We may therefore properly, for the present, treat the rotation of the heavens, not as an established fact, but as a supposition enabling us to account for, and represent all these appearances.

SECTION 2.—*First Observations on the apparent Motions of the Sun—Diurnal Motion—Annual Motion.*

Having thus ascertained the apparent rotation of the heavens and the stars round an axis, we may proceed to consider the more complicated appearances presented by the sun. The first notion which we gain from observation is, that he also follows the same course of revolution as the stars; for he is seen to rise in the Eastern, and set in the Western, part of the heavens, culminating in the meridian, and rising and setting at points apparently equidistant, or very nearly so, from the North point of the horizon. So far the appearances of a single day seem to correspond with those already noticed in the stars. But when we register the observations of a long period, a year for example, we find a striking difference between the two cases. The star always appears in the meridian at the same spot, it rises always in one point of the East, and sets in one point of the West; but all these things are different in the case of the sun. If we observe his height when he crosses the meridian, we find that it is less on the 21st day of December, than at any other period during the year; that it is greatest on the following 21st day of June; and that in the interval it is on each succeeding day greater than on the preceding one: and again, after the 21st of June the sun passes the meridian at points successively lower on each day, until, on the 21st of December, he again returns to his least elevation. The variation of this elevation is not uniform; it is most rapid about the 21st of March and the 21st of September; and for a few days before and after the 21st of June and the 21st of December it is hardly perceptible. At these periods therefore the sun has loosely been said to stand still; and they have in consequence, and from the seasons at which in Europe they happen, gained the name of the summer and winter *solstices*. The same observation may be made also with respect to the points where the sun rises and sets; indeed it is a necessary consequence of the sun rising from day to day higher on the meridian, that his points of rising and setting should also approach the pole which is above the horizon; and, on the other hand, when he is falling on the meridian, that the points of rising and setting should in like manner recede from it. It is also found by nice observation, that during the period in which his meridian heights are continually increasing, he always, when it is the North Pole which is above the horizon, sets nearer to the Northern point than he rises, and rises on the following day still nearer the North; and conversely, that when his meridian heights are continually diminishing, his point of setting on any day is more Southward than that of his rising that day, but less so than that of his rising on the following day. All these observations correspond exactly to the supposition, that for the six months extending from the winter to the summer solstice, the sun, besides partaking of the general rotation of the heavens, has a proper motion of his own in the heavens which continually carries him Northward; and again, that in the interval between the summer and the winter solstice, he has also a proper motion, but that its direction is opposite to the former, that is to say, from North to South. The same observations may be made where the South Pole is above the horizon, with similar results; the only difference being, that in that case his motion Southwards produces effects corresponding to those of motion Northwards in the other.

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* The argument in that treatise deduced from the appearances of *heavenly* bodies cannot properly be applied to the purposes of this treatise, as the reader must be supposed hitherto ignorant of the nature of these appearances. Omitting these, we have sufficient evidence, in p. 3 of that Treatise, of the globular figure of the earth in a very general sense, and that is all which we want for the present purpose.

This however is not the only motion which we can discover in the sun. Every one who has observed the nightly appearances of the heavens with any attention, is aware that they continually differ. On each succeeding night, or it may be more convenient, for the sake of marking the changes more strongly, to take nights at a considerable interval, as for instance a month, from each other, some stars become visible which had not been so at the last time of observation, and others cease to appear which had then been seen. The new stars which from time to time make their first appearance, do so invariably in the Eastern portion of the heavens a little before sun-rise; those which have ceased to appear always were last seen in the West a little after sun-set. It is found also, that stars which rose a little before the sun at the former observations, rise longer before him at the latter; and, in the same manner, that those which set a certain time after him at the former, set a less time after him at the latter observations. It is easily seen that these changes can only be occasioned in one way. We have already seen that the stars themselves keep the same positions with respect to each other; but the sun evidently approaches the regions of the heavens which are above, but near his point of setting in the Western part of the horizon, at the time of his setting; for having previously set before stars there situated, he now sets at the same time with them. In the same manner, he recedes from the regions of the heavens which are near, but above his point of rising in the eastern part of the horizon when he rises; for having previously risen at the same time with stars there situated, he now rises after them. He evidently therefore has a motion from West to East. These observations hold alike in every period of the year, and at the close of the year the sun is in the same position relatively to the stars, as at its beginning. We have therefore ascertained that he has a proper motion from North to South, and back from South to North, which restores him at the end of the year to the same altitude when on the meridian which he had at its beginning; and that he has also a motion from West to East, which in the same period brings him back to the same place, but which, being continually in one direction, can only do so by making him describe in that direction the complete circuit of the heavens.

SECTION 3.—*Mode of ascertaining the motions and places of heavenly Bodies —Measure of time—Pendulum—Solar and sidereal day—Measure of place—Right ascension—Declination —Sun's path, the ecliptic —Equinoctial points—Longitude--Latitude.*

We are still however far from a sufficient knowledge of the motions either of the sun or stars. We see that they revolve round the earth, but that they do not do so in the same time, and we cannot yet tell whether either revolve uniformly, or with a variable motion. To know any thing certain concerning motion we must know the space passed over and the velocity with which it is so. But the velocity can only be measured by the space passed over in a given portion of time, and the first great object therefore is to find out some method of computing and measuring time. If we knew that the motion either of the sun or of the stars was equable, we might adopt that motion as a standard; but it is evident that we have as yet no reason for so supposing. An hour appears a period of a very different length according to the mode in which we pass it: almost imperceptible sometimes in sleep, short in agreeable mental or bodily exertion or amusement, long in irksome employment or tedious idleness; and we have no power, by mere sensation, of accurately discriminating or comparing the lengths of several unconnected periods. But the periods of the successive returns of a heavenly body, whether it be sun or star, to the meridian are unconnected; and we consequently have no means by mere mental perception of comparing their durations, and saying whether they are equal or unequal. And this being the case, we can have no right to assume either as a standard of time; for time, like everything else, can only be measured by reference to something of a fixed and determinate value. Our notions of time indeed are so complicated with the words describing the periods into which we find it convenient to divide it, that it requires some attention to feel the full force of the difficulty. A day is the period to which we commonly refer everything, and which we consider as of a fixed and uniform duration; a day is also the period between the successive appearances of the sun on the meridian; and in common language it is also the period between his successive risings or settings. Being thus in the habit of using the word both as an expression for a certain fixed and uniform period of time, and for the interval between certain events which do in fact happen very nearly at that distance of time from each other, we cannot readily separate the two meanings of the word, or perceive how difficult it would be to discover the equality of the intervals between these events unless we had some certain and definite standard, independent of themselves, whereby to measure them. The difficulty however certainly exists; and we may in some degree be convinced of its force, even by an argument drawn from the difference between the two latter uses of the word *day* which we have mentioned. During the period in which the sun's motion is from South to North, we have already seen that he rises farther North on each successive morning, and it necessarily follows (a consequence which will be fully proved hereafter), that on each successive day a larger portion of his circle of rotation is above the horizon. If his motion be such that each successive interval between his appearances on the meridian is equal, or nearly so, his successive appearances on the horizon will be rapidly accelerated for a considerable period, then slowly so, and finally retarded. This will be more fully explained hereafter; but we all know the fact from the common tables of the time of sun rise. Thus, in our climate, in March, sun-rise precedes the arrival of the sun on the meridian by a period increasing from day to day by about two minutes; in June the alteration is hardly perceptible; and in September the interval diminishes as rapidly as it increased in March. In March then, the day, understood as the interval from sun-rise to sun-rise, is about two minutes less than the day, understood as the interval from noon to noon; while in June they are equal, and in September again, the day from sun-rise to sun-rise is as much longer than that from noon to noon, as it was shorter in March. No man could pretend to be conscious of these minute differences of duration; and if therefore we were to depend on sensation merely for our measure of time, we might just as well take the interval from sun-rise to sun-rise for our standard, as that from noon to noon; and either as that from the successive appearances of a given star on the meridian. The differences between the actual lengths of these different periods may seem trifling; but such is the nicety required in astronomical observations and computations, that no inaccuracy can safely be overlooked; nor, if we once force ourselves to give up the appearance of security given by the popular use of the words "day," "year," &c., are we justified in believing that our errors will be confined within any narrow limits.

We have however one standard of time independent of any assumption whatever, except that of the truth of those general principles, called the laws of motion, which are involved in all the deductions of mechanical science, and which, although inferred from observation and reasoning, do not admit of strict and absolute demonstration. All bodies are found, by experiments made anywhere on the earth's surface, to fall there towards the earth in a straight line, called a vertical line, in consequence of the action of some force acting on them in that direction, which we call the force of gravitation. If at that point the force act at every instant of time with the same intensity, a pendulum properly adjusted will perform its successive oscillations in equal times; that is to say, it will always take the same time in swinging backwards and forwards. This depends on no assumption whatever; it is a necessary mathematical consequence of the action of a constant force, tending vertically downwards, on such a body so suspended. If therefore we can ascertain that the force by which a body falls to the ground at a particular place is always equal, we know that the oscillation of a pendulum there will give a fixed standard of time. We might perhaps venture at once to assume that this force is invariable, on the principle that, all the circumstances under which it acts being, as far as we can perceive, the same at all times, there is no reason whatever for supposing it to vary. It is, however, satisfactory, if possible, to ascertain by actual experiment, whether the force be variable or not. Of course, when the object is to ascertain the mode of measuring time, no measure of time must be used in the process. Nothing, therefore, which depends upon the measurement of velocity can be admitted, for velocity only means space described *in a certain time.* We have however sufficient, though not very ready means of ascertaining the fact. Space is easily measured; and the momentum of a body can be, although not very accurately, yet sufficiently estimated by the effects it produces. If the quantity of matter in a body be represented by A, and the force acting on each particle of matter in it by F, and the space through which it falls, by S, it may be proved, as a necessary consequence of the meaning of these terms, that the momentum varies as $A. F.^{\frac{3}{2}}, S.^{\frac{1}{2}}$* If therefore we try the experiment with the same body falling through the same space at different times, the momentum will vary as $F^{\frac{3}{2}}$, and will increase and diminish with the increase and diminution of the force. If then we find that the body always sinks to the same depth in similar bodies placed to receive it, that it produces the same contraction of a spring attached to the point on which it falls, and generally that it produces effects always equal in manners which may be varied according to the pleasure of the experimenter, we are led to the conclusion that the force always acts alike at the same point. Now this is the conclusion found by experiment. We have therefore in the pendulum a standard of duration: not a convenient one as yet, because we have nothing whereby to fix on any particular length of pendulum as the standard; nor a general one, for we have as yet no reason to form an opinion whether the force of gravity at different points on the earth's surface is the same or different, and we therefore cannot tell whether our pendulum of one place would swing at the same rate at another. We know only that at the place where the experiments were tried its successive oscillations are always of equal lengths, and consequently, that we may there ascertain by it whether the revolutions of the sun, or stars, or either of them, be of uniform length; for whatever be the duration of the uniform oscillation of our pendulum, if we observe that either of these periods of revolution always includes the same number of these oscillations, that is an uniform period, and may itself be adopted as a standard of duration. If the number of oscillations be different in different revolutions, the period is not uniform or fit for a standard*.

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* [Momentum $\propto$ W.V.

$\propto$ A.F.V. (but $V. \propto \sqrt{F.S.}$)

$\propto A.F.\sqrt{F.S.}$

$\propto A.\ F.^{\frac{3}{2}}\ S^{\frac{1}{2}}.$

Now the period of revolution of the stars is found by this sort of comparison to be always accurately the same. Each star is found to have precisely the same interval between its successive appearances on, or as they are also termed, *appulses* to, the meridian, and this interval is called a *sidereal* day. It is also found that the different portions of each revolution are described in proportional periods: thus, if two different stars are in the same circle of rotation, but the one distant 180° from the other, the half of a sidereal day elapses between their appulses to the meridian; if the distance of two such stars be 90°, the interval between their appulses is a quarter of a sidereal day; and so in like manner for every proportion of distance.† We find therefore, not only that the duration of a sidereal day is constant, but that during every part of it the rotation goes on uniformly; or in other words, *that the heavens revolve round their axis continually with an uniform velocity.* The discoveries of an advanced state of science furnish the most complete confirmation of this result, by shewing that none of the causes which produce variations and disturbance in other motions, produce any in that from which the apparent motion of the heavens round an axis proceeds.

The interval between the successive appearances of the sun upon the meridian, or from noon to noon, is necessarily longer than that between those of a star; for as the motion of the heavens is from East to West, and the proper motion of the sun from West to East, the sun on each successive day, when the point of the heavens where he was at noon on the day before returns to the meridian, is to the Eastward of that point, and, consequently, to the Eastward of the meridian; and he therefore only returns to the meridian after the rotation of the heavens has continued for some additional period, long enough to bring his new place to the meridian. The interval between two successive appulses of the sun to the meridian, is called a *solar day*. We find by observations with our pendulum, that the length of the solar day is continually varying, but still that its variations succeed each other in a regular succession, and go through all their changes in a certain period of time, a year. Although therefore the solar day is of variable length, we can, as we know all its variations and their period, ascertain its *mean* or average length; and this quantity is called a *mean solar day*.

The pendulum, although it furnishes us with the means of ascertaining that the motions of the stars are uniform, and those of the sun variable, is not, for the reasons we have given, adapted in the first instance for furnishing the common standard of time. The solar day, again, does not seem well suited to the purpose, on account of its variable length. On the other hand, as the continual appearances and re-appearances of the sun above the horizon furnish the most remarkable distinctions between different portions of time, and do practically regulate the occupations of men, and determine the periods of labour and those of rest, it would be inconvenient so to fix the standard of time used respecting the common occurrences of life, that the periods by which we measure it should continually have their commencement at different parts of this, which we may call *the working day*. This inconvenience however would clearly occur if we took the sidereal day for our ordinary standard; that is, if we fixed the commencement of the day, at the instant when a particular star is on the meridian: for as the sun, moving continually Eastward, is successively at all distances East of the star, he must come to the meridian at all intervals of time after it; and consequently the star will be on the meridian at one period of the year when the sun is so also, or as we say at noon; at others when he is just rising, just setting, or midway below the horizon, or we say at midnight. The *mean* solar day, however, is liable to no such objection: being a period deduced from computation of the *average* of the actual solar days, it is a fixed and invariable period; and as no solar day differs much from it, and the differences are some in excess, and others in defect, the period of the observed commencement of the real solar day never differs so far from the computed or registered commencement of the mean solar day as to occasion the inconvenience which must result when these standards materially differ. In fact, the difference between the commencement of the real and the mean solar day never much exceeds sixteen minutes*.

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* It must not be understood that the considerations stated in the text were those from which the expediency of using the pendulum as a standard of time was in fact deduced, though they seem to furnish the mode of deducing it, which involves the fewest assumptions. It is, indeed, probable that the experiments mentioned in the text have never been made with any degree of minuteness. Huygens, who first demonstrated the mathematical principles of the motion of a pendulum, though Galileo had accidentally observed that its oscillations appeared to be all of the same length, seems to have proceeded on the principle mentioned in the text, of assuming that the force of gravitation was constant, from the absence of any apparent reason for its varying. From this assumption he deduced the laws of the motion of a pendulum; and it has ever since been adopted as the standard of time.

† These observations of distance cannot be made with any great degree of accuracy, unless by means, hereafter pointed out, which proceed on the supposition that the heavens do revolve continually with an uniform velocity. They may, however, be made with certainty enough to convince us that the limits within which any variation must be confined must be exceedingly small: for, by multiplying observations, and taking *a mean*, or average, between their results, we may be sure, where there is no cause affecting all the observations in the same way, to obtain a value very near the truth. This is an obvious and necessary consequence from the supposition that there is no constant cause of error affecting all the observations *alike*: any accidental cause is likely to affect them in different manners, sometimes increasing and sometimes diminishing the apparent distance, and the errors thus compensating each other, the mean result, even where the individual errors are considerable, will not differ much from the truth.

Taking then the *mean* solar day as our standard of time, it is divided into twenty-four hours, each hour into sixty minutes, and each minute into sixty seconds; and these are each of fixed and determinate length. It is established by the principles of mechanics, that we can, by varying the length of a pendulum, make its oscillations of any exact length that we please: and as the second is the smallest division of time in common use, it is usual to make the pendulum of a clock of such a length that its oscillations are of a second each. If the force of gravity is different at different places on the earth (as we shall hereafter see that it is), the lengths of the pendulums vibrating seconds at these different places will differ; but they may be so adjusted that the period of the oscillation shall be accurately the same in all. The length of a pendulum vibrating seconds in the latitude of London, 51° 31.′1, in a vacuum at the level of the sea, is 39,13929 inches.

Taking these common divisions of time into hours, minutes, and seconds, the length of the sidereal day is found to be uniformly 23 hours, 56 minutes, or more accurately, $23^h\ 56^m\ 4^s.092$. This period may be divided into 24 equal portions, and each of those subdivided into 60, and those again into 60 other equal portions; and these divisions will be respectively *sidereal hours, minutes, and seconds*, bearing the same relation to the sidereal day that the common hour, minute, and second do to the mean solar day. Of course a pendulum may be made of such a length that its oscillations will be of a sidereal second each; and in fact astronomical clocks are usually made so. Computations and observations made by sidereal time are of course easily transferred to common (or mean solar) time, and the reverse.

Having now got our measures of time, we proceed to see how, by means of them, we can ascertain the actual position of the sun in the heavens on each particular day. His light so far overpowers that of every other body, that there is some difficulty in observing his situation accurately, by immediate comparison with that of others; for, although we can see the stars through telescopes in the day time, they are not very readily found, and cannot easily be observed unless we have some previous knowledge of their positions, and adjust the telescope accordingly; they consequently are ill suited for the earliest observations. The moon indeed is often visible at the same time with the sun, and may be compared in her position with him by day, and afterwards with known stars by night; but she has, as we shall see, a very complicated proper motion of her own, in the interval between the two observations; and consequently furnishes only an inconvenient method of determining the position of another body. We must therefore have recourse to other means, and for this purpose we must explain a few of the simpler properties of the sphere.

In *fig.* 2. let P*p* represent the axis of the heavens, P, *p*, being the poles, and let PE*p*Q be the meridian of the observer. Suppose also that a plane is

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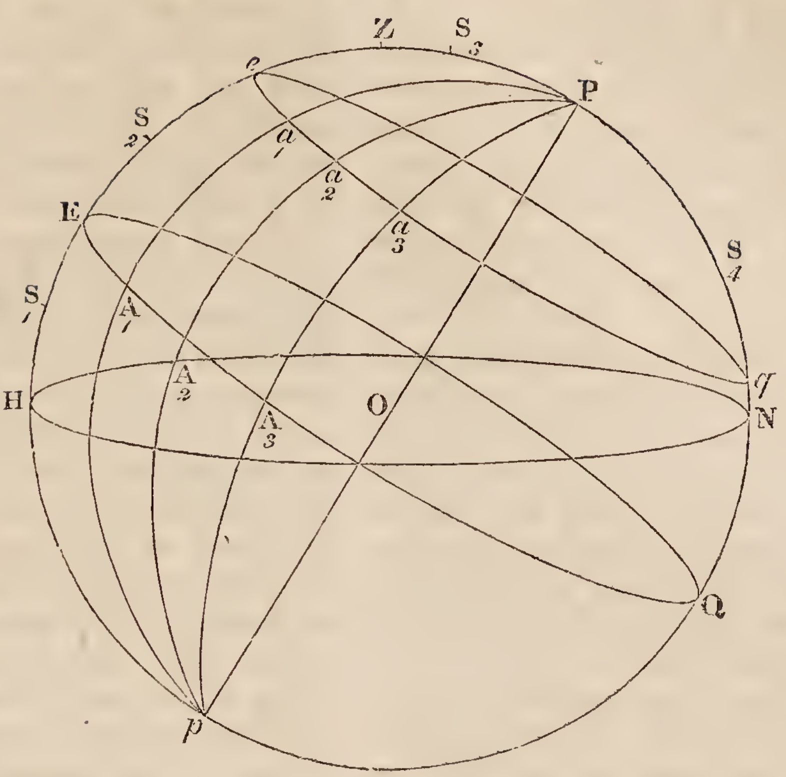
* Although the commencement of the mean solar day seems the most convenient division of time, for the reasons given in the text, it is by no means universally adopted. The French use the *true* solar day; that is to say, they fix the commencement of each day at the instant when the sun is actually on the meridian. The length, therefore, of the day varies, and their clocks require to be regulated accordingly.

Fig. 2.



drawn through O, the centre of the sphere, so that Pp shall be perpendicular to it; the intersection of this plane with the sphere will be a great circle of the sphere*, and this great circle will bisect the meridian $PEpQ$, and every other circle drawn through the two points P, p, as PA_1p, PA_2p, PA_3p. This circle EA_1A_2Q is called the *equator*. The meridian of every place is a great circle passing through the two poles, and every circle so passing must be the meridian of some particular places, according to our original definition of a meridian. Generally therefore they are called *meridians*, or *meridional lines*, and the equator bisects all the meridians. In this sense of the word the meridians are lines in the sphere partaking of its general revolution. The meridian of a particular place, on the contrary, is a fixed line, with which each of the others in the course of its revolution coincides. Thus, referring to the common celestial globe for illustration, lines drawn from pole to pole on the surface of the globe would be *meridians*, using that term generally: the brass meridian corresponds to the *meridian of the place*, for which the globe is adjusted.

The equator EA_1Q being a circle bisecting every meridian, has every point equidistant from P, or p: it is therefore a circle of rotation, or any point situate in the equator describes a course round the earth coincident with the equator itself. Let ea_1a_2q be any small circle parallel to the equator. It will, by the properties of the sphere, have every point equidistant from P or p, and will consequently be a circle of rotation. If e, a^1, a_2, a_3, be the several points where the different meridians $PeEp$, Pa_1A_1p, Pa_2A_2p, Pa_3A_3p, cut this small circle, and E, A, A_2, A_3, the points where the same meridians cut the equator, then it is a property of the sphere, that whatever be the proportions that the several arcs EA_1, EA_2, EA_3, bear to the whole circle EA_1Q, the arcs ea_1, ea_2, ea_3, bear the same proportions respectively to the whole circle ea_1q. But the periods of rotation of a star in the small circle ea_1q, or in the great circle EA_1Q, are equal; for they are each a sidereal day; and the motions of rotation in each are uniform: the periods of rotation therefore through the arcs a_1e, a_2e, a_3e, are to the whole sidereal day, in the proportions of the respective arcs themselves to the whole circle $e a_1 q$; and the periods of rotation through the arcs $A_1 E$, $A_2 E$, $A_3 E$, are to the same whole sidereal day in the proportions also of those respective arcs to the whole circle $E A_1 Q$, the same proportion as the preceding ones. The periods of rotation therefore through the arcs $a_1 e$, $A_1 E$, are equal; and so are those through $a_2 e$, and $A_2 E$, and again through $a_3 e$, and $A_3 E$. But when any point of one of these meridians coincides with the meridian of the place, the whole meridian coincides with it; or every point of a meridian is on the meridian of the place at the same time; and the periods of rotation through corresponding arcs being equal, every point of a meridian, when not on the meridian of the place, requires the same time to arrive at it; and that time bears the same proportion to a sidereal day that the arc of the equator, intercepted between the meridian of the place and the meridian in question, bears to the whole circumference of the equator. And conversely, if we observe the time when one star is on the meridian of the place, and that when another body appears there on the same side of the pole (for this qualification is necessary in the case of circumpolar stars which appear on the meridian of the place both above and below the pole), and note their interval, we know the interval of the equator intercepted between the two meridians; for it is the same proportion of the whole equator which the observed interval of time is of the whole sidereal day. It is of course convenient to have some fixed point of the equator as a standard of reference, and that point, which we shall presently see the reason of choosing, is called the first point of Aries, and is often marked thus ♈. The distance, measured Eastward, from ♈ to the intersection with the equator of the meridian passing through any heavenly body, is called *the right ascension* of the body, and is obviously the same for every body upon the same meridian, and upon the same side of the pole, and for none off it. The right ascension may evidently be estimated in time, as it is ascertained by it. The period of a whole revolution, or of passing through 360°, is twenty-four hours of sidereal time: in one such hour therefore a body passes through 15° of right ascension; in four minutes through 1°; and these right ascensions may be equally well called either 1° and 15°, or four minutes, and one hour. In other words, an astronomical clock marks 0^h 0^m when the first point of Aries is on the meridian; when it marks 4^m the bodies then on the meridian are said to have 1° or 4^m of right ascension: when it marks 1^h, the bodies on the meridian are said to have 15° or 1^h of right ascension.

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* Any circle formed by the intersection with the sphere of a plane passing through the centre of the sphere, is called a *great circle* of the sphere. If a diameter of the sphere be drawn perpendicular to this plane, the extremities of this diameter are called the *poles* of the great circle; and if any number of great circles be drawn through these poles, they will each be in planes perpendicular to the first circle, and are called *secondaries* to that great circle. Being in planes perpendicular to it, they are so, of course, to all other circles parallel to it. Thus, taking the circles whose names are given in the text, the north and south poles of the heavens are the *poles* of the equator, and every meridian is a *secondary* to the equator, and perpendicular to it, and to every circle parallel to it: or every meridian is perpendicular to every circle of daily rotation, for all such circles are parallel, and the equator is one of them.

It may also be convenient here to mention that all great circles in the same sphere are equal, and that they all bisect each other.

On all matters relating to the properties of the sphere, see Treatise on Geom. Book VI.

By the observation of time therefore we can ascertain the right ascension of a heavenly body, and consequently the meridian on which it is. If we can also observe its distance from the pole, or its distance from the equator, which is called *its declination,* and is called North or South declination as the body is North or South of the equator, we ascertain its position upon that meridian. The declination being measured along a meridian, the meridians have also the name of *circles of declination.* Every point in a small circle parallel to the equator, is of course at the same distance from the equator, or, in other words, has the same declination: such a small circle is therefore called a *parallel of declination.*

The observation of declination is easily made. The horizon of any given place is a fixed circle: the point therefore where a perpendicular to the horizon meets the heavens above the spectator's head (which is called the *zenith**) is a fixed point also. This point may always be ascertained, for a plumb line hangs directly in a line from it. The zenith is obviously a point in the meridian; for the plane of the meridian passes through the place of observation perpendicular to the horizon, and the perpendicular to the horizon at the place must be a line in that plane.

The zenith is the pole of the horizon, and if any great circles be drawn through the zenith, they will be secondaries to the horizon. These are called *vertical circles:* the meridian of the place is of course one of them. The arch of a vertical circle intercepted between a heavenly body and the horizon, is called the *altitude* of the body, and can always be measured by proper instruments †.

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* The opposite point, where the perpendicular produced below the horizon meets the opposite hemisphere of the heavens, is called the *nadir.* These, like many others of the terms of astronomy, are derived from the Arabian observers.

† The observer being placed apparently in the centre of the sphere, the angle which two objects in the heavens subtend at his eye will accurately measure the arc of the great circle of the heavens which lies between them. This arc therefore, or their distance, is accurately measured by observation of the angle between them; or rather, as we know nothing of their actual distances, we can only observe the angles, which give us the relative *directions* of the bodies; and then we suppose

The distance from the body to the zenith, or its *zenith distance*, together with the altitude, make up the whole distance of the zenith from the horizon, or 90°. The zenith distance, therefore, is the *complement* of the altitude.

The pole is a fixed point in the meridian; it is therefore at a fixed distance from the zenith of any given place, and from the North and South points of the horizon there. If therefore, when any heavenly body is on the meridian, its distance from either the zenith or the North or South point of the horizon can be observed, its distance from the pole, or its *North Polar distance*, can be ascertained from it. And it is obvious that if an instrument be adjusted so as to move in the plane of the meridian, these distances may be ascertained by mere observations of altitude made with it: for the altitude of a body when on the meridian, is its distance either from the North or South point of the horizon; and the complement of the altitude is the zenith distance. The adjustment of the instrument to move in the plane of the meridian, is more complicated than that required merely to make it move in a vertical circle: it is however generally made with the best instruments, which are fixed so as to move in the plane of the meridian, and in that only. Having an instrument so adjusted, the north polar distances of all the stars may be ascertained.

Let S_1, S_2, S_3, S_4, in *fig.* 2, represent any heavenly bodies on the meridian: S_1 E, S_2 E, S_3 E, S_4 Q, are obviously their declinations. Now, observing that PE, or PQ = 90°, it is obvious that whenever the North polar distance, as PS_2, PS_3, PS_4, is less than 90°, the declination S_2E, S_3E, S_4Q = 90° − North polar distance; and in these cases the declination is North. When the North polar distance, as PS_1, is greater than 90°; the declination S_1E = North polar distance − 90°. In each case therefore the declination is the difference between the North polar distance and 90°, being North when the North polar distance is less, and South, when it is greater, than 90°.

We can then always ascertain the declination of a body by observing its altitude when on the meridian of the place. Its right ascension may be ascertained in the manner already pointed out: or, if the right ascension of any particular star has been previously accurately ascertained by a sufficient number of observations, then the right ascension of any other star may be found by observing the time which elapses between its appearance on the meridian and that of the star, whose place we suppose to be already determined. The observation of right ascension gives us the meridian on which the body is situated in the heavens, and that of declination the precise point of that meridian, and the two combined give us the exact place of the body: for whenever we can ascertain the distance of a body in a known surface from a given point, measured in two directions perpendicular to each other (as in this case along the equator, and along a circle of declination), or, indeed, inclined at any given angle to each other, there is only one point which can fulfil both conditions, and that point therefore is completely ascertained.

In this manner we can determine the place of the sun on every day in the year; and the observation of all these points will give us so many of his successive positions in the heavens, and his path among the stars must, therefore, be a line passing through all these points. This line is found, when the observations are carefully made and registered, to be a great circle of the sphere, which intersects the equator at an angle of 23° 28′ nearly, and is called *the ecliptic**. All great circles of the sphere

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them ranged in the surface of a sphere, for the convenience of computation, by applying to them the principles of spherical geometry.

The altitude of a body is easily ascertained. The instrument with which the observation is made can be mechanically adjusted, so as to move in a vertical plane, and, consequently, in the plane of a vertical circle. The place of the horizon may be ascertained, though some nicety is required in the ascertainment, into the details of which it is not necessary here to enter. All instruments intended for observation have a graduated circular limb attached to them, and when the plane of the horizon is ascertained, the points on the limb which are in that plane may be found; and the angle between the diameter joining them and the direction of the instrument, which obviously measures the altitude of the body, observed by means of the graduation on the limb. There are additional and more delicate adjustments, which need not be detailed here.

* The ecliptic being a great circle of the sphere, has, of course, two poles, which are distant from the poles of the equator by an arc of 23° 28′, equal to the inclination of the one circle to the other. If from the pole of the ecliptic a secondary be drawn through any heavenly body to the ecliptic, the arc of this circle between the body and the ecliptic is called the *latitude* of the body, and the arc of the ecliptic intercepted between the first point of Aries, and the intersection of the secondary with the ecliptic, is called the *longitude* of the body. The place of a body is of course as

intersect and bisect each other; and one of the points where the ecliptic intersects the equator is called *the first point of Aries* (the point from which we have said that the right ascension is measured); the other, *the first point of Libra.* For reasons which will afterwards be seen, the former of these is also called the *vernal equinox*, the latter the *autumnal equinox*. The first point of Libra is often marked thus, ♎.

Section 4.—*Variation of Sun's apparent diameter — Sun's orbit an Ellipse, the earth being in the focus —Variation of angular velocity — Equable description of areas by Sun's radius vector.*

We have already stated, that if any plane pass through the centre of the sphere, its intersection with the sphere is a great circle; and that the ecliptic, or sun's apparent path, is such a circle. As yet we know nothing of the distance of the sun from the earth; but it is clear that we have no reason to suppose his distance equal to that of the stars, or always the same: if we have no means of comparing his distance with that of other objects, we can only ascertain the direction in which he appears. Thus the sun, the moon, the stars all appear alike situated in the concave surface of the heavens, although we shall hereafter see that their distances are almost immeasurably different; and any one who has seen a bright meteor at night, such as those commonly spoken of as shooting, or falling stars, knows that they also appear to the eye to be similarly distant. The very name indeed of shooting *stars* is evidently deduced from this circumstance. In their case indeed we know them really to be meteors engendered in the atmosphere, and therefore know their distance to be comparatively small, or we may sometimes ascertain it to be so by comparing observations made at different places; and in the case of the moon, we may perhaps imagine that she looks less distant than the sun or stars: but this notion, when it obtains, is probably really owing to our habitual knowledge that she is so, or to our being able to distinguish different parts of her surface from each other, by the different degrees of their brightness. If however we abstract ourselves from such considerations, we shall find ourselves alike unable to estimate by the eye the distances of any of these objects, and consequently unable to say which are furthest from us. All therefore that we can conclude, from finding the apparent path of the sun to be a great circle of the heavens, is, that his motion is in a plane passing through the earth; but what the shape of his circuit is, whether his distance be always the same, or be variable, and whether, if it vary, it vary gradually and regularly, or suddenly and without any fixed rule, we as yet are quite ignorant. We know only that we continually see him more Eastward at every successive observation, and consequently conclude that his motion is always in that direction, and not occasionally East, and occasionally West.

To determine the form of his orbit, we need only know the proportions of his distances from the earth at different times. For this purpose the actual distances are unimportant; for instance, if his distance is always the same, his orbit is a circle, whatever be that distance, which is the radius of the circle. Now the apparent diameter of an object, or the angle which it subtends at the eye of the observer, increases in the same proportion as the distance diminishes, so long as the real diameter continues unchanged, and the apparent diameter is very small:[*] or in such cases, *the apparent diameter varies inversely as the distance.* Now the apparent diameter of the sun may be very accurately measured, and it is found to be continually varying: his distance therefore is so also, for we have no reason whatever to suppose that his actual magnitude changes; and the proportions of his apparent diameters at different times being ascertained by observation, the proportions of his cor-

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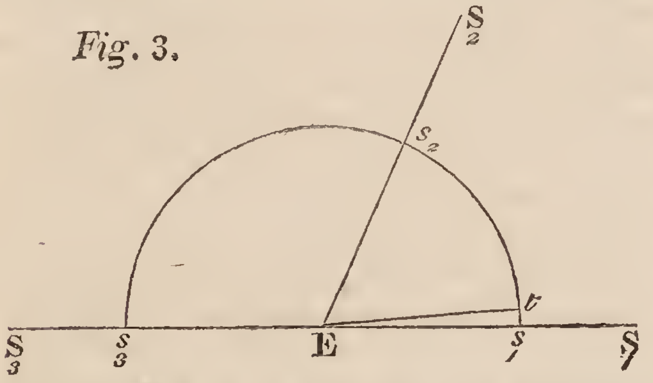
well ascertained by knowing its longitude and latitude, as by knowing its right ascension and declination; but the latter are the best adapted for observation, and the former may always be deduced from them. The terms *longitude* and *latitude*, as applied to a heavenly body, are established; but it is to be regretted that they have been chosen, as their sense in that application differs widely from their meaning with respect to places on the earth, being, indeed, analogous measures in the two cases, but referred to different circles and planes.

[*] [In strictness, the *tangent* of the apparent radius (i. e., of the angle subtended by the radius) = real radius divided by the distance: and the real radius being always the same in the same body, the *tangent* of the apparent radius varies inversely as the distance. Whenever, therefore, the apparent radius is so small, that the arc and its tangent may be treated as equal, the apparent radius varies inversely as the distance: and so of course does the apparent diameter, which is double the apparent radius.]

responding distances may be so also. If then we conceive lines drawn from the earth to every point of his orbit, the observation of his right ascension and declination will always determine on which of these lines he is situated; and if we take portions of these lines always in the inverse proportion to each other of his apparent diameters, the curve joining the extremities of these portions will be similar to his orbit. This perhaps may be rendered clearer by a figure, and an instance. Thus let us suppose the sun to be observed on any day, and that his apparent diameter is $31\frac{1}{2}'$; again, and that his place in the ecliptic is then 70° from that of the first observation, and his apparent diameter then of $32'$; and a third time, and that his place in the ecliptic is then 180° from that of the first observation (or exactly opposite to it), and his apparent diameter then of $32\frac{1}{2}'$. Take any line E S_1 to represent the direction of the sun at the first observation; and produce it, in the opposite direction, to S_3: then E S_3 will represent the direction of the sun at the third observation; and if E S_2 be drawn making an angle of 70° with E S_1 it will represent the direction of the sun at the second observation. Now the apparent diameter of the sun at the first observation is $31\frac{1}{2}'$, at the second it is $32'$; his corresponding distances, therefore, are in the proportion of 32 to $31\frac{1}{2}$, or of 64 to 63. If, therefore, we take in the line E S, a portion E s, of any magnitude we choose, and measure upon E S_2 a portion E s_2 which shall be to E s_2 in the proportion of 63 to 64, the distances E s_1, E s_2 will be proportionate to the distances of the sun at those two observations. In the same manner, the apparent diameter of the sun at the third observation being of $32\frac{1}{2}'$, his distance at that observation is to his distance at the first observation in the proportion of $31\frac{1}{2}$ to $32\frac{1}{2}$, or of 63 to 65, and it is to his distance at the second observation in the proportion of 32 to $32\frac{1}{2}$, or of 64 to 65. If, therefore, we take in the line E S_3, a portion E s_3, which shall be to E s in the proportion of 63 to 65, or to E s_2 in the proportion of 64 to 65, the distance E s_3 will be proportionate to the distance of the sun at the third observation. All the distances, therefore, E s_1, E s_2, E s_3, will be proportionate to the distances of the sun at the different observations; and the points s_1, s_2, s_3, will accurately represent his situations at those times. In the same manner his situations at other and intermediate times may be represented, and the line joining all these points (the curve $s_1\ s_2\ s^3$) will represent his orbit. It is found, by careful observation, that this line is an ellipse, of which the earth is in the focus*.

Fig. 3.



The ellipse is an oval curve, divided into two equal and similar parts by a line drawn through its foci, which is called its *major axis*, or *transverse axis*. If from one of the foci lines be drawn to every point in the curve, the greatest of these lines is that portion of the major axis which is drawn through the other focus to meet the curve, and the least is the other portion of the major axis; and the lines continually diminish from the greatest to the least, and continually increase from the least to the greatest. It follows therefore, if the sun moves in an ellipse round the earth in the focus, that his apparent diameter should continually increase from its least amount, which is when the sun is at his greatest distance, or in his *apogee* (from two Greek words signifying *away from the earth*) to its greatest amount, which is when the sun is at his least distance, or in his *perigee* (from two Greek words signifying *about* or *near the earth*); and as the two portions of the ellipse are equal and similar, the diminution of his distance in the one case will correspond with the increase of it in the other, and at the end of a complete revolution he will again be at his original distance. And so we actually find it. The least apparent diameter of the sun is of $31\frac{1}{2}'$ nearly, and is at present observed about the 30th of June; he is therefore then in his apogee: his apparent diameter then continually increases until (about the 30th of December) he arrives at his perigee, when his apparent diameter is of 32′ 35″ nearly: and then again it continually diminishes until (about the 30th of June) it is again of $31\frac{1}{2}'$, from which value it again begins to increase. The variations of the apparent diameter being so small, those of the distance are so also, or the ellipse differs but little from a circle.

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* The ellipse is one of the curves called *conic sections*. It is the curve made by cutting a cone by a plane, which passes through it without intersecting the base. Its fundamental property is this: there are two points within it, called the *foci*, such that the sum of the distances of any point in the curve from the two foci is always the same. This property furnishes an easy mode of drawing the curve. If a thread be fixed at the two foci, and a pencil carried round within the thread, so as always to keep it stretched, the pencil will describe an ellipse; for the whole length of the thread is always the same, and it constitutes the two distances from the foci.

Having thus ascertained that the sun appears to move in an ellipse round the earth, we next inquire what is the rule of his motion, whether it be regular or irregular, uniform or variable. We have already seen that we can, by ascertaining the right ascension and declination of a body, determine its place in the heavens, or the direction in which it is seen from the earth: we may therefore ascertain at each particular instant the point of the ecliptic in which the sun is, and, consequently, the arc through which he has moved in each particular interval, or the angles which the directions in which he is successively seen make with each other. We have thus the means of ascertaining his angular velocity, as referred to the earth. Now this angular velocity is found to be variable, and it is also found to be greatest when the apparent diameter is greatest or the distance least, and least when the apparent diameter is least or the distance greatest; and generally, the greater the distance the less we find the angular velocity. Its inequalities however are greater than those of the apparent diameter or distance. We have already seen that the least apparent diameter is to the greatest nearly in the proportion of 30 to 31; but the proportion of the least angular velocity to the greatest is nearly that of 30 to 32, and hardly so great. On accurate comparison of the different angular velocities with the distances, it is found that they vary *inversely as the squares of the distances*, that is, that they diminish in the same proportion as the squares of the distances increase, and increase in the same proportion as the squares of the distances diminish. For instance, if the distance is doubled, the angular velocity is reduced to a quarter of its former amount; if the distance is diminished by a third, the angular velocity is increased in the proportion of 1 to the square of two-thirds, or of 9 to 4.

These velocities however, and the distances themselves, may be considered for very short periods of time as constant, for the changes of distance in such periods are so small that they may be neglected. The whole variation of distance is only about a thirtieth part of the least distance; the greatest difference between the angular velocities only about a fifteenth part of the least angular velocity; and these differences are the accumulated differences of months: for an hour therefore, or a day, there will be no perceptible difference, and none which can at all affect the results which we shall deduce from the supposition that, during such a period, they are uniform.

In *fig.* 3, let t be the place of the sun in its orbit, when one of these very short periods has elapsed since it was at s_1. If we conceive a straight line drawn from the earth to the sun, and moving round the earth with the sun, as for instance, if they were joined by a wire, which we must suppose to be lengthened and shortened as the sun recedes from and approaches the earth, so as always to extend from the centre of the one to the centre of the other, and no farther, this line will originally have coincided with E s_1, and its position, when the sun is at t, will be E t. While the sun has moved from s_1 to t_1, therefore, or described the arc s_1 t_1 this line or wire will have described the small area E s_1 t. This line is called the *radius vector*, and it is of great importance to ascertain the areas which it describes, and these we shall find to be always equal in equal portions of time.

[This area may be considered as a triangle, for the arc s_1 t, being very small, differs insensibly from the straight line joining the points s_1 t; and the area of a triangle is equal to half the product of its base, and the perpendicular from its vertex. Again, the angle t E s_1 being very small, the perpendicular from the vertex t is indistinguishable from a small circular arc, whose centre is E, and radius E t, or E s_1; and these also may be considered as equal. The magnitude of such an arc varies as the product of the angle t E s_1, and the radius E t, or E s_1. The area t E s_1 therefore (which is equal to half the product of its base and the perpendicular from its vertex) varies as the distance E s_1 multiplied by the product of the angle t E s_1 and the radius E s_1 (for its base is the distance E s_1, and the perpendicular we have already seen to vary as the product of the two latter quantities); or it varies as the square of the distance $E s_1$, multiplied into the angle $t E s_1$. If, however, the area $E s_1 t$ be supposed to be that described in some given portion of time, as an hour or a minute, and the angular velocity be uniform during the time of describing it, the angle $t E s_1$, varies as the angular velocity; for it will increase or diminish exactly in the same proportion as that velocity increases or diminishes. The area $t E s_1$ therefore, described in such a period of time, varies as the square of the distance $E s_1$, multiplied by the angular velocity. But we have seen that the angular velocity varies inversely as the square of the distance: the area $t E s_1$ therefore is not affected by the distance at all, or it is constant; for in whatever proportion it is increased by reason of the increase of the distance $E s_1$, in the same proportion it is diminished by the diminution of the angle $t E s_1$.]

If however the areas described by the radius vector in small equal portions of time be equal in every part of the orbit, the whole areas described by the radius vector in equal portions of time of any magnitude will also be equal; for they will be the sums of equal numbers of these small equal portions: and generally, *the areas described by the radius vector in any times whatever will be proportional to those times.* This therefore is ascertained to be a law of the motion of the sun round the earth, and it is one which we shall hereafter find of the very greatest importance.

We have now ascertained these facts with respect to the sun: that its annual motion round the earth takes place in an ellipse, of which the earth is in one of the foci; that the plane in which he moves is inclined at an angle of 23° 28′ to the plane of the equator; and that his radius vector describes areas proportional to the time. These are the principal phenomena of his motions: they require however some qualifications, which we shall hereafter mention; and we shall also see that we may explain his apparent motions on the supposition that he is at rest, and the earth moves round him, just as we have said that we shall be able to explain the apparent diurnal motions of the stars, by supposing them at rest, and the earth spinning on an axis.

We may however at once proceed to explain how these facts produce some of the most important phenomena which we observe, especially the difference of the seasons in the same country, the different lengths of the day (as distinguished from the night) at different periods of the year, and the different climates of different countries; and also the equation of time, or the manner and degree in which the real differs from the mean solar time.

SECTION 5. — *Comparative length of time during which bodies at different Declinations continue above the horizon—Variation and length of Day at the same place—Tropics—Equinoxes — Seasons produced by the Sun.*

WE have already seen that the apparent diurnal motions of the heavenly bodies are performed in circles, every point of which is equidistant from the pole: of these circles the equator is one, and is a great circle of the sphere; the others are all small circles parallel to the equator. The horizon, being a great circle as well as the equator, bisects that circle. It will immediately appear, also, from inspection of any of the figures, as *fig.* 1, or *fig.* 2, that of the parallel circles a larger portion is continually above the horizon, and a smaller continually below, as they approach nearer that pole which is above the horizon. Thus, in *fig.* 1, $P q$ being less than $P Q$, $t q r$ is a larger portion of the circle $t q r b$ than $T Q R$ is of $T Q R B$: and the portions continually go on increasing, until, finally, the whole circle is above the horizon, as in the case of $e a_1 a_2 a_3 q$ in *fig.* 2, or of $X V Y U$ in *fig.* 1, when the distance, $P q$ or $P U$, of the body from the pole is less than the elevation of the pole above the horizon. In the same manner, if a body move in a circle distant from the other pole, p, less than its depression below the horizon, or than $H p$, it will never appear above the horizon at all. We have also seen that the diurnal motions of the heavenly bodies are equable; that is, that the times of passing through every successive portion of their circles of motion bear the same proportion to the whole time of rotation, that the portion described bears to the whole circle; and that the times of rotation in every circle are the same. The larger, therefore, the portion of each circle of rotation which is above the horizon, or the nearer that circle approaches to the pole which is above the horizon, the longer the body is above it. If then the North Pole be above the horizon, as it is here, those bodies which have the least South, or the greatest North declination, are longest above the horizon: and in like manner, where the South Pole is above the horizon, the most Southerly bodies are longest visible. The equator is in all cases bisected by the horizon; in every place therefore a body in the equator is for an equal length of time above and below the horizon; and it is from this property that the circle itself derives its name.

These positions, which are deduced from the supposition that the diurnal motions take place in parallel circles, and uniformly, are not strictly and accurately true for the sun, whose place on each succeeding day is either North or South of that of the day before, and whose diurnal motion, being compounded of the diurnal rotation of the whole heavens and his own motion in his orbit, cannot be strictly uniform, because his motion in his orbit is not so. Neither of these inequalities however need here be taken into consideration, so small is the proportion which they bear to the general diurnal motion which they affect. The heavens complete their rotation, or move through 360°, in a day. In that time the sun's motion Northwards or Southwards, or his motion in declination, is never so much as 24′, and seldom near it: a quantity so small that we cannot err in considering, for purposes of general explanation, the diurnal motion of the sun to take place in a circle parallel to the equator, and distant from it by the sun's distance from it when he is on the meridian. The other inequality arising from the inequality of the sun's motion is still less perceptible: the whole motion of the sun in his orbit is less than 1° in a day on an average, and very little more when greatest; the whole difference between the greatest and least motions is only about a fifteenth part of this, or about 4′, a quantity which, for all purposes of explanation, though not of computation, may safely be neglected in comparison with 360°. Even this, which occasions some difference of length in days taken at periods thus distant from each other, is the accumulated difference of half a year; the difference in a single day will be quite imperceptible, and the motion during one day may therefore be considered as perfectly uniform. Thus we may with safety apply to the sun our conclusions with respect to the length of time for which a body is above the horizon*, and say that, where the North Pole is above the horizon, he is longer above the horizon as he comes farther North, and less so as he returns Southward; or generally, as his being above or below the horizon constitutes the distinction between day and night, that the days are longer as he is more to the North, and shorter as he is more to the South. Where the SouthPole is above the horizon, the days are of course longer when he is more to the South, and shorter as he moves Northward. Whatever conclusions, indeed, are true with respect to the parts of the earth where the North Pole is above the horizon, or of which the latitude is North, are true also for those where the South Pole is so, or of which the latitude is South, if we change the positions of the words, and attribute the same results to South declination, &c., which, in the other case, we ascribe to North; and we shall not in future express both, but suppose the North Pole above the horizon, and deduce our results on that supposition. The reader will find no difficulty in transferring them to the other case.

We have then the sun moving in an orbit, of which the most Southern point is 23° 28′ South of the equator, and the most Northern 23°28′ North of it. Taking his motion from the first of these points, where he is about the 21st of December, he continually travels Northward till about the 20th of March, when he is in the equator, and still Northward till about the 21st of June, when he attains his greatest Northern declination; he then returns towards the South, passes the equator again about the 22nd of September, and returns to his greatest South declination about the 21st of the following December, when he again returns Northward. His successive places, from December the 21st to June the 21st, being continually more and more Northward, his time of continuing above the horizon, or the day, continually lengthens; and in the same manner it continually shortens from that time till the 21st of December. The longest day is the 21st of June, the summer solstice, or the day on which he has his greatest Northern declination; the shortest, the 21st of December, the winter solstice, or the day on which he has the greatest Southern declination; and the circles which on those days he describes are called *the tropics*, from a Greek word signifying *to turn*, because, although still proceeding in the same direction in his orbit, he has reached the extreme North or South point of it, and, therefore, as it were, turns back, and travels in an opposite direction (as to North and South) to his former course. At the two periods when the sun is in the equator, or when he is at the first point of Aries, and the first point of Libra, the parts of his daily course above and below the horizon are equal, or the day and night are equal; these therefore are called *equinoxes*, and taking place in spring and autumn, they are called the *vernal* and the *autumnal* equinox, and the equator itself is sometimes called *the equinoctial* on the same account. The rate of the sun's motion Northward or Southward is most rapid at the time that his orbit intersects the equator, and is hardly perceptible just when he is at his greatest distance from that circle*: the variation of the length of the day is therefore most rapid about the equinoxes, and least so about the solstices. The rate of variation is also affected by the rate of his motion in his orbit; for as his motion in declination is produced by that, the greater his motion in his orbit is, the greater, all other things continuing the same, is his motion in declination. But his motion in his orbit is greatest when he is in and near his perigee, and least when in his apogee; and as he is in his apogee nearly at the summer solstice, and in his perigee nearly in the winter solstice, his motion is slower in summer than in winter, and the days vary still more slowly in length at the summer than at the winter solstice. For the same reason, as the two portions of the ellipse from perigee to apogee, and from apogee to perigee are equal and similar, the sun will take longer in describing those 180°, or that half of his orbit in the course of which he is in apogee, than that half during which he is in perigee: and as he is in apogee between the vernal and autumnal equinox, and in perigee between the autumnal and the vernal, the former interval must be longer than the latter, or the summer in these parts of the world longer than the winter. In fact, the difference at present is of 7 days, 16 hours, and 50 minutes.

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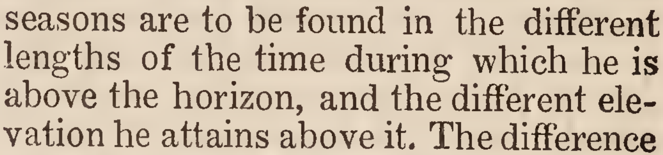
* The sun also being thus considered to move on any given day in a circle parallel to the equator, will have the part of that circle which is above the horizon bisected, like any other heavenly body, by the meridian; or he will be an equal time above the horizon on each side of the meridian. He will, therefore, be on the meridian at mid-day; and this is the origin of the word meridian, which is derived from *meridies*, a Latin word signifying mid-day. The primary application of the word therefore was to the meridian of the place, the line on which the sun appears at noon; the general application to all great circles passing through the two poles is only secondary, and derived from the circumstance, that each of these must correspond at some time with the meridian of the place.

Having thus seen how the motion of the sun produces the various lengths of the day, it is easy to perceive how it also produces the different seasons of the year. Light and heat being essential to all the great operations of nature, and the sun being the great source of light and heat, his position and the length of time he is above the horizon must be the great causes of the variations of season. The variation of his distance would of course tend to make his influence greatest when he is nearest, and this would seem contrary to observation, as he is nearest in the winter. In fact however other circumstances are much more powerful in increasing and modifying his effect than this, which could not in its extreme variations produce a difference of more than about one-fifteenth*. The main causes of the different warmth of the seasons are to be found in the different lengths of the time during which he is above the horizon, and the different elevation he attains above it. The difference of the length of the day evidently tends to increase the temperature by giving the sun a longer time to act; and as with us the longest day is more than twice the length of the shortest, this cause by itself is far more than enough to counterbalance the effect of the increased distance of the sun. The other cause, the difference of elevation, requires a little more explanation. In the first place, the greater the elevation of the sun, the more directly do his rays strike upon the earth, and the greater proportion of them does a given surface receive. Thus, in *fig.* 4 let the parallel

Fig. 4.



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* We have already stated this as a fact ascertained by observation: it will be hereafter (in the section on the equation of time) shewn to be a necessary consequence of the inclination of the ecliptic to the equator.

* All other things continuing the same, the intensity of light and heat diminishes in the same proportion that the square of the distance from their source increases. In the case of the sun and earth, the extreme difference is about one-fifteenth.

and equidistant lines represent rays of light proceeding from the sun, and A B an object of any given magnitude exposed to them in a direction perpendicular to them, so as to intercept them all. Now, if A B be gradually turned round the point A, so as to form an angle continually more and more acute with the direction of the rays, as at A C, A D, A E, A F, or continually more obtuse as at A *c*, A *d*, A *e*, A *f*, it is evident, from inspection of the figure, that it will in each successive position receive fewer of them; and this will be the case however close the rays be to each other, if they are uniformly so; for, however we may multiply the number of rays which pass between A F and A G, or which fall on the object in the position A F, the number which it ceases to receive by passing from the position A B to A F, or the number of those above F in the figure, is increased in the same proportion. Besides this, we know that both light and heat are absorbed in passing through the atmosphere; and as the height of the atmosphere above the surface of the earth is the same, or nearly so, in all places, the course of a ray through the atmosphere is shortest when it falls most directly upon it, and longer as its deviation from the perpendicular increases*. On this account also, the greater the elevation of the sun, the greater is the light and heat which he affords: and we accordingly observe, that the mid-day sun does actually give more light and heat than that of the morning or afternoon, and the summer sun than that of winter.

It is true that the greatest heat of the day is generally after noon, and the greatest heat of the year after Midsummer; but these facts are not inconsistent with the principles we have stated. There is a continual tendency in nature to equalize the heat of different bodies; those more heated parting with their heat to the cooler. This process is continually going on; but, as long as the sun is above the horizon, and furnishing new supplies of heat, the heat will increase if he supplies it faster than by the ordinary processes of cooling it is diminished; and it will diminish as soon as his supplies are less than the ordinary consumption. For a considerable period after noon he generally supplies heat faster than it is distributed, and consequently, although the actual supplies diminish, the whole stock increases: just as if you pour water rapidly into a vessel with a hole at the bottom, the water will rise in the vessel as long as you pour it in more rapidly than it runs out, although you may supply it less copiously after a time than you did at first; or, as the mercury in a thermometer brought from a frosty air, and placed near a fire, will continue to rise in the room after it is removed from the fire, if it has not been kept there so long as to render its temperature equal to that of the atmosphere of the room. The same reason also explains the greater heat of July and August than of June. As long as the surface of the earth and the atmosphere in the course of twenty-four hours receive more heat than they part with, the general heat of the weather increases; and this is the case long after the summer solstice, though the actual quantity of heat then received is greatest. The heat of particular days is principally influenced by other local and accidental causes; but the general difference of the seasons depends on these principles, and on the different degrees of moisture and cloud in the air, and the different quantity of evaporation and radiation which consequently takes place at different times. Any theory of the seasons resting merely on the sun's position, without reference to these other causes, would be imperfect and fallacious; but, as far as the sun's position affects them, they are thus to be explained.

SECTION 6. — *Differences as to the length of Day and the Seasons at different places—Days at the Equinoxes everywhere of the same length—Longest Days greater, and shortest Days less, as the latitude increases—If latitude above 66° 32′, there are some periods of twenty-four hours during which the Sun never sets, others, during which he never rises; and these are longer as the latitude is greater—At the latitude 90°, there is only one Day, and one Night—At the latitude 0°, the Days and Nights are all equal—Length of Day increased by the Sun's magnitude, and by refraction—diminished by parallax—Effect of the Sun on climate at different places.*

WE have thus far spoken of the difference of days and seasons at a particular place; we have still to shew how different places are affected, to explain the diversity of climates, and the exceptions which particular parts of the earth (the circumpolar and intertropical regions) offer to the general account which we have given.

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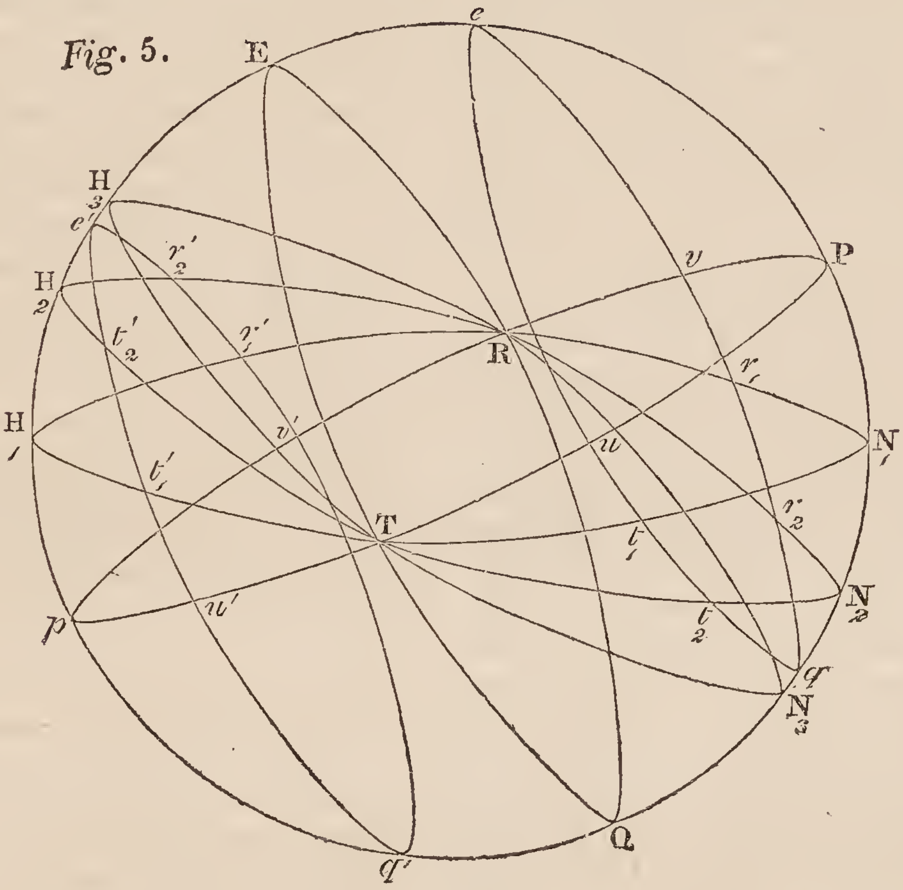
* See this more fully explained in a note in the section on Refraction.

For this purpose, let us suppose several places, at each of which the sun, or any other heavenly body, or any point of the heavens, comes upon the meridian at the same time, or where the meridian of the place is the same great circle of the heavens. This supposition is only made for the sake of simplifying a necessarily complicated figure; for every conclusion which is true for a place having this meridian, will be equally so for a place having any other, if the elevation of the sun when on the meridian is the same at each. We have already seen, that at every place half the equator is always above, and half below the horizon; the equator and horizon being both of them great circles. Now as half the equator is above the horizon at each place, and the point of the equator on the meridian is the middle point of that half, and that point, on the supposition we have made, is the same at each place, the points of the equator which are upon the horizon must be the same at each place; or, in other words, the horizons of different places under the same meridian always intersect the equator at the same points. If, therefore, in *fig*. 5, we take the circle

Fig. 5.



E R Q T to represent the equator, and H_1 R N_1 T to represent the horizon of a particular place, R and T, being the points where the equator and this horizon intersect each other, will also be the points where the equator and the horizon of any other place which has the same meridian intersect each other; and consequently, if through the points R and T, we draw other great circles H_2 R N_2 T, H_3 R N_3 T, these will represent the horizons of other places under the same meridian. If P be the pole, the elevation of the pole at these different situations will be P N_1, P N_2, P N_3. Now, let *e q*, *e' q'* be two small circles parallel to the equator E Q, the one North of it, the other South of it; they therefore are circles of diurnal rotation: and let the points where they respectively cut the different horizons, be r_1, t_1, r_2, t_2, r_1', t_1', r_2', t_2'; and we shall immediately see, by inspection of the figure, how the elevation of the pole, or the latitude of the place, affects the time during which any heavenly body is above the horizon, and consequently the length of the day at different periods of the year.

The equator is bisected by all the horizons; and of course a body moving in it is everywhere half its time above, and half below the horizon; when the sun therefore is in the equator, day and night are everywhere equal, so that those periods are called the equinoxes, not from any accidental equality affecting a particular place of observation only, but from an universal fact.

Let us next take the case of a circle

of rotation North of the equator, as $e\,q$, and let us conceive a great circle described through the points R and T, and passing also through the poles P and p. By a known property of the sphere, this secondary to the equator E Q bisects it, and every circle parallel to it. If, therefore, u and v be the points where it cuts the circle $e\,q$, $u\,e\,v$ is half that circle. Now the part of the circle $e\,q$ which is above the horizon H N_1, is $t_1\,e\,r_1$, which is evidently greater than $u\,e\,v$, or more than half the circle of rotation $e\,q$ is above the horizon; and the part of the circle $e\,q$ which is above the horizon H_2 N_2 is $t_2\,e\,r_2$, which is evidently greater not only than $u\,e\,v$, but also than $t_1\,e\,r_1$. But the elevation of the pole above the horizon H_1 N_1, is P N_1, its elevation above the horizon H_2 N_2 is PN_2; the greater portion, therefore, of the circle $e\,q$ is above that horizon above which the pole is most elevated, or the horizon of that place which has the greater latitude. Now $e\,q$ being a circle of rotation North of the equator, may represent the sun's line of diurnal motion when his declination is North; and we consequently deduce the following general rule, that *wherever the North Pole is above the horizon, and the sun's declination is North, the day is longer than the night; and that in different places, the day is longer where the North Pole is more elevated above the horizon, and shorter where it is less so.*

In the same manner, taking the case of a circle of rotation South of the equator, as e' q', the circle P R p T divides it also into equal parts u' e' v', u' q' v', and t' e' r', the part above the horizon H_1 N_1 is evidently less than u' e' v'; and $t_2{}'$ e' $r_2{}'$, the part above the horizon H_2 N_2, is evidently still less than t' e' r'. *Whenever the North Pole, therefore, is above the horizon, and the sun's declination is South, the day is shorter than the night; and in different places, the day is shorter where the North Pole is more elevated above the horizon, and longer when it is less so.*

As the days both lengthen and shorten more and more as the elevation of the pole increases, of course the inequality of their lengths increases. Thus at London, where the elevation of the pole is of 51° 31′, the longest day is of 16^h 34^m; the shortest of 7^h 44^m, and the difference of 8^h 50^m. At Paris, where the pole is only elevated 48° 50′, the longest day is of 16^h 7^m; the shortest of 8^h 11^m, and the difference of 7^h 56^m. And at Edinburgh, where the elevation is of 55° 57′, the longest day is of 17^h 25^m; the shortest of 6^h 53^m, and the difference of 10^h 32^m.

We have, in our figure, a third horizon, H_3 N_3, of which we have yet made no use, but which will furnish us with some very important observations. It will be seen from the figure, that neither the circle $e\,q$, nor the circle e' q', ever meet this horizon at all; the circle $e\,q$ being entirely above it, the circle e' q' entirely below it. A body moving in the former circle is then always above, in the latter always below, this horizon. The elevation of the pole P is greater above the horizon H_3 N_3, than above either H_1 N_1 or H_2 N_2: as this elevation increases therefore, some circles of rotation become entirely above the horizon, which were not so before; and others disappear entirely below it. It is easy to ascertain when this takes place for any particular circle. Taking the case of the circle $e\,q$, the point most elevated above the horizon is e, where it cuts the meridian on one side of the pole; the point least elevated above, or most depressed below, the horizon is q, where it cuts the meridian on the other side of the pole. Whenever therefore q is above the horizon, the whole circle is so. Now, the zenith is 90° from the horizon, and the pole 90° from the equator; and these equal arcs of 90° are made up in one case of the distance of the pole from the zenith, and the latitude of the place, or elevation of the pole above the horizon, in the other of the same elevation of the pole above the horizon, and the depression of the point Q of the equator below it. The zenith distance of the pole, therefore, and the depression of the equator at the point Q below the horizon are equal. Now, it is clear that whenever the North declination of a body, as Q q, is less than the depression of the equator below the horizon, as it is in the cases of the horizons H_1 N_1, H_2 N_2, the body when it comes to the meridian at q is below the horizon; but that when the declination Q q is greater than that depression, as it is in the case of the horizon H_3 N_3, the body at q is above the horizon; and then its whole circle of rotation is so. When the declination of the body, and the depression of the point Q are exactly equal, the body just touches the horizon at q, and all the rest of its course is completely above it. Generally therefore, *wherever* *the North declination of a body is not less than the depression of the point* Q *of the equator, or than the zenith distance of the pole, no part of its whole course of diurnal rotation is below the horizon.*

In the same manner, taking the case of the circle $e' q'$, we shall find that the most elevated point e' never rises above the horizon $H_3 N_3$, when the South declination E e' exceeds the elevation of the point E of the equator above the horizon; and as the 90° from the horizon to the zenith are made up of this elevation and the zenith distance of the point E, and the 90° from the point E to the pole are made up of the same zenith distance of the point E and the zenith distance of the pole, the elevation of the point E, and the zenith distance of the pole are equal. Generally therefore, *wherever the South declination of a body is not less than the zenith distance of the pole, no part of its whole course of diurnal rotation is above the horizon.*

The greatest North and South declinations of the sun are of 23° 28′ each: for, although there is some little difference in the observations of the two, they may without sensible error, for this purpose, be treated as equal. These are his declinations at the solstices. Where therefore the zenith distance of the pole is of 23° 28′, or the latitude of 66° 32′, the sun at the summer solstices, where he is at his greatest North declination, will, at his lowest point, only just touch the horizon, and all the rest of his course for that day will be above it; or there will be one day of 24 hours, with no night: and, in the same manner, there will be one period of 24 hours, when the sun is at the winter solstice, during which he will, at his highest point, only just touch the horizon, and the rest of his course will be below it; or there will be a night of 24 hours, and no day. If the zenith distance of the pole be less than 23° 28′, or the latitude greater than 66° 32′, (as, for instance, if the zenith distance be 15°, or the latitude 75°,) then, as soon as the sun attains a North declination equal to that zenith distance, (in this instance 15°,) his daily course will only just touch the horizon; and from that time forward till he attains his greatest North declination, and again till he returns to the same North declination (15°), his whole course will be above the horizon; or, for a considerable period (in the instance put, from the 30th of April to the 12th of August), there will be uninterrupted daylight. In the same manner, as soon as the South declination is equal to this zenith distance (here 15°), the sun's course will only just touch the horizon; and from that time till he attains his greatest South declination, and again till he returns to the same South declination (15°), his whole course will be below the horizon; so that for a considerable period (in this instance from the 2d of November to the 8th of February), there will be uninterrupted night. These intervals of uninterrupted day and night are obviously longer as the latitude increases; for then the zenith distance of the pole, or the declination at which the sun begins to be continually above or continually below the horizon, diminishes, and the sun in consequence attains that declination sooner after one equinox, and does not return to it till a shorter time before the other; or the interval during which he has that or greater declination is longer.

When the pole coincides with the zenith, or the latitude is 90°, the equator coincides with the horizon. In this case therefore, every circle parallel to the equator, and North of it, is entirely above the horizon, and every point of it at the same elevation; and all circles parallel to the equator, and South of it, are entirely below the horizon. Here then all bodies which have North declinations are always above the horizon, and describe circles in their daily rotation parallel to it; and all bodies which have South declination are always below the horizon. At the vernal equinox therefore, the sun, which then comes upon the equator, just coincides with the horizon; he is then continually above it while his declination is North, or until he returns to the equator at the autumnal equinox, and from that time continually below it, while his declination is South, or until he returns again to the equator at the succeeding vernal equinox. In this case, therefore, there is uninterrupted daylight for half the year, and then uninterrupted night for the remainder.

The pole may also coincide with the horizon, which, in this case, will be represented by the circle P p, which, as we have already observed, bisects, and is perpendicular to, every circle of rotation, as E Q, $e\ q$, and $e'\ q'$. In this case then every heavenly body is an equal time above and below the horizon; and the sun therefore is so whatever be his declination, or at every period of the year.

We have now generally ascertained the manner in which the length of the day differs at different places. There are however two causes which make the length of the day greater and that of the night less than we have stated it. In speaking of the sun, as in the equator or horizon, we mean that his centre is so. At these instants therefore, his Northern limb, or the Northern part of his visible circumference, is North of the equator, his upper limb above the horizon; and as the mean apparent semidiameter of the sun is about 16′, the sun begins to appear on the horizon, or the equator or any circle parallel to the equator, sooner than we have stated, by the time corresponding to that difference of elevation in the one case, and of declination in the other; that is to say, his upper limb appears on the horizon when his centre is 16′ below it, and thus the length of each day is increased; and the greatest declination of the Northern limb being 16′ greater than the declination of the centre, or being 23° 44′, some part of the sun is always above the horizon at the summer solstice, where the zenith distance of the pole is 23° 44′, instead of 23° 28′. So also, where the pole and zenith coincide, as the North limb of the sun is on the equator, when the declination of the centre is 16′ South, some part of the sun is continually seen on the horizon, not merely from the time of his coming to the equinox, but from the time that his South declination is less than 16′. There is another cause also, which produces a similar effect, and to a greater degree. In speaking of the place of the sun, we have hitherto given the results of observation, as they are obtained after allowing for the operation of certain causes which materially complicate them in the first instance, and which we shall hereafter explain under the heads of parallax and refraction. Parallax tends to make the apparent place of a body lower than that which we consider it really to occupy, and which we call its real place; refraction to make it higher; and as the effect of refraction is much greater than that of parallax upon the sun, the joint effect of the two is to render the apparent higher than the real place. When the sun therefore is really upon the horizon he appears above it; and when he appears on it, he is really about 33′ below it. The length of daylight is increased by the time corresponding to this difference of elevation, and the zenith distance of the pole at the points where some part of the sun is first seen never to set at the solstice, and the South declination of the sun when first seen on the horizon where the pole and zenith coincide, are increased in exactly the same manner by this cause, as we have already seen that they are by the apparent magnitude of the sun himself.

A short notice of the manner in which the climates of different places differ will be sufficient. We have already seen that the sun's influence depends on the length of time during which he continues above the horizon, and the elevation he attains above it. The greatest elevation of the sun, like that of all other heavenly bodies in their daily rotation, is always when he is on the meridian; and his distance at that time from the Southern point of the horizon is always equal to the distance of the intersection of the meridian and equator from that point, increased by the declination, when North, and diminished by it when South *; or, as the distance of the intersection of the meridian and equator from the South point of the horizon is equal to the zenith distance of the pole, the sun's distance from that point when he is on the meridian is equal to the zenith distance of the pole increased by his declination when North, or diminished by it when South. When the declination is South, and greater than the zenith distance of the pole, this expression becomes negative; and the sun never rises. When it has a positive value, it continually increases as the South declination diminishes, or the North declination increases. Now, the whole arch of the meridian from South to North is 180°; and consequently when the meridian distance of the sun from the South point exceeds 90°, he is nearer the North point than the South point: his meridian elevation, therefore, in that case is his distance from the North point, and dimi-

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* These conclusions will, perhaps, appear more plainly by a reference to a figure. In *fig*. 2, H being the South point of the horizon, and E the intersection of the equator and meridian, S, and S_2 may represent two situations of the sun in the meridian, S_1 being a situation where his declination is South, S_2 where it is North. S_1 H and S_2 H are evidently his distances at those times from H, the South point of horizon; and S_1 H is the distance, E H, of the intersection of the meridian and equator from that point, diminished by E S_1, the sun's South declination; S_2 H is the same distance E H, increased by E S_2, the sun's North declination.

nishes as his declination increases, after it has once attained that value which makes it equal to 90°.

The greatest value of the declination is 23° 28′: whenever, therefore, the zenith distance of the pole is less than 66° 32′, the sun's meridian distance from the South point can never exceed 90°: this distance therefore is the sun's altitude at the time, and increases for all places as the sun moves Northward; and it is greatest on the same day, for different places, as the zenith distance of the pole increases, or the elevation of the pole diminishes. The influence of the sun then, as far as it is determined by his elevation, increases as the elevation of the pole diminishes, within these limits. We have already seen that the length of the day increases, when the sun's declination is North, as the elevation of the pole increases. These two causes of heat therefore are opposed to each other, and we cannot easily tell in what degree they may counteract each other; and in fact we know that in high latitudes, or places where the elevation of the pole is great, the summers are often very hot, although the elevation of the sun is small.

Where the zenith distance of the pole is greater than 66° 32′, we are still less able to arrive at any satisfactory conclusion. In this case we have already seen that the meridian elevation of the sun may have to be measured Northward, and that it will then decrease as the declination increases, or as the length of the day increases. Thus the two causes which affect the solar power are here opposed, so as to prevent us from even saying when it is greatest at the place itself; and of course we have another difficulty, added to that already mentioned, in comparing it with the corresponding power in other situations. When the pole is in the horizon, the length of the day, as we have already seen, is always equal. In this case the power of the sun to communicate heat depends only on his elevation. Now here the equator passes through the zenith, and the meridian elevation of the sun is therefore greatest when he is in the equator, and continually less as he recedes from it, either towards the North or the South. Here therefore the equinoxes are the periods of the greatest, the solstices of the least solar power.

The periods of the least solar power, which correspond to the winter of different places, are more easily ascertained. The least distance from the South point is always the difference between the elevation of the equator where it crosses the meridian, and the greatest South declination: and this is necessarily the least meridian elevation, except where the poles are in the horizon, and there are equal elevations on the North and South side of the zenith. The least elevation therefore increases, as the elevation of the point of intersection of the equator and meridian increases, or as the zenith distance of the pole increases; and the greater this zenith distance the greater is the least meridian elevation of the sun. But we have already seen that, as the zenith distance of the pole increases, or its elevation diminishes, the length of the shortest days increases; and consequently on both accounts the influence of the sun, during the period of his South declination, is increased as the elevation of the North Pole is diminished. We should expect, therefore, to find the winters diminish in severity, as the elevation of the North Pole diminishes.

It may seem also, that, notwithstanding the difficulty of comparing the sun's extreme power at different places, his average power may be accurately compared. The whole time for which the sun in the course of a year is above the horizon, is everywhere the same or nearly so. If, in *fig.* 5, $e\,q$ be a circle of rotation North of the equator, and $e'\,q'$ be one South of it, and at the same distance from it, the portion $t_1\,e\,r_1$ of the first circle which is above the horizon $H_1\,N_1$, and the part $t_1'\;q'\;r_1'$ of the second circle which is below it, are in all cases equal; and so, of course, are the parts remaining. The parts above the horizon therefore, $t_1\,e\,r_1$ and $t_1\;e'\;r_1'$ are together equal to the whole of one of these circles; or the time for which the sun is above the horizon in the two days during which he describes the two circles $e\;q$, $e'\;q'$, is equal to the time of describing one of the circles, or to a day, and the time during which he is below the horizon is so also; and, in this manner, as the sun in his course has equal North and South declination, two days may continually be found, in which, taken together, the whole time of the sun's being above the horizon is equal to that of his being below it; and the whole year may be divided into these pairs of days, or only not so, because the sun's motion being rather more rapid in the Southern than the Northern part of his orbit, the days during which he is longer below than above the horizon, are not, in places North of the equator, quite so many as those when he is longer above it. In the same manner, if the circle $e q$ is entirely above the horizon $H_3 N_3$, the circle $e' q'$ is entirely below it; and thus the period, during which the sun never sinks below the horizon, is counterbalanced by a period during which he never rises above it.

Taking the extreme case, when the pole is in the zenith, these periods are each of them from equinox to equinox, or they comprehend each half the sun's course, and they only differ therefore, like the others, by the small inequality occasioned by his variable rate of motion. When the pole is in the horizon we have already seen that the day and night are always equal. Neglecting therefore the slight inequality we have mentioned, (which makes the whole period during which, in the course of a year, the sun is above the horizon, somewhat longer when the North Pole is so, and shorter when the South Pole is, and thus tends to render the heat of the former greater than that of the latter climates) we may consider that everywhere the sun is half the year above, and half below the horizon; and his influence to produce heat throughout the year, and consequently the average heat, as far as he occasions it, will depend upon his average elevation above the horizon, and be greatest where that is greatest, and least where that is least. It would however lead to very complicated investigation, if we were to endeavour to determine the manner in which this average elevation differs at different places. It is sufficient to state generally that it is greatest when the pole is in the horizon, and continually diminishes as the elevation of the pole increases; or the influence of the sun for the whole year is greatest when the latitude is nothing, it gradually diminishes with the increase either of North or South latitude, and is least where the latitude is of 90°. And it is familiarly known to every one that these deductions actually correspond with the general results of observation; that, generally, the countries near *the equinoctial line* (for so the line where the latitude is nothing is called, the day and night being there always equal) are the hottest, those in high latitudes the coldest. The ancients indeed put so much faith in these considerations, that they divided the whole earth into climates accordingly; and having no actual knowledge of the high Northern regions, nor of those near the equinoctial line, or South of it, they conceived the former to be uninhabitable from extreme cold, and the countries near the equinoctial to be equally so from extreme heat. The experience of modern times has proved that both suppositions were extravagant: and it has farther shewn that the heat of different places where the elevation of the pole is the same is very different; that it is indeed so much affected by local causes, as the elevation of the country above the level of the sea, its degree of cultivation, its geological constitution, the average moisture of the soil and atmosphere, the prevalent winds, the extent of continent with which it is connected, and other similar circumstances, that the mere consideration of the average influence of the sun, as deduced from his average elevation, is quite inadequate to give any information as to the comparative temperature of different places. Thus Edinburgh, Moscow, and Copenhagen, have all nearly the same latitude; but the Baltic is frozen up every year, while the sea near Edinburgh is unencumbered with ice, and the severities of a Russian winter, and the early period at which they commence, have too lately been the subjects of history, for us to want any other proof of the comparative mildness of the climate of Scotland*. It would therefore be useless to enter into any laborious research to discover the exact laws according to which a power, which we find not to have the importance once attributed to it, varies; nor should we have given to it even so much consideration as we have, unless the investigation had involved in it some facts and principles which are really of value; the explanations, we mean, of the manner in which the greatest and least solar power vary at different places, and the important general fact that the whole period of daylight during the year is equal, or very nearly so, at every place on the earth's surface.

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* Our intercourse with our North American possessions furnishes a still more striking example. The mouth of the river St. Lawrence is in the latitude of about 49° N., more than 2° south of London; and this is the most Northerly point of the river; yet all access to Canada is stopped by the frost which closes the river at an early period of the winter, and the ice does not break up, and navigation recommence, until the month of May.

SECTION 7.—*On the equation of Time—Inequality arising from unequal motion in his Orbit—that from the inclination of the Ecliptic—Deduction of the nature of the equation throughout the Year.*

A GREAT number of the most important elements involved in astronomical researches are variable in their amount. Their variations, however, generally succeed each other in a certain order, and are confined within certain limits; and when these limits, and all the varying values are ascertained, it is of course possible to take an average among them, and this average value is termed *a mean value.*

It is indeed always possible to take an average between any number of observations, or of ascertained values of a particular element; but unless the observations are so taken that the whole course or cycle of the variation is included, it is not usual to call the average a mean value; or rather it is not the absolute mean value of the thing itself, it is only its average or mean value for a certain time. For instance, we have already seen that the length of the day, considered as the interval from sunrise to sunset is continually varying, but that it goes through all its changes in the interval from one solstice to another. Its average duration for this whole time, then, is its mean value: it is a little more than twelve hours, and it is very nearly the same everywhere; it would be everywhere exactly twelve hours, if the sun always moved at the same rate, and there were no parallax or refraction. An average, however, might be taken of the lengths of this day for a portion only of this interval, for instance, from the vernal equinox to the summer solstice; in this case, the shortest length of the day would be a little more than twelve hours, and the average length at London about fourteen hours and three quarters. This would be a correct average of the lengths observed; but as the time of observation would not comprehend all the variations of the element in question, it would not be the mean length of the day absolutely, though it might be called the mean length for the period of observation.

In the same manner as we have taken an instance of mean duration, we might have an instance of mean motion; that is to say, if a body moves with a variable motion; but if the whole course of its variation is ascertained, its average rate of motion during this whole course may be found, and this will be called *its mean motion.* A body moving with this mean motion, and of course moving uniformly, for the whole time occupied by the whole series of the real motions, would move through the same space as the real body, but its place at many, or all intermediate periods, would be different from the place of the real body, on account of the difference between the real and mean motions. The place of a body so moving, or the place which the real body would occupy on the supposition that it moved uniformly, and described in the time occupied by the whole series of its real motions the same spaces which it actually does, is called *the mean place of the body.* In the same manner an event which happens at various intervals which succeed each other in a certain and recurring order, will have *a mean time* of occurrence.

Now it very generally happens in astronomy that it is less inconvenient first to compute the mean place of a body, or the mean time of an event, and then to ascertain the difference between the mean and the true, than to go through the computations necessary to find the true time and place in the first instance.

When once the mean values have been ascertained, the mean motion of a body during a known period, its mean place at a known time, the mean time of the occurrence of a given event, are easily found; for the intervals of the mean time, and the rate of the mean motion being always the same, we only want to know how often the event has occurred, or how long the motion has been continued. If, from consideration of the manner in which the difference between the true and mean values arises, we can ascertain the amount of that difference in each particular instance, we can find what is to be added to or subtracted from the mean value to arrive at the true; and the quantity so added or subtracted is called *an equation.* The mean value thus leads to the true value, and of course it furnishes an approximation to it; and as the subjects of astronomical inquiry generally have their variations confined within narrow limits, so that the difference between the true and mean motion's times and places is not very great, the approximation is not very distant.

We shall find several instances of the application of the terms above explained, and of the use made of these mean values and results in treating of the equation of time, of which we have still to speak, and then the more obvious appearances of the sun, and their principal effects, will be for the present sufficiently explained.

The solar day is longer than the sidereal day in consequence of the motion of the sun Eastward in his orbit. It is evident that the degree of its excess above the sidereal day must be affected by the quantity of that motion, and must, when other circumstances are the same, be greatest when that motion is greatest, or when the sun is in his perigee, and least when that motion is least, or the sun is in his apogee. The motion of the sun goes through all its variations in the course of one revolution of the sun in his orbit; it admits, therefore, on the principles already explained, of a mean value. Let us call the sun S, and let us suppose a fictitious body, which we call S_1, to move uniformly in the ecliptic, and to perform a complete revolution in the same time as S: the motion of S_1, therefore, will be the *mean motion*, and its place, the *mean place*, of S. The difference between the places of S and S_1 will be *an equation;* it is called *the equation of the centre.* Let us suppose the two bodies, S, and S_1, to be together when the sun is in apogee. As the revolution of the supposed body S_1, is completed at the same time as that of S, they will be again together at the end of the year, or when the sun returns to his apogee. Besides this, the times of the sun's motion from apogee to perigee, and from perigee back to apogee, are equal; they are therefore each equal to half the time of his whole revolution, or to half the time of the revolution of S, or to the time taken by S_1, which moves uniformly, to pass through half the ecliptic. But the sun's apogee and perigee are at the distance of half the ecliptic from each other; consequently, as S and S_1, were together at the apogee, they are so also at the perigee, each taking the same time (half of the year) to pass through half of their orbits. We find therefore that the real and mean places of the sun coincide at the apogee and perigee.

It is also plain that they correspond nowhere else. The sun's distance from the earth continually decreases from apogee to perigee, and his angular velocity continually increases during the same period. It is evident then that as his whole real angular motion for that period is equal to his mean angular motion for the same time, the real motion will at first be less, and afterwards greater than the mean motion. His real place therefore will at first fall behind his mean place, and the distance between them will increase day by day until his real motion becomes equal to his mean motion; the distance will then diminish as the real motion becomes greater than the mean motion, and this excess will finally bring them together again. It will not however do this until they reach the perigee, for we have already seen that they are then together, and this could not be the case if they had been so before; for as they are only brought together by the real motion exceeding the mean motion, and as the real motion continually increases from apogee to perigee, if at any period before the perigee S had come up with S_1, at the following instant S would have passed S_1 by the excess of its motion, and would have continued from day to day to increase the distance between them by the continuing and growing excess of the real above the mean motion; and the consequence would be, contrary to the fact, that S would arrive at the perigee before S_1. We arrive therefore at this conclusion, that the real place of the sun is *behind* his mean place, as he passes from apogee to perigee, the distance between them continually increasing for a certain time, then continually diminishing till the two places again coincide at the perigee. Exactly in the same manner we find that the real and mean place never coincide from perigee to apogee, only with this difference, that as in this half of the orbit the real motion is at first greater, and afterwards less, than the mean motion, the real place is always *before* the mean place, until, on the return to the apogee, they both again correspond. The greatest difference in the time of the approach of S and S_1 to the meridian cannot exceed $8^{m}\ 24^{s}$.

It is not however only by the variation in the sun's rate of motion, that the length of the solar day, or the interval between his successive appearances on the meridian, is affected; it is also influenced by the inclination of the ecliptic to the equator. The motion of rotation of the heavens is uniform; and consequently, if we suppose another fictitious body, S_2, to be at the point ♈ at the same time with the supposed body S, to

LUBBOCK, Sir John William (9) and
DRINKWATER [afterwards] BETHUNE, John Elio

ON PROBABILITY.

1. In considering any future event, we are generally unable to determine whether or not it will happen; yet, we can often conjecture the number of cases which are possible, and of these how many favour the production of the event in question. In our uncertainty, we say that there is a chance it will happen; and thus our idea of chance arises from our wanting data which might enable us to decide whether or not the event will take place. If, for instance, a bag contain one white and two black balls, it is impossible to decide whether or not a black ball will be drawn in one trial; but we know that there are three cases possible, of which two favour the appearance of a black ball and one the contrary, and of these, we have no reason to think one more probable than another.

2. The operations of the mind are of two kinds; the one consists in acquiring data, the other consists in making deductions from data previously acquired. Our data are only probable; our deductions from these are also probable. The subject, therefore, of this treatise is intimately connected with every science, and, whether on account of its numerous and useful applications, or of the exact reasoning by which its principles are established, carries with it the highest degree of interest. In the sequel we shall explain the method of applying it to the calculation of life annuities, a few tables of which will be subjoined.

3. To avoid circumlocution, those cases which embrace the production of a particular event are called the favourable cases, and those which do not, the unfavourable cases. It is usual to apply the word belief to the past, and the word expectation to the future; but the theory of probability is in all respects the same, whether it be applied to past or to future events. When we endeavour to discover whether an event $\left\{\begin{matrix}\text{did}\\ \text{will}\end{matrix}\right\}$ happen, we review the different cases which are possible. If the favourable cases are more numerous than the unfavourable, we $\left\{\begin{matrix}\text{believe}\\ \text{expect}\end{matrix}\right\}$ that the event $\left\{\begin{matrix}\text{did}\\ \text{will}\end{matrix}\right\}$ take place. The words believe and expect, and those to which they correspond, are placed between brackets, in order to show that the reasoning is the same in both cases. Let us suppose that a bag contains one black and two white balls: if I am asked whether a white ball will be drawn, or if, a ball being already drawn, but concealed from my view, I am asked whether a white ball has been drawn, it is clear that the judgment formed in both cases will be the same. I answer in both cases, that it is more probable that the ball which is drawn is white than black, yet if the ball be already drawn, but concealed from my view, the event is already determined and certain. We perceive, therefore, that the estimation of probability has no necessary reference to actual occurrence, but only to the means of judging which a given individual possesses.

4. We have used the word probability before giving its definition, because its popular meaning has hitherto sufficed for our purpose. We now give its mathematical definition, which is this: the probability of any event is the ratio of the favourable cases to all the possible cases which, in our judgment, are similarly circumstanced with regard to their happening or failing. Thus, if a bag contain one white and two black balls, the probability of drawing a white ball is $\frac{1}{3}$; the probability of throwing ace with a die at the first

throw is $\frac{1}{6}$; or, in common language, we should say of the first of these events that the odds are 2 : 1 against it, and of the second 5 : 1.

Generally, if $m + n$ is the whole number of cases, and if
m is the number of cases which are favourable to the event P, $m : n$ are the *odds*
n .... unfavourable ....
in favour of the event P, and the probability of the event P is $\dfrac{m}{m+n}$

5. Simpson has defined the probability of an event to be the ratio of the chances by which the event in question may happen to all the chances by which it may happen or fail. In this definition the word chance must be understood a way of happening; we, however, frequently say, "I left such a thing to chance," or, "such a thing is entirely chance;" these expressions, which are in some measure sanctioned by common use, are intended to signify that we are ignorant of the causes which produce the event in question, or that we do not influence its occurrence.

6. When there are many events, such that one must, and only one can happen on any given trial, we shall call them conflicting events; and it is evident, from the definition of probability, that the probability that either of two conflicting events will happen on any single trial is equal to the sum of their respective probabilities; it is also evident that the sum of the probabilities of all the conflicting events which can happen on any single trial is expressed by unity; for, by the supposition, one of them must happen. Our $\left\{\begin{array}{c}\text{belief}\\ \text{expectation}\end{array}\right\}$ is founded upon the probability of the event under consideration. It will often happen that our judgment is influenced by circumstances of too complicated or delicate a nature to be submitted to numerical calculation; and the conclusions with which this science furnishes us are true only within the limits of the errors which arise from neglecting these considerations. We experience the same difficulty in applying to physical phenomena the theories deduced from abstract principles of measure and motion. When all the cases which are possible, are favourable, the event is certain and belief becomes certitude, or knowledge. *Certainty*, which is the greatest probability, is therefore represented by unity; it must be distinguished from the highest degree of belief which we have called *certitude;* they are often confounded with each other, while they differ in the same manner as probability and belief differ. In fact, we want two words for every stage as much as for unity, the one to express the ratio of the favourable to all the cases possible, the other denoting the opinion consequent on the perception of that ratio.

7. If a bag contain no white and ten black balls, the probability of drawing a white ball is $\frac{0}{10}$, or, zero; if, on the other hand, the bag contain 10 white and no black balls, the probability of drawing a white ball is $\frac{10}{10}$, or unity, and whatever be the number of black balls, the probability of drawing a white ball must be some fraction between 0 and 1, which are its limits. When the fraction which expresses the probability of an event is little different from unity, we say the event is very probable, or nearly certain; when it is but little greater than $\frac{1}{2}$, we say it is probable; when $\frac{1}{2}$, doubtful; when rather less than $\frac{1}{2}$, improbable; when much less than $\frac{1}{2}$, very improbable; and when zero, impossible.

8. We habitually assent to propositions which have in their favour a probability less than unity: this degree of probability is vulgarly called moral certainty, an expression which is at variance with every analogy of language. The state of mind of a man who is aware of unfavourable events

which are possible, but who disregards them by reason of their reputed improbability, is perhaps what is meant. Some philosophers have endeavoured to fix the numerical fraction to which this moral certainty is equal by observing the risks of which men are in general careless. Buffon chose the fraction $\frac{9999}{10000}$; Condorcet estimated it in a different manner, and of course obtained a very different result. Indeed it is obvious that this fraction is arbitrary, and we shall therefore not enter more minutely into this question. There may, perhaps, be a practical utility for each man to determine the risk his own temperament enables him to disregard, in order to obtain a standard with which to compare the results of occasional theorems: without some such comparison they might fail in their abstract numerical form to determine his judgment.

9. We have said that probability does not exist in the abstract, but always refers to the knowledge possessed by some particular individual. Let us suppose that a bag contains one white and two black balls, and that A having drawn a white ball holds it so that he can see what colour it is, but so that B cannot. Here three cases appear possible to B, of which two favour the drawing a black ball; the probability therefore that it is a black ball to B is $\frac{2}{3}$, while the probability to A that it is a white ball, is unity, or certainty. Again: suppose a bag contain one white, one black, and one red ball; A, having drawn a white ball, whispers to B that the ball which is drawn is not red. Three cases appear equally possible to C, of which one only favours the drawing a white ball; C therefore estimates the probability at $\frac{1}{3}$, while B (if he believes the information given him by A) has only two alternatives to choose between: he therefore estimates the probability at $\frac{1}{2}$. Even if B do not implicitly believe the information given him by A, it is clear that his judgment will be formed on grounds different from those on which C decides.

10. It is thus that the same fact related before a numerous audience obtains from different individuals different degrees of belief: this is chiefly to be attributed to the different degrees of knowledge possessed by different individuals of circumstances which bear on the fact in question. An inhabitant of the torrid zone has difficulty in believing that water freezes; and the recovery of a sick person may appear probable to one unacquainted with medicine, while the skilful physician despairs of effecting a cure.

11. It follows from the definition of probability, that to determine the probability of any event, it is only necessary to enumerate the cases which are favourable and those which are unfavourable to its production, in order to form the fraction which expresses its probability. In order that this may be well understood, we shall begin with some very simple examples, and for these it will be necessary, at first, to have recourse to games of chance, in which the whole number of possible occurrences is most readily ascertained.

Ex. 1. Suppose a piece of money is thrown into the air, and that the probability of its falling on the obverse side twice successively is required: here the following cases present themselves.

Case 1. The obverse both times.
2. The obverse the first time and the reverse the second.
3. The reverse the first time and the obverse the second.
4. The reverse both times.

These are the only cases possible; and if we are ignorant of the existence of any cause tending to make the piece fall on one side rather than on the other, they are all similarly circumstanced, and therefore the probability of each case is $\frac{1}{4}$.

The probability of the obverse once and the reverse once in any order is $\frac{1}{2}$, because the second and third cases favour the production of this event; and the probability of the obverse arising at least once is $\frac{3}{4}$, because the first, second, and third cases are favourable to this event.

Ex. 2. Again, suppose of two bags one contains 5 white balls and 2 black, and the other 7 white balls and 3 black. The number of cases possible in one drawing from each bag is $(5+2) \times (7+3)$ or 7×10, because every ball in one bag may combine with every ball in the other, which cases, if we are ignorant of any cause favouring the appearance of a white rather than a black ball, are all similarly circumstanced.

The number of cases which favour the drawing a white ball from both is 5×7, for every one of the 5 white balls in one bag may combine with every one of the 7 white balls in the other. For a similar reason, the number of cases which favour the drawing a white ball from the first bag and a black ball from the second, is 5×3; a black ball from the first bag and a white ball from the second, is 7×2; and a black ball from both is 3×2. Therefore,

$\frac{5 \times 7}{(5+2)(7+3)} = \frac{1}{2}$ is the probability of drawing a white ball from both.

$\frac{5 \times 3}{(5+2)(7+3)} = \frac{3}{14}$ . . . . $\left\{\begin{array}{l}\text{a white ball from the first and}\\ \text{a black ball from the second.}\end{array}\right.$

$\frac{7 \times 2}{(5+2)(7+3)} = \frac{1}{5}$ . . . . $\left\{\begin{array}{l}\text{a black ball from the first and}\\ \text{a white ball from the second.}\end{array}\right.$

$\frac{3 \times 2}{(5+2)(7+3)} = \frac{3}{35}$ . . . . a black ball from both.

The probability of drawing one white ball, without reference to the bag from which it comes, is

$$\frac{5 \times 3 + 2 \times 7}{(5+2) \times (7+3)} = \frac{29}{70},$$

for both the second and third cases favour the production of this event.

The probability of drawing at least one white ball is

$$\frac{5 \times 7 + 5 \times 3 + 2 \times 7}{(5+2) \times (7+3)} = \frac{32}{35},$$

for the first, second, and third cases favour the production of this event.

Let the number of white and black balls in each bag be the same, say 5 white and 2 black, then the probability of drawing

a white ball from both . . . . $= \frac{5 \times 5}{(5+2) \times (5+2)} = \frac{25}{49}$

a white ball from the first and a black ball from the second $= \frac{5 \times 2}{(5+2)\ \ (5+2)} = \frac{10}{49}$

a black ball from the first and a white ball from the second $= \frac{2 \times 5}{(5+2)\ \ (5+2)} = \frac{10}{49}$

a black ball from both . . . . $= \frac{2 \times 2}{(5+2)\ \ (5+2)} = \frac{4}{49}.$

Ex. 3. Two dice are thrown; required the probability that the sum of the numbers on the sides which fall uppermost, or the throw, is any given number, say 7.

Since every one of the six numbers on one of the dice may combine with every one of the six on the other, the number of throws on the dice is 36.

$$\text{The number 7 may be made up of} \left\{\begin{matrix} \text{1 and 6} \\ \text{or 3 and 4} \\ \text{or 2 and 5} \end{matrix}\right\},$$

and as these numbers may be on the one die or the other, there are in all six ways which favour the number 7, and therefore the probability required is $\frac{6}{36}$ or $\frac{1}{6}$.

Ex. 4. A, the dealer in a party at whist, desires to know the probability of his partner holding a given card. The number of cards which are held by the other three players is 39; therefore the probability that the card in question is any given card in A's partner's hand is $\frac{1}{39}$, but it may be any one of the 13 cards which A's partner holds, therefore the probability is $\frac{13}{39} = \frac{1}{3}$. Or thus, there are three cases possible: either the card is in the hand of A's partner, or of one of the other two players; and as these three cases are similarly circumstanced, the probability of either of them is $\frac{1}{3}$, the odds against it being of course 2 : 1.

A desires to know the probability of his partner holding 2 given cards.

The number of combinations of 39 things taken two and two together is $\frac{39 \times 38}{1.2}$, therefore the probability that these two cards are any given two cards in A's partner's hand is $\frac{1}{\frac{39 \times 38}{1.2}} = \frac{1}{39 \times 19}$; but they may be any two cards in A's partner's hand; therefore, since the number of combinations of 13 cards taken two and two together, is $\frac{13 \times 12}{1.2} = 13 \times 6$, the probability required is $\frac{13 \times 6}{39 \times 19} = \frac{2}{19}$, the odds against are therefore 17 : 2.

Similarly, the probability that he holds any three given cards, is $\frac{22}{703}$; the odds against are, therefore, 681 : 22.

Ex. 5. Required the probability, that in a deal at whist each player holds an honour.

The number of permutations of 52 cards taken all together is $52 \times 51 \ldots \times 3 \times 2 \times 1$, and the number of permutations of 13 cards taken all together is $13 \times 12 \times 11 \ldots\ldots 3 \times 2 \times 1$, therefore the number of different deals is

$$\frac{52 \times 51 \times 50 \quad \ldots \quad \times 3 \times 2 \times 1}{(13 \times 12 \times 11 \quad \ldots\ldots\ldots \quad 1)^4},$$

because the 13 cards may be permuted in each player's hand separately, without altering his hand.

The number of permutations of 48 cards taken all together is $48 \times 47 \times 46 \ldots\ldots 3 \times 2 \times 1$, therefore the number of different ways in which 48 cards can be dealt to four persons is

$$\frac{48 \times 47 \times 46 \ldots\ldots\ldots 3 \times 2 \times 1}{(12 \times 11 \times 10 \ldots\ldots\ldots\ldots 1)^4}.$$

Let A, B, C, D be the hands dealt to each player out of 48 cards,
a, b, c, d, the four honours.
It is evident that A, B, C, D may be combined with a, b, c, d in as many different ways as a, b, c, d can be permuted, that is, in $4 \times 3 \times 2 \times 1$, or 24 different ways; therefore the probability that

$$\text{each hand has an honour} = \frac{\dfrac{48 \times 47 \times 46 \, . \, . \, . \, . \, . \, 3 \times 2 \times 1}{(12 \times 11 \times 10 \, . \, . \, . \, . \, . \, 3 \times 2 \times 1)^4}}{\dfrac{52 \times 51 \times 50 \, . \, . \, . \, . \, . \, 3 \times 2 \times 1}{(13 \times 12 \times 11 \, . \, . \, . \, . \, . \, 3 \times 2 \times 1)^4}} \times 24.$$

Odds against. About

$$= \frac{13^4}{52 \times 51 \times 50 \times 49} \times 24 \qquad = \frac{2197}{20825}, \qquad 17 : 2$$

So it may be found that the probability that

$$\left\{\begin{array}{l}\text{one hand has the four}\\ \text{honours . . .}\end{array}\right\} = \frac{13 \times 12 \times 11 \times 10}{52 \times 51 \times 50 \times 49} \times 4 \qquad = \frac{220}{20825}, \qquad 94 : 1$$

$$\left\{\begin{array}{l}\text{two hands have each}\\ \text{two honours . .}\end{array}\right\} = \frac{13 \times 12 \times 13 \times 12}{52 \times 51 \times 50 \times 49} \times 6 \times 6 \quad = \frac{2808}{20825}, \qquad 13 : 2$$

$$\left\{\begin{array}{l}\text{one hand has two ho-}\\ \text{nours and two hands one}\end{array}\right\} = \frac{13 \times 13 \times 13 \times 12}{52 \times 51 \times 50 \times 49} \times 12 \times 12 = \frac{12168}{20825}, \qquad 5 : 7$$

$$\left\{\begin{array}{l}\text{one hand has three ho-}\\ \text{nours and one hand one}\end{array}\right\} = \frac{13 \times 13 \times 12 \times 11}{52 \times 51 \times 50 \times 49} \times 12 \times 4 \quad = \frac{3432}{20825}, \qquad 5 : 1$$

The number of deals essentially different is

$$\frac{52 \times 51 \times 50 \, . \, . \, . \, . \, . \, . \, . \times 3 \times 2 \times 1}{(13 \times 12 \times 11 \, . \, . \, . \, . \, . \, . \, .3 \times 2 \times 1)^4} \times \frac{1}{24}.$$

This number, of which the logarithm is 27.34935, is so great, that if 1,000,000,000 persons, about the population of the earth, were to deal the cards incessantly day and night for 100,000,000 years, at the rate of a deal by each person a minute, and even if each of these deals were essentially different, they would not have exhausted a $\frac{1}{100000}$th part of the number of essentially different ways in which 52 cards can be distributed equally between four players.

12. Let a bag contain 10 slips of paper, each having the name of a different individual written upon it, and suppose that the individual whose name is drawn is to receive 10*l.*

In estimating the value of the expectation of these individuals it is evident that the sum of the values of all their expectations is equal to 10*l.*, the sum which one of them must receive, for if they each sold their chance of winning the 10*l.* to another person, this person would be sure of receiving 10*l.*; and since their expectations are equal, for their chances of winning are the same, if e be the expectation of one of them

$$10\,e = 10, \; e = 1$$

that is, the expectation of each is worth 1*l.* It is evident, also, that if there were $m + n$ slips of paper, each having the name of a different individual written upon it, and if the individual whose name was drawn should receive a pounds, the value of the expectation of each individual would be $\dfrac{a}{m+n}$. The value of the expectation of two individuals is the sum of the values of their separate expectations, and is, therefore, $\frac{2a}{m+n}$; of three individuals $\frac{3a}{m+n}$; and of m individuals is $\frac{ma}{m+n}$; but the expectation of m individuals must be the same as the expectation of one individual holding m tickets; therefore, the expectation of an individual holding m tickets is $\frac{ma}{m+n}$, or $\frac{m}{m+n} \times a$, but $\frac{m}{m+n}$ is the probability of this individual winning; therefore, generally, the value of the expectation of any individual with respect to any particular event is the product of the probability that the event will take place by the gain that will accrue to the individual if it does take place.

Ex. 6. Suppose the probability that a horse wins a race is p: A gives B £a: what odds should B bet A that the horse wins, in order that, on the whole, their situation may be the same? Let $x : 1$ be the odds required; then the expectation of $B = a + p - (1 - p)\,x$: the expectation of $A = (1 - p)\,x - a - p$; in order that they may be in the same situation,

$$a + p - (1 - p)\,x = (1 - p)\,x - (a + p); \quad x = \frac{a + p}{1 - p}.$$

13. All wagers are founded upon this principle, of multiplying the probability of the event by the contingent gain. Generally, however, one party does not pay down to the other a sum of money in order to receive another sum of money if a certain event takes place; but A engages to pay B a certain sum if the event takes place, and B engages to pay A a certain sum if it does not take place. Let a be the sum which A is to pay B, on the event of which the probability is p; and b the sum to be paid by B to A, if that happens of which the probability is q.

The expectation of A is equal to $b\,q - a\,p$.

The expectation of B is equal to $a\,p - b\,q$.

In order that the wager may be fair, the expectations of A and B must be equal.

$$b\,q - a\,p = a\,p - b\,q \qquad a\,p = b\,q$$

$\frac{a}{b} = \frac{q}{p}$, $\frac{b}{a+b} = p$, $\frac{a}{a+b} = q$, since $p + q = 1$: a and b are the odds.

Since the sum of the probabilities of any number of conflicting events is equal to unity, we have an equation of condition between the odds; and whenever they do not satisfy this equation, it is possible to bet with the certainty of gain.

Ex. 7. Suppose three horses, A, B, C, are entered for a race, and that I bet £12. to £5. against A; £11. to £6. against B; and £10. to £7. against C.

If A wins, I gain £6. + 7 − 12 = £1.
B wins, I gain £5. + 7 − 11 = £1.
C wins, I gain £5. + 6 − 10 = £1.

Thus I gain £1. whichever horse wins, from having taken the field against each horse.

$$\text{Here the odds quoted in favour of}\left\{\begin{array}{l}\text{A are } 5 : 12\\ \text{B} \;\;,,\;\; 6 : 11\\ \text{C} \;\;,,\;\; 7 : 10\end{array}\right\}\left\{\begin{array}{c}\text{the correspond-}\\ \text{ing probability}\\ \text{would be}\end{array}\right\}\left\{\begin{array}{l}\frac{5}{17}\text{ in favour of A.}\\ \frac{6}{17}\;\;\ldots\;\;\text{B.}\\ \frac{7}{17}\;\;\ldots\;\;\text{C.}\end{array}\right.$$

$$\frac{5}{17}+\frac{6}{17}+\frac{7}{17}=\frac{18}{17}=1+\frac{1}{17}.$$

The odds are often, as in this case, far from satisfying the equation of condition to which we have alluded.

The odds quoted just before the race upon the horses entered for the Oaks Stakes,* May, 1828, were so enormous, that the corresponding probability of one of the first seven favourites winning, was $1+\frac{31}{336}$, exclusive of the winner and six others which started.

14. Although it is easy in similar cases to see whether or not to *back the field*, it is not possible to determine the proportions which should be betted against each event in order that the advantage may be the greatest, unless the probabilities of each are known; and it may be observed, that the odds offered and taken will sometimes depend upon the incertitude of the better, whether or not he will receive the amount of his wager in case of a favourable issue. This chance enters as legitimately into the estimation of the odds as any other, though it may be more difficult to estimate its influence. It may also happen that the odds will be affected by those who, having almost secured a certain advantage in the manner we have just described, will prefer to diminish that advantage by offering more than the real odds against the events which yet remain out of their list, rather than leave any contingency unprovided against. But this last consideration verges on a branch of the subject which we shall afterwards treat more in detail: sufficient reason has been given why, practically, the odds vary so much from the result of a theory, which has not estimated these causes of discrepancy.

15. Let P_1, Q_1 represent two conflicting events.
P_2, Q_2 any other two conflicting events.

$$\text{and let }\left.\begin{matrix}m_1\\m_2\\n_1\\n_2\end{matrix}\right\}\text{ be the number of cases favourable to the event }\left\{\begin{matrix}P_1\\P_2\\Q_1\\Q_2\end{matrix}\right.$$

P_1 cannot happen with Q_1, nor P_2 with Q_2: the whole number of cases possible, therefore, is $(m_1+n_1)\,(m_2+n_2.)$ Of these

$$\left.\begin{matrix}m_1\,m_2\\m_1\,n_2\\n_1\,m_2\\n_1\,n_2\end{matrix}\right\}\text{ are favourable to the production of the event }\left\{\begin{matrix}(P_1\,P_2)\\(P_1\,Q_2)\\(Q_1\,P_2)\\(Q_1\,Q_2)\end{matrix}\right\}\text{ that is, of the events }\left\{\begin{matrix}P_1,\,P_2\\P_1,\,Q_2\\Q_1,\,P_2\\Q_1,\,Q_2\end{matrix}\right.$$

simultaneously, and therefore by previous definition

$$\left.\begin{matrix}\dfrac{m_1\,m_2}{(m_1+n_1)\,(m_2+n_2)}\\[2ex]\dfrac{m_1\,n_2}{(m_1+n_1)\,(m_2+n_2)}\\[2ex]\dfrac{n_1\,m_2}{(m_1+n_1)\,(m_2+n_2)}\\[2ex]\dfrac{n_1\,n_2}{(m_1+n_1)\,(m_2+n_2)}\end{matrix}\right\}\text{ is the probability of the event }\left\{\begin{matrix}(P_1,\,P_2)\\[2ex](P_1,\,Q_2)\\[2ex](Q_1,\,P_2)\\[2ex](Q_1,\,Q_2)\end{matrix}\right.$$

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* Betting for the Oaks, Morning Chronicle, May 24, 1828.
5 to 2 against Ridicule.
5 . 2 .. Zoé.
4 . 1 .. Rosalie.
7 . 1 .. Trampoline.
14 . 1 .. Delta.
14 to 1 against L'Estelle.
15 . 1 .. Ruby.
25 . 1 .. Turquoise, who won.
6 others started.

$$\left.\begin{array}{c}\dfrac{m_1}{m_1+n_1}\\[2ex] \dfrac{m_2}{m_2+n_2}\end{array}\right\}\text{ is the probability of the event }\left\{\begin{array}{c}P_1\\[2ex] P_2\end{array}\right\}\text{ considered by itself,}$$

and as P_1, P_2 may represent any independent events whatever, we conclude that the probability of the concurrence of any two independent events is equal to the product of the probabilities of each considered separately.

It is easy to extend the same theorem to any number of independent events: for we may consider the event (P_1, P_2) as one event of which the probability is $\frac{m_1 m_2}{(m_1+n_1)(m_2+n_2)}$, and if $\frac{m_3}{m_3+n_3}$ be the probability of any other independent event P_3, the probability of the concurrence of (P_1, P_2) and P_3 or $(P_1, P_2, P_3,)$ is equal to $\frac{m_1 m_2}{(m_1+n_1)(m_2+n_2)} \times \frac{m_3}{m_3+n_3}$, or $\frac{m_1 m_2 m_3}{(m_1+n_1)(m_2+n_2)(m_3+n_3)}$; but $\frac{m_1}{m_1+n_1}$, $\frac{m_2}{m_2+n_2}$, and $\frac{m_3}{m_3+n_3}$, are the probabilities of the events P_1, P_2, P_3, considered separately.

The same reasoning may be extended to any number of independent events, and hence this general and important theorem: the probability of the concurrence of any number of independent events is equal to the product of the probabilities of each considered separately. Before we proceed farther, it may be well to illustrate this by an example.

Ex. 8. Let us assume p_1 to be the veracity of any witness A_1, or the probability that A_1 tells the truth; p_2 the veracity of any other witness A_2. A_2 asserts that A_1 has asserted that a certain event took place; what is the probability that it did take place?

The event took place, if both A_1 and A_2 tell the truth; the probability of this is $p_1 \times p_2$.

It also took place if both A_1 and A_2 lie, that is, if A_1 said it did not take place, and if A_2 says that A_1 said it did take place: the probability of this is $(1-p_1)(1-p_2)$; and the probability that the event did take place, is $p_1 p_2 + (1-p_1)(1-p_2)$, or $1 - p_1 - p_2 + 2 p_1 p_2$.

Suppose, for instance, p_1 and p_2 were equal to $\frac{9}{10}$ths, the probability that the event took place which A_1, on the authority of A_2, asserts to have taken place, would be $\frac{82}{100}$ths, a probability which is considerably less than $\frac{9}{10}$ths.

The same might be extended to any number of testimonies; and it is thus that events, probable in the first instance, by passing through many relaters, may at last become extremely improbable. Laplace compares this diminution of probability to the diminution of light in passing through a succession of transparent mediums: the analogy does not strike us as being very forcible.

16. Since the odds are inversely as the probabilities of the events, whenever the odds in favour of two independent events are known, the odds that they will both happen may be found.

Ex. 9. In May, 1828, the odds were 3 to 1 against Rapid Rhone winning the Gold Cup, and 6 to 1 against Bessy Bedlam winning the St. Leger. According to these odds, the probability of Rapid Rhone winning the Gold Cup is $\frac{1}{4}$, and of Bessy Bedlam winning the St. Leger is $\frac{1}{7}$. Therefore, the probability of both these events taking place is $\frac{1}{28}$, and the odds against it 27 to 1, or about 1000 to 37. The odds were given at 1000 to 60.

It may be remarked that if the *same* horse as Rapid Rhone were to run in both races, and if the probability of his winning the first race were $\frac{1}{4}$, and of his winning the second $\frac{1}{7}$; the probability of his winning both is not $\frac{1}{28}$, but rather more. This very important distinction will be more fully explained in treating of probabilities *à posteriori*.

17. Let P_n, Q_n be any two conflicting events, of which the probabilities are p_n, q_n.

The probability of the concurrence of all the events $P_1, P_2, \ldots P_n$, is $p_1 \times p_2 \ldots p_n$, and the probability of any other event is that term in the product

$$(p+q_1)\,(p_2+q_2)\ldots\ldots(p_n+q_n);$$

in which the indices of p and q are the same as those of P and Q in the composite event which is considered.

If we consider repeated trials of the same event, so that $P_1, P_2, \ldots P_n$ are all the same, $(p_1+q_1)\,(p_2+q_2)\ldots\ldots\ldots(p_n+q_n)$ becomes $(p+q)^n$, and the probability of any event composed of a times P and b times Q in any given order is $p^a q^b$, the probability of having a times P and b times Q, without any regard to the order in which they occur, is the sum of the terms which are equal to $p^a q^b$ in the developement of $(p+q)^n$, or, what is the same thing, the term which has $p^a q^b$ for its argument in the expansion of $(p+q)^n$.

By the binomial theorem, this term is $\dfrac{n\,.\,(n-1)\ldots\ldots\ldots\ldots 1}{1\,.\,2\,.\,3\ldots a\ldots 1\,.\,2\,.\,3\ldots b} \times p^a q^b$.

Ex. 10. Thus, if n shillings are thrown into the air, in order to find what is the probability of any particular combination, it is only necessary to take the corresponding term in the developement of $\left\{\frac{1}{2}+\frac{1}{2}\right\}^n$.

Suppose there are five shillings

$$\left\{\frac{1}{2}+\frac{1}{2}\right\}^5=\left(\frac{1}{2}\right)^5+5\,.\left(\frac{1}{2}\right)^5+\frac{5\,.\,4}{1\,.\,2}\left(\frac{1}{2}\right)^5+\frac{5\,.\,4}{1\,.\,2}\left(\frac{1}{2}\right)^5+5\,.\left(\frac{1}{2}\right)^5+\left(\frac{1}{2}\right)^5.$$

The probability that they will all fall heads $= \left(\frac{1}{2}\right)^5$

4 heads and 1 tail $= 5 \times \left(\frac{1}{2}\right)^5$

3 heads and 2 tails $= 10 \times \left(\frac{1}{2}\right)^5$

2 heads and 3 tails $= 10 \times \left(\frac{1}{2}\right)^5$

1 head and 4 tails $= 5 \times \left(\frac{1}{2}\right)^5$

5 tails $= \left(\frac{1}{2}\right)^5$

18. The same theorem may be extended to any number of conflicting events, so that if P, Q, R, S represent any conflicting events of which the probabilities are p, q, r, s, respectively, the probability of the event which is composed of a times P, b times Q, c times R, d times S, &c. in $a+b+c+d$ repeated trials is the term in the expansion of $(p+q+r+s,)^{a+b+c+d}$, which has $p^a q^b r^c s^d$ for its argument; by the multinomial theorem this term is

$$\frac{(a+b+c+d)\,(a+b+c+d-1)\ldots\ldots 3\,.\,2\,.\,1\,.}{1\,.\,2\,.\,3\ldots a\,1\,.\,2\,.\,3\ldots b\,1\,.\,2\,.\,3\ldots c\,1\,.\,2\,.\,3\ldots d}\,p^a q^b r^c s^d.$$

Ex. 11. Let a jury be composed of n jurymen, and let p be the probability that each juryman separately will give a right decision, q the probability that he will give a wrong decision. The probability of a unanimous verdict, that is, that they all will voluntarily give a wrong or all a right decision, is $\frac{p^n + q^n}{(p+q)^n}$: if $p = \frac{9}{10}$, $q = \frac{1}{10}$ and $n = 12$, this is equal to $\frac{282429536482}{1000000000000}$, or about $\frac{13}{46}$; This probability must not be confounded with the probability, after an unanimous decision has been given, that it is a correct one, which is a very different question.

Ex. 12. Let A and B be two gamesters, and let p be the probability of A's winning a game, and q the probability of B's winning a game; required the probability of A's winning m games out of $m + n$, the set being supposed to finish as soon as A has won m games.

The probability of A's winning the first m games running, is p^m. The probability of A's winning m games out of $m + 1$, is $(m + 1)\, p^m q$. The probability of A's winning the first m games, and B the $(m + 1)^{th}$ out of $m + 1$ games, is $p^m q$; therefore, the probability of A's winning the set in exactly $m + 1$ games is $(m + 1)\, p^m q - p^m q$, or $(m + 1 - 1)\, p^m q = m\, p^m q$, and the probability of A's winning the set in not more than $m + 1$ games is $p^m + m\, p^m \,.\, q$.

The probability of A's winning m games out of $m + 2$, in any order, is

$$\frac{(m + 2)\,.\,(m + 1)}{1\,.\,2}\, p^m q^2,$$

but the probability of A's winning m games in the first $m + 1$ is $(m + 1)\,.\,p^m q$, and the probability of B's winning the $(m + 2)^{th}$ game is q; therefore, the probability of A's winning m games in the first $m + 1$ out of $m + 2$ games is $(m + 1)\,.\,p^m q^2$; and the probability of A's winning the set exactly in $m + 2$ games is

$$\left\{\frac{(m + 2)\,.\,(m + 1)}{1\,.\,2} - (m + 1)\right\} p^m q^2 = \frac{m\,.\,(m + 1)}{1\,.\,2}\, p^m q^2;$$

and the probability of A's winning the set in not more than $m + 2$ games is

$$p^m + m\, p^m q + \frac{m\,.\,(m + 1)}{1\,.\,2}\, p^m q^2.$$

The same reasoning may be applied to the general term: thus the probability that A will win m games out of $m + n$ in any order

$$= \frac{(m + n)\,.\,(m + n - 1)\,.\,.\,.\,2\,.\,1}{1\,.\,2\,.\,.\,.\,m\,.\,1\,.\,2\,.\,.\,.\,n}\, p^m\,.\,q^n.$$

The probability that A will win m games out of $m + n - 1$, and that B will win the $(m + n)^{th}$

$$= \frac{(m + n - 1)\,.\,(m + n - 2)\,.\,.\,.\,2\,.\,1}{1\,.\,2\,.\,.\,.\,m\,.\,1\,.\,2\,.\,.\,.\,.\,(n - 1)}\, p^m\,.\,q^n.$$

Therefore the probability that A will win m games in exactly $m + n$ games

$$= \frac{(m + n - 1)\,.\,(m + n - 2)\,.\,.\,.\,.\,2\,.\,1}{1\,.\,2\,.\,.\,.\,.\,m\,.\,1\,.\,2\,.\,.\,.\,.\,(n - 1)} \left\{\frac{m + n}{n} - 1\right\} p^m q^n$$

$$= \frac{m\,.\,(m + 1)\,.\,.\,.\,.\,.\,(m + n - 1)}{1\quad 2\,.\,.\,.\,.\,.\,.\,.\,.\,.\,n}\, p^m q^n,$$

and the probability of A's winning the set in not more than $m+n$ games will appear to be

$$p^m\left\{1+mq+\frac{m.(m+1)}{1.2}q^2\ldots\frac{m.(m+1).(m+2)..(m+n-1)}{1.2.3.\;..\;n\;.}q^n\right\}.$$

If A, in order to win the set, must win m games before B wins n games, A must win m games out of $m+n-1$, and the probability of this event is

$$p^m\left\{1+mq+\frac{m.(m+1)}{1.2}q^2\;..\;\frac{m.(m+1).(m+2)..(m+n-2)}{1.2.3\;..\;(n-1)}q^{n-1}\right\}.$$

and the probability of B's winning n games out of $m+n-1$ is

$$q^n\left\{1+np+\frac{n.(n+1)}{1.2}p^2\ldots.\frac{n.(n+1).(n+2)..(n+m-2)}{1.2..3\;\;(m-1)}p^{m-1}\right\}.$$

The same result may be obtained from the following considerations. If the play be supposed to continue without end, the probability that A will gain a single game

$$=p+pq+pq^2+\ldots.\infty=\frac{p}{1-q}, \text{ or } 1,$$

and the probability that A will win any finite number of games as m will be represented by $\left(\frac{p}{1-q}\right)^m$

$$=p^m\{1+mq+\frac{m.(m+1)}{1.2}q^2+\ldots.\infty\}.$$

This probability is made up of the partial probabilities that A will win m games in m exactly, in $m+1$ exactly, &c. The probability that A will win m games in $m+x$ exactly must have $p^m.q^x$ for its argument, and, therefore, since the above-written series contains all these partial probabilities, and no others, and consists solely of terms whose arguments are of the form $p^m.q^x$, each of these partial probabilities will be rightly represented by the corresponding term in that series, for it can be exhibited in no other shape, having the arguments of all its terms of this necessary form. Therefore the probability that A will win m out of $m+n$ games consists of the first terms of that series up to that inclusively, whose argument is $p^m.q^n$, the same as before obtained.

If A wants m games of being up, and B n games, and they agree to leave off playing, the stakes should be divided between them in the proportions of their probabilities.

This problem is celebrated in the history of the theory of probabilities, and was the first of any difficulty which was solved. It was proposed to Pascal by the Chev. de Méré, with some others relating to games with dice.

Ex. 13. A bag contains $n+1$ balls which are marked by the numbers $0.1.2.3...n$, a ball is drawn and afterwards replaced in the bag. Required the probability that after i drawings the sum of the numbers drawn is equal to s.

A little consideration will show that the probability required will be the coefficient of x^s in the expansion of $\left\{\frac{x^0+x^1+x^2\ldots.+x^n}{n+1}\right\}^i$ because that coefficient will be made up of all the different ways in which the different indexes of x can be combined in this developement, so as to equal the required index s.

$$x^0 + x^1 + x^2 \ldots\ldots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

$$\left\{1 - x^{n+1}\right\}^i = 1 - i\,x^{n+1} + \frac{i.(i-1)}{1.2}\,x^{2(n+1)} - \frac{i.(i-1).(i-2)}{1.2.3}\,x^{3(n+1)} + \&c.$$

$$\left\{1 - x\right\}^{-i} = 1 + i\,x + \frac{i.(i+1)}{1.2}\,x^2 + \frac{i.(i+1).(i+2)}{1.2.3}\,x^3 + \&c.$$

The coefficient of x^s is obtained by multiplying the coefficient of the first term of the upper series by the coefficient of x^s in the lower, that of the second in the upper by that of x^{s-1} in the lower, &c. and

$$= \frac{i.(i+1)\ (i+2)\ldots.(i+s-1)}{1.2.3\ldots.s\,(n+1)^i} - i\,\frac{i.(i+1).(i+2)\ldots(i+s-n-2)}{1.2.3\ldots.(s-n-1)\ (n+1)^i}$$
$$+ \frac{i.(i-1)}{1.2}\,\frac{i.(i+1).(i+2)\ldots.(i+s-2n-3)}{1.2.3\ldots.\ (s-2n-2)}\,(n+1)^i\ \&c$$

This series is equivalent to

$$\frac{(s+1).(s+2)\ldots(s+i-1)}{1.2\ldots\ldots(i-1)\ (n+1)^i} - i\,\frac{(s-n).(s-n+1)\ldots(s-n+i-2)}{1.2\ldots\ldots\ldots.(i-1)\ (n+1)^i}$$
$$+ \frac{i.(i-1)}{1.2}\,.\,\frac{(s-2n-1).(s-2n)\ldots.(s-2n+i-3)}{1.2\ldots\ldots\ldots.\ (i-1)\ (n+1)^i}\ \&c.$$

It is to be remarked, that all the terms of the developement of $(1 - x^{n+1})^i$ in which x is involved to a higher power than s may be rejected, because we have no negative powers of x in the other factor $(1 - x)^{-i}$ by which to reduce them.

This serves to show how many terms of the resulting series are to be taken, for if l represent the rank of the last term, we must have $(l - 1)(n + 1) =\!< s$. Therefore the series is to be continued only so long as $s - (l - 1).(n + 1)$ is positive. As a numerical example, we may take the problem already solved in page 4, to find the chance of throwing 7 with two dice. Dice have no side marked with 0, therefore, before the formula can be applied to this and similar cases, each face or ball must be supposed to import one less than is marked on it, which amounts to substituting in the formula $s - n$ for s. We shall, however, not substitute in the formula so altered, but deduce its value in the particular case, as we have done in the general one.

The probability required will be the coefficient of x^7 in

$$\left\{\frac{x^1 + x^2 + x^3 + x^4 + x^5 + x^6}{6}\right\}^2 = \left\{\frac{x - x^7}{6.(1-x)}\right\}^2$$

$$= \frac{1}{36}\,(x^2 - 2\,x^8 + x^{14})\ (1 + 2\,x + 3\,x^2 + 4\,x^3 + 5\,x^4 + 6\,x^5 + \&c.)$$

which is $\frac{6}{36} = \frac{1}{6}$ as before.

If we want the probability of throwing 10 with three dice, we must find the coefficient of x^{10} in

$$\left\{\frac{x^1 + x^2 \ldots. + x^6}{6}\right\}^3 = \left\{\frac{x - x^7}{6.(1 - x)}\right\}^3$$

$$= \frac{1}{216}\,(x^3 - 3\,x^9 + \&c.)\ (1 + 3\,x + 6\,x^2 \ldots + 36\,x^7 + \&c.)$$

which is $\frac{36-9}{216}=\frac{1}{8}$, the required probability.

19. If we give to s the different values 0, 1, 2, . . . s, successively, the sum of all the $s+1$ resulting series will of course be the probability that the sum of the numbers drawn will not be greater than s. In the last example, if we collect the coefficients of all the terms to x^{10} inclusive, we shall find the probability of not throwing more than 10 with three dice. It appears to be

$$\frac{1+3+6+10+15+21+28+36-3-9}{216}=\frac{108}{216}=\frac{1}{2}.$$

This probability is the foundation of the game of *Passe dix.*

20. If, instead of thus giving to s values which are increased by unity at each step, we suppose that s may have any value whatever between 0 and s, the probability must be estimated according to the rules of continuously varying quantities. The result in that case will be found identical with that given by our series, if in it we suppose s and n both infinite. The series then becomes

$$\frac{1}{1.2\,..\,(i-1)\,n}\left\{\left(\frac{s}{n}\right)^{i-1}-i\left(\frac{s}{n}-1\right)^{i-1}+\frac{i.(i-1)}{1.2}\left(\frac{s}{n}-2\right)^{i-2}-\&c.\right\},$$

and the sum of all these series will be found by integrating this expression between the limits 0 and s, which gives

$$\frac{1}{1.2\,....\,i}\left\{\left(\frac{s}{n}\right)^{i}-i\left(\frac{s}{n}-1\right)^{i}+\frac{i.(i-1)}{1.2}\left(\frac{s}{n}-2\right)^{i}-\&c.\right\},$$

the last or l^{th} term here being the last in which $\frac{s}{n}-(l-1)$ is positive.

This taken between the proper limits will give the probability that the sum of the numbers drawn is contained between those limits.

21. Laplace applies this equation, Théorie Anal. des Prob. p. 257, to finding the probability that the sum of the inclinations of the orbits of the planets would be contained within given limits, if all inclinations were similarly circumstanced.

There are 10 planets besides the Earth, namely, Mercury, Venus, Mars, Pallas, Juno, Ceres, Vesta, Jupiter, Saturn, and Uranus; therefore, $i=10$ and the sum of their inclinations to that of the Earth at the beginning of 1801 was 82° 16′ 36″, therefore

$$\frac{s}{n}=\frac{82\cdot 27683}{90}=\cdot 914187.$$

Retaining only the first term of the preceding expression, because $\frac{s}{n}-1$ is negative, the probability that the sum of the inclinations of the orbits of the planets would be comprised between the limits zero and 82°·27683, if all inclinations were similarly circumstanced, is

$$\frac{1}{1.2.3\,....\,10}\,.(\cdot 914187)^{10}=\cdot 00000011235.$$

The question we have just solved is nearly the same as to determine the probability of the losses of an insurance company upon i policies upon persons of the same age being contained within certain limits.

22. A shilling is tossed into the air; B gives A a certain sum, in consideration of which A engages to pay B 2 pounds if the shilling falls head the first time; 4 pounds if it falls head the second time, and not before; 2^n pounds if it falls head the n^{th} time, and not before.

The expectation of B is

$$2 \times \frac{1}{2} + 4 \times \frac{1}{4} + 8 \times \frac{1}{8} \dots + 2^n \times \frac{1}{2^n} = £n.$$

This is called the Petersburg problem, probably from the mention made of it by Daniel Bernoulli in the Transactions of the Petersburg Academy; it was first proposed by Montmort in the Analyse des Jeux de Hasard, and has been generally considered as involving a great paradox, because if it is agreed that the game shall not be discontinued till the shilling falls head, n must be made infinite, and the expectation of B is infinite; still no man of prudence would be disposed to venture even a small part of his fortune at this game. On account of the celebrity of this problem we have inserted it, but there is nothing paradoxical in the result more than in any other case of *long odds*: it shows, however, that in order to estimate the value of a contingent advantage to an individual, other elements must be considered, and that the moral expectation, as it is called, to distinguish it from the mathematical expectation, depends on a great many circumstances which it is very difficult to submit to calculation.

23. It is evident that the value of any sum is much greater to an individual who lives by his daily labour and is without any capital, than it is to an individual possessed of £100,000; we may, therefore, meet this difficulty by supposing that the value of a given sum to any individual, is proportional to and may be represented by that sum, divided by the whole of the fortune which he possesses.

Let a be the sum A is to pay B on the event of which the probability is p,
b . . . . B . . . . A . . . . q,
and let f be the fortune possessed by A. On the preceding hypothesis the expectation of A is

$$\frac{b\,q}{(f+b)} - \frac{a\,p}{(f-a)}.$$

If $b\,q = a\,p$, which must be the case in order that the wager may be *fair*, the expectation of A will be $> = < 0$ as

$$\frac{1}{f+b} - \frac{1}{f-a} > = < 0$$

$$f - a > = < f + b, \text{ or } a + b < = > 0;$$

a and b are both necessarily positive, therefore $a + b > 0$. It is, therefore, evident that according to this theory of the value of money, a wager under the most favourable circumstances consistent with honesty, injures the fortune of the gamester, because he loses more by losing than he can gain by gaining any wager, since the amount of it must be compared in the former case with his diminished, in the latter, with his augmented fortune.

24. Suppose the fortunes of A and B to be both equal to f, the expectation

of A is $$\frac{b\,q}{(f+b)} - \frac{a\,p}{(f-a)},$$

of B is $$\frac{a\,p}{(f+a)} - \frac{b\,q}{(f-b)}.$$

In order that the expectations of A and B may be equal

$$\frac{b\,q}{(f+b)} - \frac{a\,p}{(f-a)} = \frac{a\,p}{(f+a)} - \frac{b\,q}{(f-b)}$$

$$\frac{2\,f\,a\,p}{f^2-a^2} = \frac{2\,f\,b\,q}{f^2-b^2} \quad \frac{a}{b} = \frac{q}{p} \cdot \frac{f^2-a^2}{f^2-b^2}$$

$\frac{a}{b} > = < \frac{q}{p}$ as $f^2 - a^2 > = < f^2 - b^2$, as $b > = < a$; therefore if $a > b$, that is, if $p < q$, (a and b are the odds) a is not so much greater than b, as it would be if the moral expectation were not taken into account. In practice we believe this is always the case, as the reader may have observed in the example we took, page 9.

Ex. 14. Let p be the probability of a vessel coming into port, q the probability of a total loss, a the value of that part of the cargo in any vessel, which belongs to A, let there be n vessels, and f, as before, A's fortune. The mathematical value of A's expectation, the $n\,a$ goods being equally distributed over n vessels, is

$$n\,a\,p^n + (n-1)\,a\,n\,p^{n-1}\,q + (n-2)\,a\,.\,\frac{n\,(n-1)}{1\,.\,2}\,p^{n-2}\,q^2 + \&c.$$

$$= n\,a\,p\,\{\,p+q\,\}^{n-1} = n\,a\,p, \text{ since } p+q=1,$$

which is the same as if they were all on board the same vessel; but, on the preceding hypothesis, the value of A's expectation is

$$n\,a\,\frac{p^n}{f+n\,a} + (n-1)\,a\,.\,\frac{n\,p^{n-1}\,q}{f+(n-1)\,a} + (n-2)\,a\,.\,\frac{n\,.\,(n-1)}{1\,.\,2}\,\frac{p^{n-2}\,q^2}{f+(n-2)\,a} + \&c.$$

Where every denominator but the first is less than $(f+n\,a)$, and, therefore, this series is greater than the sum of the numerators divided by $(f+n\,a)$, that is, greater than $\frac{n\,a\,p\,.\,(p+q)^{n-1}}{f+n\,a}$, that is, than $\frac{n\,a\,p}{f+n\,a}$.

The value of A's expectation is, therefore, greater than $\frac{n\,a\,p}{f+n\,a}$, which is the value of his expectation when all his goods are on board the same vessel.

This principle of the distribution of risk is well known to every merchant, and, in fact, affords him all the advantages derived from insurance. It is on this principle that the East India Company never insure their vessels.

The hypothesis we have adopted in the preceding problems was suggested by the celebrated naturalist, Buffon.

25. Daniel Bernoulli supposes that the value of the fortune of any individual is made up of an infinite number of indefinitely small elements, the value of each of which is inversely as the capital already formed, so that if ϕ represent a small element of the fortune f, and f_1, f_2, &c., its successive amounts, the value of the whole fortune f will be

$$\frac{k\,\phi}{f_1} + \frac{k\,\phi}{f_2} + \ldots\ldots\ldots + \frac{k\,\phi}{f-\phi}$$ k being some constant quantity.

When ϕ is indefinitely diminished, this series $= k \log. \left(\frac{f}{f_1}\right)$. The divisor f_1 denotes that part of the fortune of the individual which is absolutely unalienable, and below which his fortune cannot sink. This theory affords us solutions to the problem preceding, and similar problems analogous to those obtained on the supposition of Buffon.

We shall not dwell longer on this subject, as these hypotheses, although they may serve, in some measure, to show the difference which exists between the mathematical value of any sum and its value in practice, are quite arbitrary.

26. The search after a method of enabling a gamester to win with certainty from his antagonist, who has a greater probability in his favour, has wasted as much ingenuity as the attempt to discover perpetual motion. At all gaming tables an advantage is given by the laws of the game to the banker; and many infatuated persons, in the vain hope of detecting some scheme for rendering that advantage nugatory, have spent years in registering the course of the play with a degree of patient industry which, exerted in another direction, might have made them useful and distinguished members of society. One favourite scheme is so celebrated as to have acquired a particular name; it is called the Martingale, or Double or Quits, and consists in doubling the last stake after every loss. In order that this may be permanently successful, the player requires not only an immense capital, but an unlimited permission of staking. It is not very easy to show mathematically the amount of the player's expectation who uses the martingale, on account of the various order in which the gains and losses may be supposed to follow each other; instead of attempting it, we shall give an analysis of another scheme, in which the same difficulty does not occur. This consists in increasing the stake by a fixed sum after every loss, and diminishing it by the same after every gain. The inventors of this mode of betting looked upon it as infallible, and indeed there is something in it which might easily deceive the unwary; for it can be shown, that if the number of games won and lost be the same, no matter in what order this takes place, the result is always a gain to the player who bets upon this principle. Notwithstanding this specious circumstance, we shall show that the value of the player's expectation of gain, when his probability of winning a single game exceeds $\frac{1}{2}$, is never so great as his expectation of loss when this probability falls short of $\frac{1}{2}$ by the same quantity.

27. Let a be the original stake, b the quantity by which it is increased or diminished after every loss or gain, $m+n$ the whole number of games played. The first thing to determine is the player's gain who wins m and loses n games.

His first stake $= a$, therefore his first gain $= \pm a$, and may be represented generally by $a(-1)^\alpha$ which becomes $\pm a$ as a is even or odd. If he wins, i. e. if a be even, his second stake is $a - b$, and it is $a + b$ if he loses, i. e. if a be odd: therefore his second stake generally is $a - b(-1)^\alpha$ and his second gain may be represented in the same general manner by $\{a - b(-1)^\alpha\}(-1)^\beta$. For the same reason his third stake will be $a - b.(-1)^\alpha - b.(-1)^\beta$, and therefore his third gain is $\{a - b((-1)^\alpha + (-1)^\beta)\}(-1)^\gamma$, and so on.

We have, therefore, the following table of gains:

$$\begin{aligned}
&\text{1st gain} = a(-1)^\alpha,\\
&\text{2d} \;.. \quad = a(-1)^\beta - b\{(-1)^\alpha\}(-1)^\beta,\\
&\text{3d} \;.. \quad = a(-1)^\gamma - b\{(-1)^\alpha + (-1)^\beta\}(-1)^\gamma,\\
&\text{4th} \;.. \quad = a(-1)^\delta - b\{(-1)^\alpha + (-1)^\beta + (-1)^\gamma\}(-1),\\
&\quad \&\text{c.} \qquad\qquad \&\text{c.} \qquad\qquad \&\text{c.}
\end{aligned}$$

There are $m + n$ stakes, and we have determined nothing of the quantities a, β, γ, &c. except that m are even, and n odd, because by supposition the player wins m games, and loses n. Therefore m of the quantities $(-1)^\alpha$,

$(-1)^{\beta}$, &c. will each $= +1$, and the remaining n each $= -1$. The coefficient of a in the sum of the gains is the sum of all these quantities, and therefore $= m - n$. The coefficient of b is the sum of the products of them all two by two, and therefore is equal to the coefficient of the third term of an equation which has m roots $= +1$, and n roots $= -1$, *i. e.* of $(x-1)^m (x+1)^n$, *i. e.*

$$= \frac{m \,.\, (m-1)}{1 \,.\, 2} - m\,n + \frac{n \,.\, (n-1)}{1 \,.\, 2} = \frac{(m-n)^2 - (m+n)}{2}.$$

Therefore, the player's gain $= (m-n)\,a + \dfrac{(m+n) - (m-n)^2}{2}\,b$; if $m = n$ this reduces itself to $\dfrac{(m+n)\,b}{2}$, and it is plain that these results are quite independent of the order in which the gains and losses follow each other. This very elegant solution was given by Mr. Babbage, in the Edinburgh Transactions for 1821; it remains now to estimate the player's expectation, whose probability of winning any single game $= p$. Let $m + n = i$. The player's gain, if he wins m games, $= (m-n)\,a + \dfrac{m + n - (m-n)^2}{1 \,.\, 2}\,b$ $= -\left\{i\,a + \dfrac{i\,(i-1)}{1 \,.\, 2}\,b\right\} + 2\,m\,\{a + (i-1)\,b\} - 2\,m \,.\, (m-1)\,b$, since $n = i - m$; and in order to get the player's expectation we must multiply this into the term of $\{p + (1-p)\}^i$ of which the argument is $p^m \,.\, (1-p)^{i-m}$, and take the sum of all those products; giving m, which is now the only variable, every value from 0 to i both inclusive. This product is

$$-\frac{i \,.\, (i-1) \,.\,.\,.\,.\,.\, 1}{1 \,.\, 2 \,.\,.\, m \,.\, 1 \,.\, 2 \,.\,.\, n}\,p^m\,(1-p)^{i-m}$$

$$\times \left\{\left\{i\,a + \frac{i\,(i-1)}{1 \,.\, 2}\,b\right\} - 2\,m\,\{a + (i-1)\,b\} + 2\,m\,(m-1)\,b\right\}.$$

Therefore the sum of all the values of this product is

$$\left\{i\,a + \frac{i \,.\, (i-1)}{1 \,.\, 2}\,b\right\} \times \text{the sum of all the values of} \,.\,.\, \left\{\frac{i \,.\, (i-1) \,.\,.\,.\,.\, 1}{1 \,.\, 2 \,.\,.\, m \,.\, 1 \,.\, 2 \,.\, n}\,p^m\,(1-p)^{i-m}\right.$$

$$+ 2 \,.\, \{a + (i-1)\,b\}\,i\,p \times \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\, \frac{(i-1) \,.\, (i-2) \,.\,.\,.\, 1}{1 \,.\, 2 \,.\, (m-1) \,.\, 1 \;\; 2 \,.\, n}\,p^{m-1}(1-p)^{i-m}$$

$$- 2\,b \,.\, i \,.\, (i-1)\,p^2 \times \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\, \frac{(i-2) \,.\, (i-3) \,.\,.\, 1}{1 \,.\, 2 \,.\, (m-2) \,.\, 1 \,.\, 2 \,.\, n}\,p^{m-2}(1-p)^{i-m}.$$

When every value from 0 to i inclusive is given to m, the sums of all the values of these three right-hand factors, rejecting those in which the index of p is negative, severally become $\{p + (1-p)\}^i$, $\{p + (1-p)\}^{i-1}$, and $\{p + (1-p)\}^{i-2}$, all equal to unity.

Therefore the sum of all the values of the products

$$= -\left\{i\,a + \frac{i \,.\, (i-1)}{1 \,.\, 2}\,b\right\} + 2\,i\,p \,.\, \{a + (i-1)\,b\} - 2\,i\,(i-1)\,p^2\,b$$

$$= i \,.\, (2\,p - 1) \,.\, a - \frac{i \,.\, (i-1)}{1 \,.\, 2}\,(2\,p-1)^2\,b.$$

If $p = \dfrac{1}{2} + x$, this expectation of gain $= 2\,i\,x\,a - 2\,i\,(i-1)\,a^2\,b$, and if $p = \frac{1}{2} - x$ the expectation of loss $= 2\,i\,x\,a + 2\,i\,(i-1)\,x^2\,b$. The expectation of a player, who is entirely ignorant of the value of p, is found by integrating the expression here found

$$\int \left\{ i\,a\,.\,(2\,p-1) - \frac{i.(i-1)}{1\,.\,2}\,b\,.\,(2p-1)^2 \right\} d\,p$$

$$= \frac{3\,i\,a\,.\,(2\,p-1)^2 - i\,.\,(i-1)\,.\,b\,.\,(2\,p-1)^3}{12}$$

and from $p = 0$ to $p = 1$, $= -\frac{i\,.\,(i-1)\,.\,b}{6}$.

It should be observed that this solution only applies to the case when there is a limiting equation between a and b, such that $a-(i-1)\,b > 0$, otherwise there might be a conjunction in the game, in which the player could not follow the rules of this scheme, and consequently would alter his expectation. If this be not attended to, the theorem supposes, what can never take place in practice, that the player has the power of reducing his stake below zero, that is, of taking his adversary's situation in some point of the game.

28. Let there be two conflicting events P and Q, of which the probabilities are p and q respectively, and let $m+n$ trials take place; the probability that the event P will happen m times, and the event Q, n times, without regard to the order in which they succeed each other, is

$$\frac{(m+n)\,.\,(m+n-1)\,\ldots\ldots\,2\,.\,1}{1\,.\,2\,\ldots\ldots\,m\,.\,1\,.\,2\,\ldots\ldots\,n}\,p^m\,.\,q^n,$$

which we shall represent by p_m. Similarly, the probability that the event P will happen $m+1$ times and the event Q, $(n-1)$ times, is

$$\frac{(m+n)\,.\,(m+n-1)\,\ldots\ldots\,2\,.\,1}{1\,.\,2\,..\,(m+1)\,1\,.\,2\,..\,(n-1)}\,p^{m+1}\,.\,q^{n-1} = p_{m+1}.$$

If $p_{m+1} > p_m$, $\frac{p_{m+1}}{p_m} > 1$; $p > \frac{(m+1)\,.\,q}{n}$; and since $p+q=1$, $m > (m+n)\,p - q$, and this continues until $m = (m+n)\,p - q$, in which case $p_m = p_{m+1}$; and, since m must be a whole number, the greatest term in the developement of $(p+q)^{m+n}$ is that when $m = (m+n)\,p - q$, if $(m+n)\,p - q$ be a whole number, or. if not a whole number, the next greater.

Suppose, for instance, $p = \frac{2}{3}$ and consequently $q = \frac{1}{3}$, and let $m+n = 17$, m is $\frac{17 \times 2 - 1}{3}$, or $\frac{33}{3} = 11$; so that the most probable event, as compared with any other event which can occur in 17 trials, is a repetition of P 11 times, and Q 6 times.

If $m+n = 18$, m is the whole number next greater than $\frac{18 \times 2 - 1}{3}$, $= \frac{35}{3}$; which is 12, and the most probable event as compared with any other which can happen in 18 trials, is a repetition of P 12 times, and of Q 6 times.

When $m+n=\frac{w}{p}$, w being a whole number; since m is the next whole number greater than $(m+n)\,p-q$, and since $(m+n)\,p=w$, and q is necessarily a proper fraction, $(m+n)\,p$ or w is the next whole number greater than $(m+n)\,p-q$, and, therefore, $m=w=(m+n)\,p$, $n=(m+n)\,(1-p)$ $=(m+n)\,q$; $\frac{m}{n}=\frac{p}{q}$, that is to say, the event most likely to happen is a combination in which the number of repetitions of p and q is proportional to the simple probability of the happening of each.

29. It is only as compared with any other single combination, that the one we have just mentioned increases in probability with the number of trials: if we estimate the abstract probability of the event corresponding with this *maximum* term, we shall easily find that it diminishes as the number of trials increases.

For this purpose let the maximum term in $m+n$ trials be represented by p_m, we have already seen that

$$p_m=\frac{(m+n).(m+n-1)\ldots.2.1}{1.2\ldots\ldots m\,..\,1.2\ldots.n}\,p^m\,q^n,$$

where $m>=(m+n)\,p-q$, that is $>=(m+n+1)\,p-1$,
and $<(m+n+1)\,p$.

In one more trial

$$p_{m'}=\frac{(m+n+1).(m+n)\ldots2.1}{1.2\ldots m'\ldots1.2\ldots n'}\,p^{m'}.q^{n'},$$

m' being limited in the same manner,

$$m'>=(m+n+2)\,p-1, \text{ that is } >m+p-1,$$
$$<(m+n+2)\,p, \qquad \text{that is } <m+p+1,$$
$$\therefore\ m' \text{ is either } m \text{ or } m+1.$$

If $m'=m$, $\quad \frac{p_{m'}}{p_m}=\frac{m+n+1}{n+1}q=\frac{n+1+m-(m+n+1)\,p}{n+1}<1.$

If $m'=m+1$, $\frac{p_{m'}}{p_m}=\frac{m+n+1}{m+1}\,.\,p=<1,$

$\therefore$ in both cases, which will go on occurring successively, $p_{m'}=<p_m$.

30. There is, however, another probability connected with the most probable combination, which does increase continually with the number of trials, for it is always possible to assign a number of trials, such as to give any required probability that the difference between the ratio of the number of repetitions of the events, and the simple probabilities of the events, shall lie within any given limits. Thus, if there are 3 white balls in a bag, and 2 black balls, we can always assign a number of trials such as to give any probability as near as we please to certainty, that the difference between $\frac{3}{2}$ and the ratio of the number of white balls drawn to the number of black balls, shall lie between given limits, however near those limits may be assumed. This is a theorem of the highest importance in the theory of probability, and indeed, it is on the converse of it that the value of experience depends. We shall endeavour to prove it as shortly as possible, and it will facilitate this object if, instead of representing the simple probabilities by p and q, as we have hitherto done, we express them by the two fractions $\frac{a}{a+b}$ and $\frac{b}{a+b}$, in which a and b are both whole numbers.

31. The events of which the probabilities are $\frac{a}{a+b}$, $\frac{b}{a+b}$ would be repeated in $m\,(a+b)$ trials exactly $m\,a$ and $m\,b$ times, if they were combined in the ratio of their simple probabilities. Let us suppose that the number of times that the first will be repeated in the observed event lies between the limits $m\,a - m$ and $m\,a + m$, that is to say, that there will not be fewer than $m\,a - m$ and not more than $m\,a + m$ recurrences of that event in the $m\,(a+b)$ trials. The probability that this will be the case is the sum of the $2\,m + 1$ terms in the developement of $\left(\frac{a}{a+b}+\frac{b}{a+b}\right)^{m\,(a+b)}$ from the term whose argument is a^{ma-m} to the term whose argument is a^{ma+m}, both inclusive. We shall call these two last-mentioned terms the first and second limiting terms, and it is clear that the maximum term, the argument of which is $a^{m\,a}$, lies between them.

32. The whole series may thus be written out at length:

$$\left(\frac{a}{a+b}+\frac{b}{a+b}\right)^{m\,(a+b)} = p_{m\,(a+b)} + p_{m\,(a+b)-1} \cdots\cdots + p_{ma+m-1}$$

First limiting term. Maximum. Second limiting term.

$$\boxed{\begin{array}{l} + p_{ma+m} \cdots\cdots + p_{ma+1} \\ \text{Parcel of the first limit.} \end{array}} + p_{m\,a} \boxed{\begin{array}{l} + p_{ma-1} \cdots\cdots + p_{ma-m} \\ \text{Parcel of the second limit.} \end{array}}$$

$$+ p_{ma-m-1} \cdots\cdots\cdots\cdots\cdots\cdots + p_1 + p_0,$$

the index at the foot of p always denoting the power to which a is involved in that term. Our object is to show that the $2\,m + 1$ terms within the limits, may be made as many times greater than the rest of the series as we please; and we shall do this by showing that the first m of these $2\,m + 1$ terms, which we will call the parcel of the first limit, can be made as many times greater as we please than all which precede them, and the last m terms, or the parcel of the second limit, as many times greater as we please than all which follow them.

33. There are $m\,b - m$ terms which precede our first limit, which may be classed in $(b - 1)$ parcels, each containing m successive terms, and similarly the $m\,a - m$ terms, after the second limit, may be classed in $(a - 1)$ parcels, each containing m successive terms. As the maximum term $p_{m\,a}$ is in the middle of our limits, and as the values of the terms increase from each end of the series up to the maximum term, the sum of all the $(b - 1)$ parcels before the first limit will be less than $(b - 1)$ times the parcel next before the first limit; and the sum of all the $(a - 1)$ parcels beyond the second limit will be less than $(a - 1)$ times the parcel next following the second limit. It is also plain, for the same reason, that the parcel of the first limit is greater than the parcel which immediately precedes the first limit, and, since the ratio of the maximum term to the first limiting term, or the ratio of p_{ma} to p_{ma+m}, is less than the ratio of any term in the parcel of the first limit, to the corresponding term in the parcel next preceding the first limit, (because these ratios continually approach nearer to a ratio of equality as they approach the maximum term,) it follows that the ratio of $p_{m\,a}$ to p_{ma+m} is less than the ratio of the whole parcel of the first limit to the whole parcel immediately preceding it.

34. If, therefore, we can show that $p_{m\,a}$ can, by a proper assumption of m, be made greater than $i.(b - 1)$ times p_{ma+m}, however great i is taken, it will follow that this value of m will make the parcel of the first limit still

greater than $i.(b-1)$ times the parcel immediately preceding it, and very much greater than i times the sum of all the parcels which precede it. Exactly the same reasoning will show that if p_{ma} can be made greater than $i.(a-1)$ times p_{ma-m}, that is, $\frac{p_{m,a}}{p_{ma-m}} > i.(a-1)$ the parcel of the second limit, will, with the same value of m, be more than i times greater than all the parcels which follow it.

35. Let a be that one of the quantities a and b, which is not the least. A value of m, which will make p^{ma} greater than $i.(a-1)$ times p_{ma-m}, and than $i.(a-1)$ times p_{ma+m}, will evidently satisfy both the above-mentioned conditions.

$$\frac{p_{ma}}{p_{ma+m}} = \frac{(ma+1)\,..\,(ma+m)}{(mb-\{m-1\})\,..\,mb}\left(\frac{b}{a}\right)^m = \frac{(mab+b)\,..\,(mab+mb)}{(mab-\{m-1\}a)\,..\,mab}.$$

The last pair of factors here is $\frac{mab+mb}{mab}$, or $\frac{a+1}{a}$, and any other pair, such as the r^{th}, is

$$\frac{mab+rb}{mab-(m-r)a}, \text{ which is} > \frac{a+1}{a},$$

if
$$mab+rb > (a+1)(mb-m+r),$$

that is, if
$$b < a+1,$$

which by supposition it is. Therefore

$$\frac{p^{ma}}{p_{ma+m}} > \left(\frac{a+1}{a}\right)^m,$$

for there are m factors in this continued product.

$$\frac{p_{ma}}{p_{ma-m}} = \frac{(mb+1)\,..\,(mb+m)}{(ma-\{m-1\})\,..\,ma}\left(\frac{a}{b}\right)^m = \frac{(mab+a)\,..\,(mab+ma)}{(mab-.\{m-1\}b)\,...\,mb}.$$

Any factor, as the r^{th}, here

$$= \frac{mab+ra}{mab-(m-r).b}, \text{ which is} > = \frac{a+1}{a},$$

if
$$a^2(mb+r) > = (a+1).b(ma-\{m-r\}),$$

that is, if
$$ra(a-b)+(m-r).b > = 0,$$

which it must always be, for neither $a-b$, nor $m-r$, can ever become negative.

$$\therefore \frac{p_{ma}}{p_{ma-m}} > \left(\frac{a+1}{a}\right)^m. \quad \text{Assume} \left(\frac{a+1}{a}\right)^m = i.(a-1),$$

or
$$m = \frac{\log. i + \log. (a-1)}{\log. (a+1) - \log. a},$$

and with this value of m both the necessary conditions are satisfied.

36. Therefore the probability

$$= \frac{\text{1st parcel} + \text{2d parcel} + a_m}{\text{1st parcel} + \text{2d parcel} + a_m + \text{sum of other parcels}},$$

or since 1st parcel + 2d parcel $> i.$(sum of other parcels)

$$= i \text{ (sum of other parcels)} + k,$$

our probability

$$= \frac{i.(\text{sum of other parcels}) + k + a_m}{(i+1)\,(\text{sum of other parcels}) + k + a_m} > \frac{i}{i+1},$$

since every proper fraction is diminished by taking the same quantity (in this instance $k + a_m$) from its numerator and denominator.

37. We thus have a probability, $> \frac{i}{i+1}$, that the number of recurrences of the event, whose probability is $\frac{a}{a+b}$, will, in $m.(a+b)$ trials, lie between the numbers $(m\,a + m)$, $(m\,a - m)$, however great i be taken, and the limits of the ratio of the number of repetitions of this event to the number of trials are $\frac{m\,a+m}{m.(a+b)}$ and $\frac{m\,a-m}{m.(a+b)}$, or $\frac{a+1}{a+b}$ and $\frac{a-1}{a+b}$, the difference of which is $\frac{2}{a+b}$; and since the only restriction on the value of a and b is, that they must be in the proportion of p to q, we may increase $a+b$ at pleasure, and thus bring the limits of this ratio as close as we please, and yet have a probability, $> \frac{i}{i+1}$, that the observed ratio will be between them. If all that is required is that there will not be fewer than $m\,a - m$ recurrences, the probability clearly becomes much greater, as also the probability that there will not be more than $m\,a + m$.

It only remains to give an example to show fully how this theorem is applied.

Ex. 15. A bag contains three white balls and two black balls, from which a ball is repeatedly drawn and replaced. Required the number of trials in which the odds will be at least 1000 to 1 that the ratio of the number of times that a white ball is drawn to the whole number of drawings is not less than $\frac{29}{50}$, and not greater than $\frac{31}{50}$.

The value of i is here 1000; $a = 30$, and $b = 20$.

$$\therefore m = \frac{\log. i + \log. (a-1)}{\log. (a+1) - \log. a} = \frac{\log. 1000 + \log. 29}{\log. 31 - \log. 30}$$

$$= \frac{4{\cdot}462398}{0{\cdot}0142404} > 313 < 314.$$

Therefore we must take $m = 314$, and the number of trials $= m\,(a+b) = 15700$, in which number of drawings the odds will be more than 1000 to 1 that a white ball will have been drawn not less than $m\,a - m$, or 9106 times, and not more than $m\,a + m$, or 9734 times.

If the odds had been required to be at least 10,000 to 1, or 100,000 to 1, we need only change the value of i, which gives, in the first case,

$$m = \frac{5{\cdot}462398}{0{\cdot}0142404} < 384 \text{ and } m\,(a+b) = 19200;$$

and in the second case,

$$m = \frac{6{\cdot}462398}{0{\cdot}0142404} < 454 \text{ and } m\,(a+b) = 22700,$$

which serves to show how much more rapidly the probability increases than the number of trials; for after the first 15,700 trials, the addition of 3500 new trials increases the odds tenfold, and 3500 more increases them tenfold again.

38. This highly important theorem is due to James Bernoulli, a celebrated mathematician of the last century, whose name it bears.

39. We have supposed known the number of favourable and unfavourable cases similarly circumstanced in the problems we have hitherto considered. Thus, in the problems relating to dice, we took for granted the form of the dice, and also their homogeneity.

40. However nearly any die may fulfil these given conditions, it will not do so strictly; and if we investigate the probability of any throw upon the principles hitherto developed, we obtain a result approximately correct, and of which the error depends on the inaccuracy of our hypothesis. The knowledge that a defect of homogeneity is possible, renders the return of the same face in several repeated trials more probable than it would be, if the die were known to be homogeneous. In tossing up a shilling, the probability of its falling heads, or the reverse, twice successively, is rather greater than $\frac{1}{4}$.

41. If such considerations apply to these very simple questions, it will readily be seen how difficult it is to estimate mathematically those probabilities which depend on more complicated circumstances; as the probability of an individual living a given number of years, or the probability of the truth of any assertion. Truths of definition are the only certain propositions. We shall not stop to inquire whether any limit separates truths of definition from propositions which rest upon experience; the distinction however may be admitted.

42. It is impossible to suppose that "*a part is greater than the whole*" without involving a contradiction to the sense in which the words forming this sentence are understood, so that the truth of the proposition, that "*a part is less than the whole*" results from the very definition of the words which compose this sentence. If on the other hand we consider the proposition that the sun will rise to-morrow; the number of times we believe the sun to have risen daily without interruption induces us to believe that the sun will rise again. Most of our opinions arise from our experience of the past, and rest upon probabilities of this kind.

43. In order to obtain mathematical solutions of problems similar to these we must revert to games of chance. Any problem in chances may be represented by throws with dice of different forms, or by drawings from bags containing balls of different colours. Nor is it any objection to the results we obtain by this means, that no dice can be formed which exactly fulfil the conditions we suppose them to do, any more than it is to the theorems in Euclid that the lines which compose the diagrams are not mathematically straight.

44. When our knowledge of the number of cases similarly circumstanced is imperfect, the probability of an event is still deduced upon the same principles as those hitherto developed. We have recourse to *hypotheses*, and having estimated their probability, the probability of any future event which depends on them, is easily deduced.

45. The probability of each hypothesis by definition is the number of cases which favour this hypothesis divided by the whole number of cases possible.

Ex. 16. Let us suppose a bag to contain three balls, and that we are uncertain whether of these balls two are black and one is white, or one is black and two are white, and that a white ball has been drawn.

Let us call these balls 1, 2, 3, and let us also suppose that the uncertainty is with respect to the colour of No. 2.

	No. 1.	No. 2.	No. 3.
1st hypothesis,	black,	black,	white.
2nd ,,	black,	white,	white.

On the first hypothesis No. 3 must have been drawn; we have, therefore, only one case which favours this hypothesis. On the second hypothesis, either No. 2 or No. 3 has been drawn, so that we have two cases which favour this hypothesis, and, therefore, the probabilities of these hypotheses respectively are $\frac{1}{1+2}$ and $\frac{2}{1+2}$, or $\frac{1}{3}$ and $\frac{2}{3}$.

In order to extend the principle of this reasoning to the general case, let us suppose that an event has been observed which must have resulted from one of a given number of causes. Let the probability of the existence of one of these causes have been estimated at P before the observed event took place, and let the probability of the observed event calculated upon the certitude of that cause be called p.

The probability that the event will happen in consequence of that cause $= P\,p$, and the probability that the event will happen, without reference to any particular cause $= \Sigma . P\,p$; extending the sign of summation to all the possible causes.

The probability that the event will happen in consequence of the selected cause, (or $P\,p$), may be estimated in a different manner: it equals the product of the probability that it will happen, (or $\Sigma . P p$), by the probability that if it does happen, it will be in consequence of that cause: the latter is evidently the probability of the selected cause derived from the observation. Therefore the derived probability of that cause $= \frac{P\,p}{\Sigma . P\,p}$, which result may be stated in the following important theorem.

The probability of any hypothesis is the probability of the observed event upon this hypothesis multiplied by the probability of the hypothesis antecedently to the observation divided by the sum of the products which are formed in the same manner from all the hypotheses.

46. The probability antecedent to the observations under consideration is called the *à priori* probability; but in using this term it must be remembered that it is relative to a given epoch.

Ex. 17. Thus in the instance of a bag containing three balls, of which a white one has been once drawn and replaced. There are three possible suppositions: first, all three are white; second, two only are white; third, only one is white. On the first supposition the probability of the event observed is certainty or 1, on the second the probability is $\frac{2}{3}$, on the third it is $\frac{1}{3}$.

Therefore the probability of the first hypothesis $= \dfrac{1}{1+\frac{2}{3}+\frac{1}{3}} = \dfrac{1}{2}$,

,, ,, second ,, $= \dfrac{\frac{2}{3}}{1+\frac{2}{3}+\frac{1}{3}} = \dfrac{1}{3}$,

,, ,, third ,, $= \dfrac{\frac{1}{3}}{1+\frac{2}{3}+\frac{1}{3}} = \dfrac{1}{6}$.

It is worth observing that these conclusions would not be affected by retaining all the four hypotheses which might have been made before the observation. For the probability of the observed event on the hypothesis we have rejected, namely, that all three balls are white, being $= 0$, the probability of the others will not be altered by including it also in the sum of probabilities which make up the denominator of the above-written fractions.

47. Let the ratio of the white balls to the whole number of balls be any of the following quantities,

$$x \dots\dots 2x \dots\dots 3x \dots\dots ix,$$

let any of these hypotheses be equally probable *à priori*, and let a white ball have been drawn.

The probability of the event observed, namely, the drawing a white ball, on the first hypothesis is x, the *à priori* probability of this hypothesis is $\frac{1}{i}$, because there are i hypotheses equally probable; therefore the probability of this hypothesis is

$$\frac{\frac{x}{i}}{\frac{1}{i}\, x\, \{1+2+3\dots i\}} = \frac{2}{i\,(i+1)}.$$

Similarly, the probability of the second hypothesis is $\frac{2 \times 2}{i\,(i+1)}$, of the third is $\frac{3 \times 2}{i\,(i+1)}$, and so on.

The probability of drawing a white ball in a future trial, after replacing the ball drawn, if the first hypothesis were the true one, would be x; the probability of this hypothesis is $\frac{2}{i\,(i+1)}$; therefore, the probability of drawing a white ball, considering only this hypothesis, is $\frac{2\,x}{i\,(i+1)}$

Similarly, considering the second hypothesis, the probability is $\frac{2\,x \times 2^2}{i\,(i+1)}$,

" third " $\frac{2\,x \times 3^2}{i\,(i+1)}$,

and the sum of all these, or the probability of drawing a white ball again, is

$$\frac{2\,x}{i\,(i+1)} \{1 + 2^2 + 3^2 \dots\dots + i^2\}.$$

In order to find the sum of this series, we shall employ a method of greater generality than is necessary in this particular case, because it furnishes the readiest method of summing a great many series of the same kind.

$$e^x = 1 + x + \frac{x^2}{1.2} + \&c.$$

$$e^{2x} = 1 + 2\,x + \frac{2^2 \times x^2}{1.2} + \&c.$$

$$e^{ix} = 1 + ix + \frac{i^2 x^2}{1.2} + \&c.$$

$$\therefore 1 + 2^2 + 3^2 \ldots + i^2 \text{ is the coefficient of } \frac{x^2}{2} \text{ in } e^x + e^{2x} \ldots e^{ix},$$

$$\text{,, \quad ,, \quad ,, \quad or } \frac{e^x(e^{ix} - 1)}{e^x - 1} \text{ or } \frac{e^{ix} - 1}{1 - e^{-x}},$$

$$\text{,, \quad ,, \quad ,, \quad or } \frac{i + \frac{i^2 x}{2} + \frac{i^3 x^2}{6} + \&c.}{1 - \frac{x}{2} + \frac{x^2}{6} - \&c.}$$

If we effect the division of $\frac{1}{1 - \frac{x}{2} + \frac{x^2}{6} - \&c.}$ the three first terms will be found to be $1 + \frac{x}{2} + \frac{x^2}{12}$, and all beyond involve higher powers of x than the square, and therefore need not be considered. Multiplying $i + \frac{i^2 x}{2} + \frac{i^3 x^2}{6}$ by $1 + \frac{x}{2} + \frac{x^2}{1.2}$, we get for the coefficient of $\frac{x^2}{2}$, $\frac{2i^3 + 3i^2 + i}{6}$, therefore

$$1 + 2^2 \ldots + i^2 = \frac{2i^3 + 3i^2 + i}{6} = \frac{i(i+1)(2i+1)}{6},$$

and the probability in question is

$$\frac{2x}{i \,.\, i+1} \left\{\frac{i(i+1)(2i+1)}{6}\right\} = \frac{(2i+1)\,.\,x}{3}.$$

When i is very great, this fraction approximates to $\frac{2ix}{3}$; if, therefore, the ratio of the white balls may be any ratio between 0 and unity, that is, if we have no data to determine that some of these values are more probable than others, $ix = 1$, and this probability is $\frac{2}{3}$.

48. Let the ratio of the white balls to the whole number of balls, be any of the following, $\Delta x, 2\Delta x, 3\Delta x \ldots\ldots i\Delta x$, and consequently the ratio of the black balls to the whole number of balls,

$$1 - \Delta x, 1 - 2\Delta x, 1 - 3\Delta x \ldots\ldots 1 - i\Delta x,$$

and let m white balls have been drawn, and n black, in any given order.

The probability of the event observed on the hypothesis that $i\Delta x$ is the ratio of the white balls to the whole number of balls is

$$\frac{(m+n)\,.\,(m+n-1)\ldots\ldots 1}{1\,.\,2\,.\,3\ldots m\,.\,1\,.\,2\,.\,3\ldots n} \times (i\Delta x)^m (1 - i\Delta x)^n,$$

and the probability of this hypothesis is

$$\frac{(i\Delta x)^m (1 - i\Delta x)^n}{\Delta x^m (1 - \Delta x)^n + (2\Delta x)^m (1 - 2\Delta x)^n + \&c.}$$

The probability of drawing m' white balls, and n' black balls, in $m' + n'$ future trials upon this hypothesis, is

$$\frac{(m'+n')\,(m'+n'-1)\,\ldots\ldots\,1}{1\,.\,2\,.\,3\,\ldots\,m'\,.\,1\,.\,2\,.\,3\,\ldots\,n'}\,(i\,\Delta\,x)^{m'}\,(1-i\,\Delta\,x)^{n'},$$

multiplying this by the probability of the hypothesis, the probability of drawing m' white balls and n' black balls, considering only this hypothesis, is

$$\frac{(m'+n')\,(m'+n'-1)\,\ldots\ldots\,1}{1\,.\,2\,.\,3\,..\,m'\,.\,1\,.\,2\,.\,3\,\ldots\ldots\,n'}$$

$$\times\,\frac{(i\,\Delta\,x)^{m+m'}\,(1-i\,\Delta\,x)^{n+n'}}{\Delta\,x^{m}\,(1-\Delta\,x)^{n}+(2\,\Delta\,x)^{m}\,(1-2\,\Delta\,x)^{n}+\&c.},$$

and the probability of drawing m' white balls and n' black balls, considering all the hypotheses, is

$$\frac{(m'+n')\,(m'+n'-1)\,\ldots\,1}{1\,.\,2\,.\,3\,..\,m'\,.\,1\,.\,2\,.\,3\,..\,n'}$$

$$\times\left\{\frac{\Delta\,x^{m+m'}\,(1-\Delta\,x)^{n+n'}+(2\,\Delta\,x)^{m+m'}\,(1-2\,\Delta\,x)^{n+n'}+\&c.}{\Delta\,x^{m}\,(1-\Delta\,x)^{n}+(2\,\Delta\,x)^{m}\,(1-2\,\Delta\,x)^{n}+\&c.}\right\}.$$

Let $\Delta\,x$ be infinitely diminished, and let $i\,\Delta\,x=1$, that is, let any ratio of white balls to the whole number of balls contained in the bag to $\Delta\,x$ be equally possible between zero and unity, then this expression becomes

$$\frac{(m'+n')\,.\,(m'+n'-1)\,\ldots\ldots\,1}{1\,.\,2\,.\,3\,..\,m'\,..\,1\,.\,2\,.\,3\,..\,n'}\,\frac{\int x^{m+m'}\,(1-x)^{n+n'}\,d\,x}{\int x^{m}\,(1-x)^{n}\,d\,x},$$

taken between the limits $x=0$ and $x=1$.

$$\int x^{m}\,(1-x)^{n}\,d\,x=\frac{x^{m+1}}{m+1}\,(1-x)^{n}+\frac{n}{m+1}\int x^{m+1}\,(1-x)^{n-1}\,d\,x$$

$$=\frac{x^{m+1}}{m+1}\,(1-x)^{n}+\frac{n}{(m+1)\,(m+2)}\,x^{m+2}\,(1-x)^{n-1}$$

$$+\frac{n\,.\,(n-1)}{(m+1)\,(m+2)}\int x^{m+2}\,(1-x)^{n-2}\,d\,x$$

$$=\frac{x^{m+1}}{m+1}\,(1-x)^{n}+\frac{n}{(m+1)\,(m+2)}\,x^{m+2}\,(1-x)^{n-1}$$

$$+\frac{n\,.\,(n-1)}{(m+1)\,(m+2)\,(m+3)}\,x^{m+3}\,(1-x)^{n-2}$$

$$\ldots\ldots+\frac{n\,.\,(n-1)\,.\,(n-2)\,\ldots\,1}{(m+1)\,(m+2)\,..\,(m+n+1)}\,x^{m+n+1}.$$

When $x=0$, all these terms vanish; when $x=1$, all vanish except the last; therefore the integral required, taken between the limits $x=0$ and $x=1$, is

$$\frac{n\,(n-1)\,(n-2)\,\ldots\ldots\,1}{(m+1)\,(m+2)\,(m+3)\,..\,(m+n+1)},$$

$\int x^{m+m'}\,(1-x)^{n+n'}\,d\,x$, taken between the same limits, is

$$\frac{(n+n')\,(n+n'-1)\,(n+n'-2)\,\ldots\ldots\,1}{(m+m'+1)\,(m+m'+2)\,\ldots\,(m+m'+n+n'+1)},$$

therefore the probability required is

$$\frac{(m+n')\,(m+n'-1)\,\ldots\ldots\,1}{1\,.\,2\,.\,3\,..\,m'\,.\,1\,.\,2\,.\,3\,\ldots\,n'}$$

$$\times\,\frac{(n+n')\,(n+n'-1)\,\ldots\ldots\,1}{(m+m'+1)\,(m+m'+2)\,..\,(m+m'+n+n'+1)}$$

$$\times \frac{(m+1)\,(m+2)\,..\,(m+n+1)}{n\,.\,(n-1)\,......\,1}$$

$$= \frac{(m'+n')\ (m'+n'-1)\,.....\,1}{1\,.\,2\,.\,3\,....\,m'\,..\,1\,.\,2\,.\,3\,....\,n'}$$

$$\times \frac{(n+1)\,(n+2)\,..\,(n+n')\,(m+1)\,(m+2)\,..\,(m+m'+1)}{(m+n+2)\,..\,(m+n+3)\,..\,(m+n+4)\,..\,(m+m'+n+n'+1)}.$$

If $(n+1)\,(n+2)\ \ ....\ \ (n+n')$ be represented by $[n+1]^{n'}$
$(m+1)\,(m+2)\,....\,(m+m')$,, ,, $[m+1]^{m'}$
and $(m+n+2)\,(m+n+3)\,......\,(m+m'+n+n'+1)$ by $[m+n+2]^{m'+n'}$, this probability is expressed by

$$\frac{(m'+n')\,(m'+n'-1)\,....\,1}{1\,.\,2\,.\,3\,....\,m'\,..\,1\,.\,2\,.\,3\,...\,n'}\ \frac{[n+1]^{n'}\,[m+1]^{m'}}{[m+n+2]^{m'+n'}}.$$

Which result in this form may be easily remembered, by observing that it is the same (with the difference of notation) as if the simple probability of drawing a white ball was $\frac{m+1}{m+n+2}$ and the probability of drawing a black ball was $\frac{n+1}{m+n+2}$.

Ex. 18. Let us suppose the sun to have risen 2000000 times, or days, then the probability that it will rise again is given by the preceding formula, by making $p = 2000000$, and $q = 0$, the probability required is $\frac{2000001}{2000002}$.

This probability, which is already very great, must be very considerably increased, if the discoveries of physical astronomy are taken into account.

49. If a white ball has been drawn p times, and a black ball q times, the probability of drawing a white ball in a future trial is $\frac{p+1}{p+q+2}$, and the probability of drawing a black ball is $\frac{q+1}{p+q+2}$; the greater p and q become, the more nearly do these fractions coincide with $\frac{p}{p+q}$ and $\frac{q}{p+q}$, which are their limits when p and q are indefinitely increased. This theorem is the converse of Bernoulli's theorem, of which we gave a demonstration, p. 21.

Ex. 19. We said, that if a shilling was tossed into the air, the probability of its falling heads, or the reverse, twice running, was rather greater than $\frac{1}{4}$.

Let the probability of the shilling falling heads, be any of the following quantities:

$$\frac{1}{2} - i\,x,\ \ \frac{1}{2} - (i-1)\,x,\ \ \frac{1}{2} - x.\ \ \frac{1}{2},\ \frac{1}{2} + x,\ \ \frac{1}{2} + (i-1)\,x,\ \ \frac{1}{2} + i\,x.$$

The number of hypotheses is $2\,i + 1$, and the probability of the shilling falling heads twice running is

$$\frac{1}{2\,i+1}\left\{\begin{array}{l}\left(\frac{1}{2} - i\,x\right)^2 + \left\{\frac{1}{2} - (i-1)\,x\right\}^2 \,......\, + \left\{\frac{1}{2} - x\right\}^2 + \left(\frac{1}{2}\right)^2 \\ + \left(\frac{1}{2} + i\,x\right)^2 + \left\{\frac{1}{2} + (i-1)\,x\right\}^2 \,......\, + \left\{\frac{1}{2} + x\right\}^2\end{array}\right\}$$

$$= \left(\frac{1}{2}\right)^2 + \frac{2\,x^2}{2\,i+1}\left\{1 + 2^2 + 3^2 \ldots\ldots i^2\right\}$$

$$= \left(\frac{1}{2}\right)^2 + \frac{2\,x^2}{2i+1}\left(\frac{i\,.\,(i+1)\,.\,(2\,i+1)}{2\,.\,3}\right) = \left(\frac{1}{2}\right)^2 + \frac{i\,.\,(i+1)\,.\,x^2}{3}$$

which probability is greater than $\left(\frac{1}{2}\right)^2$, the result when the shilling is supposed homogeneous.

Ex. 20. It follows, from what has preceded, that if an individual has made $m + n$ assertions, of which m are true and n are false; the probability of his telling the truth, in any case, is $\frac{m+1}{m+n+2}$, so far as we draw our conclusions from these assertions alone.

Let $\frac{m+1}{m+n+2} = v$, and let p be the *à priori* probability of an event which he asserts to have taken place.

The event observed is the assertion by this individual that the event took place, of which the *à priori* probability is p.

If the event did take place, the individual tells the truth; the probability of the event on this hypothesis is $p\,v$.

The probability of the event on the contrary hypothesis is $(1-p)\,(1-v)$, therefore the probability of this hypothesis is

$$\frac{p\,v}{p\,v + (1-p)\,(1-v)}.$$

If

$$\frac{p\,v}{p\,v + (1-p)\,(1-v)} > p, \quad v > \frac{1}{2}.$$

Thus we see that when a witness asserts that an event has taken place, he renders the probability that it did take place greater than the simple probability of the event only when his veracity is greater than $\frac{1}{2}$, which result might have been foreseen.

Ex. 21. A witness asserts that out of a bag containing a thousand tickets, a given ticket, say No. 70, has been drawn, required the probability that this number was drawn.

Let the veracity of the witness be v, as before.

The event observed is the assertion by the witness that the given number was drawn, the probability of this event, on the hypothesis that the witness tells the truth, is $\frac{v}{1000}$, the probability of the event on the contrary hypothesis, namely, that this ticket was not drawn, is $\frac{999}{1000}\,(1-v)$; but if the witness is supposed to have no reason or inducement for choosing the No. 70 in preference to any other of the 999 undrawn numbers, this probability must be multiplied by $\frac{1}{999}$, which is the probability of the witness choosing this number from the 999 undrawn numbers, so that the probability of the event on this hypothesis is $\frac{1-v}{1000}$.

The probability, therefore, of the first hypothesis is $\frac{v}{v + 1 - v}$, or v, the veracity of the witness.

Ex. 22. Two individuals, whose veracities are v and v', assert that an event has taken place, of which the probability is p.

Two hypotheses are admissible, namely, that the event did take place, and that both the individuals tell the truth; or, that the event did not take place, and that both individuals lie; the probabilities of the assertions on these hypotheses are $v\,v'\,p$ and $(1 - v)\ (1 - v')\ (1 - p)$, therefore the probabilities of these hypotheses are

$$\frac{v\,v'\,p}{v\,v'\,p + (1 - v)\ (1 - v')\ (1 - p)} \text{ and } \frac{(1 - v)\ (1 - v_1)\ (1 - p)}{v \times v' \times p + (1 - v)\ (1 - v')\ (1 - p)}$$

respectively.

So if n individuals, whose veracities are v_1, v_2, v_3, v_n, assert the event to have taken place, the probability that it did take place is

$$\frac{v_1\,v_2\,v_3 \ldots\ldots v_n\,p}{v_1\,v_2\,v_3 \ldots\ldots v_n\,p + (1 - v_1)\ (1 - v_2) \ldots\ldots (1 - v_n)\ (1 - p)}.$$

If $n + 1$ individuals assert the event to have taken place, the probability that it did take place is

$$\frac{v_1\,v_2\,v_3 \ldots\ldots v_{n+1}\,p}{v_1\,v_2\,v_3 \ldots\ldots v_{n+1}\,p + (1 - v_1)\ (1 - v_2) \ldots\ldots (1 - v_{n+1})\ (1 - p)},$$

which is greater than the former probability if $v_{n+1} > \frac{1}{2}$, so that the assertion of the $n + 1^{th}$ individual increases the probability of the event arising from the testimony of the other n individuals, only when his veracity is greater than $\frac{1}{2}$.

Ex. 23. Two individuals, whose veracities are v and v', assert that a given ticket has been drawn out of a bag containing a thousand tickets. The probability of the event on the hypothesis, that both the individuals tell the truth and that this ball was drawn, is $\frac{v\,v'}{1000}$; the *à priori* probability, that both the individuals lie and that the given ball was not drawn, is $(1 - v)\ (1 - v')\ \frac{999}{1000}$: if, however, these individuals have no inducement to choose the given ticket amongst the undrawn numbers, this probability must be multiplied by $\frac{1}{(999)^2}$, which is the probability of their both selecting the same number amongst the undrawn numbers; the probability, therefore, of the first hypothesis, namely, that the given ticket was drawn, is

$$\frac{v\,v'}{v\,v' + (1 - v)\ (1 - v')\ \frac{1}{999}}.$$

If v_1, v_2, v_3, … v_n are the veracities of n individuals, who all assert that a given ticket was drawn, the probability that the given ticket was drawn is

$$\frac{v_1\,v_2\,v_3 \ldots\ldots v_n}{v_1\,v_2\,v_3 \ldots v_n + (1 - v_1)\ (1 - v_2) \ldots (1 - v_n)\ \frac{1}{(999)^{n-1}}};$$

hence we see how prodigiously the probability of an event of this kind is increased by the concurrent testimony of many individuals. It must, however, be remarked, that the weight of the concurrent testimony of different individuals depends entirely upon the absence of inducement to lead these individuals to choose any one particular number, and upon the absence of collusion.

50. If we have no data to determine the veracity of an individual, and if he asserts an event to have taken place, of which the simple probability is p, in order to find the probability that the event took place, we must consider the probability of the event upon every hypothesis which can be formed; that is, we must suppose all values of his veracity between zero and unity to be equally probable, *à priori*. The probability, therefore, that the event did take place

$$=\int\frac{p\,v\,d\,v}{p\,v+(1-v)\,(1-p)}=\int\frac{p\,v\,d\,v}{1-p+(2\,p-1)\,v},$$

taken between the limits $v=0$ and $v=1$;

$$=\frac{p}{2p-1}\int\frac{\{(2p-1)\,v+1-p-(1-p)\}\,d\,v}{1-p+(2\,p-1)\,v}$$

$$=\frac{p}{2p-1}\left\{v-\frac{1-p}{2\,p-1}\log.\,(1-p+(2p-1)\,v)\right\}+c$$

$$=\frac{p}{2p-1}\left\{1-\frac{1-p}{2p-1}\log.\left(\frac{p}{1-p}\right)\right\}.$$

When $p=\frac{9}{10}$, this is equal to $\frac{9}{8}\left\{1-\frac{1}{8}\log.\,9\right\}=\cdot 816363$ or about $\frac{9}{11}$.

Ex. 24. A jury consists of n individuals; let the probability of each separately giving a right decision be p, what is the probability that a unanimous decision is a correct one? Two hypotheses can be formed, namely, that the decision is a correct one, or the contrary; the event observed is a unanimous decision, and the *à priori* probability of this event on the first hypothesis is p^n, the *à priori* probability of the event on the second hypothesis is $(1-p)^n$, therefore the probability of the first hypothesis is $\frac{p^n}{p^n+(1-p)^n}$, which is greater than $\frac{1}{2}$, only when $p>\frac{1}{2}$. Therefore it is probable that a unanimous verdict is a correct one, only when it is probable that each juryman considered separately will give a correct decision. The same rule holds *à fortiori*, when the verdict has been given by a majority only.

If $p=\cdot 9$ and $n=12$, this probability is equal to $\frac{9^{12}}{9^{12}+1}$.

A jury composed of $n-2\,m$ individuals is correct.

Similarly, the probability that the decision of a majority is correct

$$=\frac{p^n+n\,p^{n-1}\,(1-p)+n\,.\,(n-1)\,.\,p^{(n-2)}\,(1-p)^2+\ldots}{p^n+(1-p)^n+np\,.\,(1-p)\,.\,\{p^{n-2}+(1-p)^{n-2}\}+\&c.}$$

The probability that a decision given by $n-1$ is correct, is similarly

$$\frac{p^{n-1}\,(1-p)}{p^{n-1}\,(1-p)+p\,(1-p)^{n-1}}=\frac{p^{n-2}}{p^{n-2}+(1-p)^{n-2}}$$

and generally the probability that a decision given by $n-m$ of the jury is correct, is the same as the probability that a unanimous decision of a jury composed of $n - 2m$ individuals is a correct one. If $n = 12$ and $p = \cdot 9$, the probability that a decision of a majority is a correct one by the preceding expression $= \frac{999458768178}{999508948516} = \frac{19519}{19520}$ nearly.

If p is unknown, the probability that a unanimous decision is a correct one must be found by taking the integral $\int \frac{p^n \,.\, dp}{p^n + (1 - p)^n}$ between the limits through which p may be supposed to vary, multiplied by $\int dp$ taken between the same limits.

51. The decision of the jury in this country can only be considered as that of a simple majority, and the probability that this decision is a correct one is small, unless the simple probability that each juryman separately gives a correct one, is taken to be very great. If this probability is $\frac{2}{3}$, the probability of a correct decision is very little greater than $\frac{12}{13}$. The simple probability of any juryman giving a correct decision cannot be supposed to be strictly the same for each juryman composing the same jury, and it must also depend very much upon the nature of the question which is submitted to his determination. As this probability rests only on conjecture, we have considered the preceding questions relating to the decision of a jury with a view of showing how they might be solved if we were in possession of sufficient data rather than as laying any stress on the results obtained.

52. A bag contains a number of balls of i different colours:
m_1 of the 1st colour have been drawn and replaced,
m_2 ,, 2nd,
m_i ,, i^{th};
in $m_1 + m_2 \;\ldots\ldots\; + m_i$ trials; required the probability of drawing n_1 balls of the first colour, n_2 of the second, n_i of the i^{th} colour, in $n_1 + n_2 \ldots\ldots + n_i$ succeeding trials.

Let x_1 be the *à priori* probability of drawing a ball of the 1st colour,
x_2 ,, ,, ,, 2nd,
x_i ,, ,, ,, i^{th},
and let C be the coefficient of $x_1^{m_1} \times x_2^{m_2} \ldots\ldots x_i^{m_i}$ in the developement of $(x_1 + x_2 \ldots\ldots + x_i)^{m_1 + m_2 \cdots\cdots + m_i}$; then the probability of the observed event is $C \times x_1^{m_1} \times x_2^{m_2} \ldots\ldots \times x_i^{m_i}$; the probability of the hypothesis that
x_1 is the probability of drawing a ball of the 1st colour,
x_2 ,, ,, 2nd,
x_i ,, ,, i^{th},
is $C \times x_1^{m_1} \times x_2^{m_2} \times \ldots x_i^{m_i}$ divided by the sum of all the values of which this fraction is susceptible; and if C_1 is the coefficient of $x_1^{n_1} \times x_2^{n_2} \ldots\ldots x_i^{n_i}$, in the developement of $(x_1 + x_2 \ldots + x_i)^{n_1 + n_2 \cdots\cdots + n_i}$, the probability of drawing n_1 balls of the 1st colour, n_2 balls of the 2nd colour, n_i balls of the i^{th} colour is the sum of all the values of which the quantity $C \times C_1 \times x_1^{(m_1 + n_1)} \times x_2^{(m_2 + n_2)} \ldots\ldots x_i^{(m_i + n_i)}$ is susceptible, divided by the sum of all the values of which the quantity $C \times x_1^{m_1} \times x_2^{m_2} \ldots\ldots x_i^{m_i}$ is susceptible.

If x_1, $x_2 \ldots\ldots x_i$, be supposed to vary from $x = 0$ to $x = 1$, and all these values are equally probable *à priori*, then the probability required is found by taking the integral

$$\int^i x_1^{(m_1 + n_1)} \times x_2^{(m_2 + n_2)} \ldots\ldots x_i^{(m_i + n_1)} \, d\,x_1 \, d\,x_2 \, d\,x_i$$

between the limits $x_i = 0$, $x_i = 1 - x_1 - x_2 \ldots\ldots\ldots - x_{i-1}$
,, ,, $x_{i-1} = 0$, $x_{i-1} = 1 - x_1 - x_2 - x_3 \ldots\ldots - x_{i-2}$
,, ,, $x_{i-2} = 0$, $x_{i-2} = 1 - x_1 - x_2 \ldots\ldots\ldots - x_{i-3}$
,, ,, $x_1 = 0$, $x_1 = 1$;

and dividing the quantity so obtained by the integral

$$\int^i x_1^{m_1} \times x_2^{m_2} \dots\dots x_i^{m_i} \times d\,x_1\,d\,x_2 \dots\dots d\,x_i$$

taken between the same limits.

If $(m_1+1)\,(m_1+2)\,(m_1+3)\dots(m_1+n_1)$ be represented by $[m_1+1]^{n_1}$
$(m_2+1)\,(m_2+2)\,(m_2+3)\dots(m_2+n_2)$,, ,, $[m_2+1]^{n_2}$
$(\Sigma\,(m)+i)\,(\Sigma\,(m)+i+1)\,..\,(\Sigma\,(m)+\Sigma\,(n)+i)$ by $[\Sigma\,(m)+i]^{\Sigma\,(n)}$

Σ being used as a sign of collection to denote that the sum is to be taken of all quantities which are represented by a general symbol, these integrations give for the probability required,

$$\frac{C\,[m_1+1]^{n_1}\,[m_2+1]^{n_2}\dots\dots[m_i+1]^{n_i}}{[m_1+m_2\,\dots\dots\dots\dots+m_i+i]^{n_1+n_2\dots\dots n_i}};$$

which is the same, with the difference of notation, as if the simple probability of drawing a ball of the r^{th} colour was $\dfrac{m_r+1}{m_1+m_2\,..+m_i+i}$. The probability of drawing a ball of the r^{th} colour in one succeeding trial is

$$\frac{m_r+1}{m_1+m_2\,\dots\dots+m_i+i}.$$

53. One of the most interesting and useful applications of the theory of probabilities is the solution of questions connected with the duration of life and the calculation of the values of annuities and reversionary payments. The value of an annuity is the value of the sum of the annual payments made to an individual throughout his life.

Let 1 + rate of interest $= \dfrac{1}{r}$, and let $p_{m,n}$ be the probability of a given individual aged m years living at least n years; the value of any sum s to be paid to him at the expiration of n years, neglecting discount, is the value of this sum multiplied by the probability of the individual being alive to receive it, which is equal to $s\,p_{m,n}$, this must be discounted in order to obtain its present value, which reduces it to $s\,r^n\,p_{m,n}$. If a_m is the value of an annuity of £1. to be received by an individual aged m years, the value of an annuity of £s to the same individual is $s\,a_m = s\,\Sigma\,r^n\,p_{m,n}$.

54. When an individual insures his life at any office, the insurance company agrees to pay his executors a certain sum at his death, whenever that event may take place.

Let $q_{m,1}, q_{m,2}, q_{m,3}\dots\dots q_{m,n}$ be the probabilities that an individual, aged m years, dies in the first, second, third, &c., or n^{th} year, then the value of £1. to be paid whenever he dies, discounted, is

$$r\,q_{m,1} + r^2\,q_{m,2} + \dots\dots + r^n\,q_{m,n};$$

but $$q_{m,1} = 1 - p_{m,1},\quad q_{m,2} = p_{m,1} - p_{m,2},$$

therefore the value of the insurance is

$$r\,(1 - p_{m,1}) + r^2\,(p_{m,1} - p_{m,2}) + \&c.$$
$$= r\,(1 + a_m) - a_m,$$

and the value of £s to be paid when the individual dies is

$$s\,r\,(1 + a_m) - s\,a_m.$$

By means of this expression the value of an insurance at the age m may be deduced from the value of the annuity, and *vice versâ*. A person insuring his life, instead of making one payment to the office, generally pays an annuity which is called the *premium*, the value of this *premium* with present payment

$$= \frac{s\,r\,(1 + a_m) - s\,a_m}{1 + a_m} = r\,s - \frac{s\,a_m}{1 + a_m}$$

The value of any sum s to be paid if either of two individuals, aged m and m' years respectively, are alive after n years is

$$s\,r^n \{ 1 - (1 - p_{m,n})\,(1 - p_{m',n}) \}$$
$$= s\,r^n\,p_{m,n} + s\,r^n\,p_{m',n} - s\,r^n\,p_{m,n} \times p_{m',n},$$

and the value of an annuity to be paid as long as either of two individuals, aged m and m' years, are alive

$$= s \{ \Sigma\,r^n\,p_{m,n} + \Sigma\,r^n\,p_{m',n} - \Sigma\,r^n\,p_{m,n} \times p_{m',n} \}$$
$$= s\,a_m + s\,a_{m'} - s\,a_{m,m'}$$

understanding by the symbol $a_{m,m'}$ the value of an annuity of £1. on the joint lives of two persons aged m and m' years.

If life is considered valuable in proportion to its duration the expectation which any individual has of life will be measured by the sum of the probabilities of his dying after each given age, that is by an annuity without interest.

Hence we have these expressions:

Expectation of life $= \Sigma\,p_{m,n}.$

Value of annuity $s\,a_m = s\,\Sigma\,r^n\,p_{m,n}$

........insurance of a given sum s in one payment

$$= s\,r\,(1 + a_m) - s\,a_m,$$

........premium of insurance of a given sum s

$$= r\,s - \frac{s\,a_m}{1 + a_m},$$

........annuity during two joint lives

$$= s\,\Sigma\,p_{m,n} \times p_{m',n},$$

........annuity to be paid as long as either of two individuals, aged m and m' years respectively, are alive

$$= s\,a_m + s\,a_{m'} - s\,a_{m,m'}.$$

Thus if $m - 20$, $r = \frac{1}{1{\cdot}03}$, $a_m = 20{\cdot}1428$, according to Table III. for males.

Value of the single premium required for a male aged 20 to secure the payment of 1 at the end of the year in which the life shall fail

$$= \frac{21{\cdot}1428}{1{\cdot}03} - 20{\cdot}1428 = {\cdot}38419.$$

Value of the annual premium required for a male aged 20 to secure the payment of 1 at the end of the year in which the life shall fail

$$= \frac{1}{1{\cdot}03} - \frac{20{\cdot}1428}{21{\cdot}1428} = {\cdot}01817.$$

55. In calculating the values of annuities the labour is much diminished by observing that the probability of an individual aged m years living at

least n years is equal to the product of the probability of his living at least $n - q$ years, multiplied by the probability of an individual aged $m + n - q$ years living at least q years, or

$$p_{m,n} = p_{m,n-q} \times p_{m+n-q,q};$$

and therefore, putting

$$n - 1 \text{ for } q, \quad p_{m+1,n-1} = \frac{p_{m,n}}{p_{m,1}},$$

$$a_{m+1} = r\,p_{m+1} + r^2\,p_{m+2} + \&c.$$

$$= \frac{r^2\,p_{m,2} + r^3\,p_{m,3} + \&c.}{r\,p_{m,1}}$$

$$= \frac{a_m - r\,p_{m,1}}{r\,p_{m,1}} = \frac{a_m}{r\,p_{m,1}} - 1.$$

By means of this expression, which appears first to have been noticed by Mr. Barrett, the value of any annuity may be deduced from that which precedes or follows it.

Thus if $m = 20$, $r = \frac{1}{1{\cdot}03}$, $p_{m,1} = \frac{5707}{5765}$, according to Table II., and $a_m = 20{\cdot}1428$, according to Table III.,

$$a_{21} = \frac{20{\cdot}1428 \times 1{\cdot}03 \times 5765}{5707} - 1 = 19{\cdot}9580.$$

56. It has been seen that the values of annuities, reversionary payments, &c., consist of the sum of a number of separate payments. If these payments are calculated accurately at certain intervals the values of those which are intermediate may be interpolated by known methods.

In fact, if y_0, y_n, y_{2n}, &c. are successive values of the variable y, and if $\Delta\, y_0 = y_n - y_0$, $\Delta^2 y = y_0 + y_{2n} - 2\,y_n$, &c., and y_i be any value of y intermediate between y_0 and y_n,

$$y_i = y_0 + i\,\Delta\,y_0 + \frac{i\,(i-1)}{2}\,\Delta^2 y_0 + \&c.$$

When the sum only of the values of y is required it is not necessary, however, to go through the labour of calculating each particular quantity in the series, it may easily be shown that this sum is equal to

$$n\,(y_0 + y_n + y_{2n} \ldots\ldots\ldots y_m)$$

$$+ \frac{n-1}{2}\left\{y_m - y_0\right\} - \frac{(n-1)\,(n+1)}{12\,n}\left\{\Delta\,y_m - \Delta\,y_0\right\} \;\&c.$$

The problem appertains to what are called mechanical quadratures, and this method is similar to that made use of in summations which are required in calculating the perturbations of a comet. See the *Mécanique Céleste*, vol. iv. p. 206. In applications of this series to the calculation of annuities, reversionary payments, &c. y_m, $\Delta\,y_m$, &c. $= 0$. The first term in the series of the values of y or y_0 is the value of a present payment $= 1$, if we neglect the term

$$\frac{(n-1).(n+1)}{12\,n}\left\{\Delta\,y_m - \Delta\,y_0\right\}$$

and the following, and suppose the values of the annual payments to be in arithmetical progression, the value of an annuity on the life of a person aged 20, to commence at the end of the first year, supposing $n = 10$,

$$= 10 \left\{ 1 + r^{10} p_{20,10} + r^{20} p_{20,20} + \&c. \right\} - \frac{9}{2} - 1,$$

$$= 10 \left\{ r^{10} p_{20,10} + r^{20} p_{20,20} + \&c. \right\} + \frac{9}{2}.$$

57. In calculating annuities the values of the annual payments, except, perhaps, at birth, vary so gradually that the result thus obtained will be a sufficiently accurate approximation, and, probably, within the limits of the errors of which the values of p, that is of the table of mortality which is used, are susceptible; the correction, however, in all cases may be considered as constant for different tables of mortality, and may, therefore, be determined by calculating the annuity first accurately and afterwards by the approximate method from any table of mortality in which the deaths are given for every age, the difference between the two values so obtained will be the correction required.

The method of calculating annuities hitherto adopted by Dr. Price and other writers, has been, first to interpolate living between those which are actually given from ten years to ten years by the observations, to calculate probabilities of surviving each number of years from the numbers so interpolated, then to *discount* these probabilities so obtained, and finally to obtain the value of the annuity by adding together all these discounted probabilities. This labour, though diminished by means of the equation noticed by Mr. Barrett, is still unnecessary, and would lead to the same result as that given by the series of the last page. The same method is, we think, generally the simplest which can be applied to calculating annuities on two or more lives, and, in fact, to the summation of any series of which the law is too complicated to admit of the ordinary methods.

58. We have, as yet, said nothing with respect to the method of determining p and q, and this is a question of very considerable difficulty, whether as regards theory or practice.

Suppose 1000 infants to be carefully registered at birth, and the ages at which they die to be noted. If of these 900 are alive at the end of the first year the probability of an infant at birth living one year under similar circumstances would be nearly $= \frac{900}{1000}$, or $\frac{9}{10}$; and if the number of infants registered were infinite, this would cease to be an approximation, the ratio of the number alive at the end of the first year to the whole number registered at birth would be exactly equal to the probability of an infant under the same circumstances living 1 year. The problem is, in fact, similar to the one we solved, page 33, when we supposed a bag to contain a number of balls of different colours, and that a certain number of drawings had been made. The different ages at which the individuals can die correspond to the different colours in the former problem.

If we suppose 101 ages at which deaths take place, that is, if we suppose n to vary from 0 to 100 in the values of $q_{m,n}$, $p_{m,n}$, and if d_1, d_2, &c. d_n, are the number of the 1000 infants who have been observed to die in their first, second, and n^{th} years respectively, we have

$$q_{m,n} = \frac{d_{m+n} + 1}{d_{m+1} + d_{m+2} \ldots\ldots + 101 - m}$$

$$p_{m,n} = \frac{d_{m+n} + d_{m+n+1} + \ldots + 101 - m - n}{d_{m+1} + d_{m+2} \ldots\ldots + 101 - m}$$

$$q_{o,n} = \frac{d_n + 1}{d_1 + d_2 \ldots\ldots\ldots\ldots + 101}$$

$$p_{o,n} = \frac{d_n + d_{n+1} + \ldots\ldots\ \ 101 - n}{d_1 + d_2 \ldots\ldots\ldots\ldots + 101}$$

in this case $d_1 + d_2 + \&c. \ldots\ldots\ldots = 1000$

$$q_{o,n} = \frac{d_n+1}{1101},\ p_{o,n} = \frac{d_n + d_{n+1} + \&c.\ \ldots\ldots + 101 - n}{1101}.$$

59. Unfortunately, no registers of this kind have been kept, and we are obliged to have recourse to those sources which best supply this deficiency. If the population of any district were subject to no fluctuations arising from an influx of the inhabitants of the neighbouring countries, and if it were constant, that is, if the births and deaths were always the same and equal to each other, a register of the ages at which deaths took place would alone be wanted to determine the values of p and q for this place. For it is evident that if the *population were large* the probability of an individual dying at the n^{th} age would be equal to the deaths of persons at that age (n) divided by the births n years previously, but the births n years previously are, upon the hypothesis we have made, exactly equal to the present number of yearly deaths $= d_1 + d_2\ \&c. = \Sigma\, d_n$, using the letter Σ as before,

and
$$q_{o,n} = \frac{d_n}{\Sigma(d_n)};$$

and in the same way $q_{m,n}$ may be determined.

The parish books, therefore, if they were accurately kept, and if the population were subject to the conditions we have mentioned, would furnish the information required, and they were used as a first approximation.

60. When the population is not stationary, the preceding results require modification.

Let d_n be the deaths observed at the age n in a given place, b_n the births in that place n years previously; then

$$q_{o,n} = \frac{d_n + 1}{b_n + 101 - n}.$$

Let us suppose that the births m years ago were equal to the total number of deaths now, and that the births increase in a geometric progression, of which the common ratio is r; then

$$b_n r^n = b_o = b_m r^m = \Sigma\,(d)\, r^m,$$

$$\therefore b_n = \Sigma\,(d)\, r^{m-n},$$

and
$$q_{o,n} = \frac{d_n + 1}{\Sigma\,(d)\, r^{m-n} + 101 - n}.$$

m must be found, from the consideration that $\Sigma\,(q_o) = 1$, which, in the present form of the equation, would be troublesome: the labour may be much simplified by observing, that when r does not differ much from unity, this value of $q_{o,n}$ does not sensibly differ from

$$q_{o,n} = \frac{d_n + 1}{(\Sigma\,(d) + 101 - n)\, r^{m-n}} = \frac{1}{r^m} \cdot \frac{(d_n + 1)\, r^n}{\Sigma\,(d) + 101 - n}$$

In this form $q_{o,n} \cdot r^m$ may be calculated without any previous knowledge of m; let the value of $q_{o,n} \cdot r^m$ be called D_n; then

$$r^m \Sigma (q_o) = \Sigma (\mathrm{D}) = r^m$$

$$m = \frac{\log. \{ \Sigma (\mathrm{D}) \}}{\log. r}.$$

In this manner, when r is taken at 1·005, m is found, from the Chester Observations, Table I., to be somewhere about 40. This result agrees, within the errors of observation, with the period given by an actual comparison of burials and baptisms.

61. The difficulty has been seen of applying mathematical reasoning to the valuation of risks which are connected with the duration of life, but the difficulty with fire and sea insurances is greater, from the number of circumstances which are necessary to be taken into account. If underwriters only insured against total losses, the question would be comparatively easy, for it would only be necessary to ascertain the number of houses out of a given number which are annually burnt to the ground, or the number of vessels which have been lost out of a given number making the same voyage. But by far the greater part of the claims on the underwriters arises from partial losses, as when a house is damaged by fire, or a cargo is partially injured by sea water, &c.

62. In insurances upon lives the observations which we have referred to furnish data which serve to ascertain the value of the risk, but in insurances against loss by fire and sea no similar data have been published, and so various and complicated are the contingencies to which they are subject, that it would be difficult to form any tables of the values of these risks. A register might be made out by each individual underwriter, or company of underwriters, showing the result of their respective experience, which would, doubtless, be useful to them in the conduct of their business. There are many important facts which might be collected and systematically arranged, the knowledge of which would also be useful to the underwriter. Thus a register of the weather in different parts of the world, for a sufficient number of years, would be some guide in ascertaining the relative value of the risk in maritime insurances, as far as it is affected by season. The Society for the Registry of Shipping appoints competent persons to survey every ship which enters any of the principal ports in this kingdom; their detailed report, which contains the name of the ship, captain, owners, her tonnage, the port where built, the materials of which she is made, &c., is published by subscription, and an office is kept for the purpose of posting it up to the latest period. Such a plan adopted in the principal maritime ports of foreign countries would, in the course of time, give a complete register of the commercial shipping of Europe. The preceding remarks apply to maritime insurances, and although the difficulty of obtaining sufficient data to determine the risk in fire insurances is considerable, it is not so great as in sea insurances.

It is foreign to our purpose to offer any further remarks upon these questions, the solution of which is the continual business of the underwriter, and which requires great skill and experience.

63. The principal use that has hitherto been made practically of the theory of probability has been in the solution of questions connected with the valuation of annuities and reversionary payments.

The *method of least squares*, which is of very extensive application in astronomy, was proposed by M. Legendre in 1805;* it has since been shown by Laplace† to be preferable to every other, when the number of equations, which serve to determine any unknown quantities, exceed in number that of the unknown quantities themselves. Our limits do not permit us to give the analysis upon which this proof is founded, we shall, however, endeavour to explain shortly in what the method itself consists.

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* Nouvelles Méthodes pour la Détermination des Orbites des Comètes.

† Théorie des Probabilités.

The data furnished by observation lead in general to equations of this form

$$a - b\,x + c\,y + \&c. = 0,$$

in which equation a, b, c, &c. are known quantities which vary from one equation to another. Each n^{th} observation gives an equation

$$a_n - b_n\,x + c_n\,y + \&c. = 0.$$

If the number of equations is equal to that of the unknown quantities x, y, z, they may be determined by linear elimination, but generally the number of equations exceeds that of the unknown equations, and the question arises, of which the solution is in some measure arbitrary, what system of equations, equal in number to that of the unknown quantities x, y, z, is most favourable for their determination.

The *method of least squares*, which Laplace has proved to be the most advantageous, consists in determining the quantities x, y, z, so that the quantity

$$\begin{gathered} \{ a_1 - b_1\,x + c_1\,y + \&c. \}^2 \\ + \{ a_2 - b_2\,x + c_2\,y + \&c. \}^2 \\ \&c. \\ + \{ a_n - b_n\,x + c_n\,y + \&c. \}^2 \end{gathered}$$

is a minimum.

For this purpose it is only necessary to differentiate this sum separately, with respect to each of the variables x, y, z, &c., and put the results separately $= 0$.

The equation, which is obtained by making x alone vary, is

$$\begin{gathered} \{ a_1 - b_1\,x + c_1\,y + \&c. \}\, b_1 \\ + \{ a_2 - b_2\,x + c_2\,y + \&c. \}\, b_2 \\ + \&c. \\ + \{ a_n - b_n\,x + c_n\,y + \&c. \}\, b_n = 0, \end{gathered}$$

or

$$\Sigma\, a_n\, b_n = x\,\Sigma\, b_n^2 + y\,\Sigma\, b_n\, c_n + \&c. = 0:$$

each of the quantities x, y, z, &c. furnishes a similar equation, and hence a system of equations results equal in number to those quantities, from whence they may be found by linear elimination. It is thus that many thousand observations may be made to concur in the determination of one element.

Suppose, for example, the question consists in determining with accuracy the elements of the orbit of a planet, after having obtained them nearly by a first approximation.

Let λ be the geocentric longitude observed, and suppose the uncertainty is with respect to two only of the elements e and ϖ, and let $\delta\lambda$ be the error of longitude, or the difference between the longitude of the planet observed at a given time, and that which is deduced by calculation from the approximate elements; δe, $\delta\varpi$ the errors of those elements; then by Taylor's theorem, neglecting the squares, &c. of δe and $\delta\varpi$

$$\delta\lambda = \left(\frac{d\lambda}{d e}\right)\delta e + \left(\frac{d\lambda}{d\varpi}\right)\delta\varpi$$

$\left(\frac{d\lambda}{d e}\right)$ and $\left(\frac{d\lambda}{d\varpi}\right)$, may be calculated directly, but it is better to infer them by assuming an arbitrary error δe, and by finding the corresponding error $\delta\lambda$.

If a and b be put for $\left(\frac{d\lambda}{d e}\right)$ and $\left(\frac{d\lambda}{d\varpi}\right)$ the n^{th} observation furnishes the equation

$$(\delta\lambda)_n = a_n \delta e + b_n \delta\varpi,$$

and the two equations which serve to determine δe and $\delta\varpi$ by the *method of least squares* are

$$\Sigma\,(\delta\lambda)_n\, a_n = \delta e\, \Sigma\,(a_n)^2 + \delta\varpi\, \Sigma\, a_n b_n$$
$$\Sigma\,(\delta\lambda_n)\, b_n = \delta e\, \Sigma\, a_n b_n + \delta\varpi\, \Sigma\,(b_n)^2$$

If only one quantity has to be determined, this method evidently resolves itself into taking the mean of all the values given by observation.

64. We shall now, in conclusion, trace the theory of probability through the different stages of its progress, and mention the principal writers who have assisted in establishing its principles. The estimation of the probability of a future event, by enumeration of the cases supposed to be similarly circumstanced, does not appear to have been attempted until the early part of the seventeenth century; and the very elementary nature of the first problem of which the solution is on record, serves to show that the subject was then altogether new. It is contained in a fragment of uncertain date, written by the celebrated Galileo, who died in 1642. It was addressed to a friend who thought the fact that the points 9 and 10 can both be produced by six different combinations of numbers on three dice difficult to reconcile with the notorious preference given by gamesters to the latter number beyond the former. The difficulty is explained by Galileo, by taking into account the permutations of the component numbers, and the respective chances of these two numbers are thus shown to be as 25 to 27. A correct table is subjoined of the permutations of all numbers which can be thrown on three dice; and it is added, that the consideration of this table will serve for the solution of other problems of the same nature. All this must be admitted to belong to the infancy of the science, nor does it appear that Galileo thought the subject of sufficient interest to call for further inquiry.

65. The history of the theory of probability is generally made to begin several years later, when, in the year 1654, the two following problems were proposed by the Chevalier de Meré to Blaise Pascal.

1st. Two players want each a given number of points towards winning. If they separate without playing out the game, how should the stakes be divided between them?

2d. In how many trials is it an even wager to throw sixes upon two dice?

We are told in one of Pascal's letters to Fermat, that his answer to the latter question, that the odds are against twenty-four trials, and in favour of twenty-five, though undoubtedly correct, scandalized Mr. de Meré, "and made him declare loudly that the science of arithmetic is inconsistent with itself."[*] The Chevalier thought that the chances being in favour of throwing six in four trials with one die, on which are six different numbers, they ought also to be in favour of throwing two sixes in six times as many, or twenty-four trials with two dice, on which are six times as many, or thirty-six different numbers. Those who have read the preceding pages with any degree of attention, will readily perceive that the Chevalier (whose name it has become unusual to mention, without adding that he was a man of talent, but no mathematician) was thus comparing events which have no connection with each other. We shall have occasion presently to mention errors in the principles of this science committed under the sanction of a name of greater influence and authority among mathematicians.

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[*] Opera Petri de Fermat, Tolosæ, 1679.

66. The other problem (which afterwards obtained the name of the Problem of Points) appeared to Pascal of greater interest; he communicated it to Fermat, Roberval, and others; none of whom, but Fermat, returned him a satisfactory solution. The correspondence which passed on this subject between Fermat and Pascal appeared in 1679, in the posthumous edition of Fermat's works published at Toulouse, and is now also to be found in the complete edition of Pascal's works. Pascal began by considering the simplest case, in which one of two players, whom we will call A, wants one, and B, the other, two points of winning the game. He determined the required proportion from the consideration that if B win the next point, of which his chance is only $\frac{1}{2}$, these players would be in a condition of equality; and if they were then to separate, the stake ought to be equally divided between them; so that A's present share should be made up of half the stake corresponding to his equal chance of winning the next point, and one quarter corresponding to the present chance of his share, if B were to win the next; making $\frac{3}{4}$ in all. This mode of solution is very elegant, but there is some difficulty in applying the same principle to more complicated questions. It has been adopted by subsequent writers in examining the most difficult parts of the theory, but aided by a method of analysis very far superior to anything with which Pascal or his contemporaries were acquainted. In fact, Pascal was led into an error when he attempted to extend his method to the general problem, which occasioned a short controversy between him and Fermat, who had preferred the more laborious, but also more direct, method of enumerating all the possible ways in which the game might be terminated, and proportioning the division of the stakes according to the numbers which appear favourable to either party. Pascal found some difficulty in admitting that this method of Fermat's is good in every case, and confirmed himself in his mistake in consequence of an erroneous distribution which he made of the permutations of three letters, when he attempted to apply Fermat's method to the case of three players. Fermat pointed out the nature of his error, at least we may presume so from a letter of Pascal's, in which he retracts his former objection, saying, that Fermat's last remarks had been entirely satisfactory.

67. This correspondence was still unpublished, when Huyghens turned his thoughts to the theory of probability, and composed a short Latin treatise, "De Ratiociniis in Ludo Aleæ," first printed by Schooten in 1658 at the end of his "Exercitationes Geometricæ." This is the earliest regular treatise on the subject, which thenceforward continued to draw more and more the attention of mathematicians. Besides an examination in detail of Meré's questions, Huyghens' treatise contains the enunciation of the general and fundamental theorem of this branch of the science, that if any player have p chances of gaining a sum represented by a and q chances of gaining b, his *expectation* (a term then first introduced) will be rightly represented by $\frac{p\,a + q\,b}{p + q}$. Elementary as this truth may now appear, it was not received altogether without opposition.

68. In the year 1670, a Jesuit named Caramuel published the two first volumes of his course of mathematics, under the title of "Mathesis Biceps,"

in which he introduced a treatise on the theory of games at dice, which he called Kubeia, from the Greek word signifying a die. At the end of it he printed the whole of Huyghens's essay, professing to be ignorant whether it had been already published or not. Nicolas Bernoulli has characterised Caramuel's work as one continued blunder, and indeed this author has fallen into mistakes from which the reading of Huyghens's treatise ought to have preserved him. For instance, when proposing to determine the chances favourable to A and B; if the former (who is to begin) undertakes to throw six before the latter throws seven, upon two dice, his reasoning is as follows: A's chance of throwing six is $\frac{5}{36}$, and therefore, if the stake be 36, the value of his throw may be taken to be five, leaving thirty-one to B, whose chance of winning, if he have a throw, being equal to $\frac{1}{6}$, his first throw also may be bought off for $\frac{1}{6}$ of the remainder, or $5\frac{1}{6}$, leaving $25\frac{5}{6}$, still to be contended for. Caramuel's reasoning is so far correct; but instead of continuing to divide this remainder in the proportion of the chances of the two players, which would have led him to two infinite series, the sums of which would be the just proportions, he argued that the value of the first throw of each player being compensated to him by the share thus allotted to each out of the stake, this second remainder ought to be equally divided between them. Hence he deduced the shares of A and B, if they were to leave the game unplayed, to be respectively $17\frac{11}{12}$ and $18\frac{1}{12}$, instead of $17\frac{43}{61}$ and $18\frac{18}{67}$, which Huyghens had already deduced by a different and more correct analysis.

69. The Journal des Sçavans for 1679 mentions an essay published in the preceding November, by Sauveur, on the advantage of the banker at basset, a game of cards then much played in Paris, and celebrated for the duels it occasioned, to such an extent that it became necessary, solely on that account, formally to prohibit it from being played. This treatise was compiled at the request of the Marquis Dangeau, and brought Sauveur into great favour at court, where he was admitted to explain his theory to Louis XIV.*

70. It has been the misfortune of the science of probability, in consequence of the ready application made of its principles to games at cards and dice, that a prejudice has from the first existed against it as if ministering only to gambling and immorality, and available for no other purpose: accordingly the anonymous writer, who, in 1692, published the first English essay "Of the Laws of Chance," thought it necessary to protest in his preface that the design of his book was "not to teach the art of playing at dice, but to deal with them as with other epidemic distempers, and perhaps persuade a raw squire to keep his money in his pocket." This essay, which was edited, and is generally supposed to have been written, by Motte, the secretary of the Royal Society, contains a translation of Huyghens's treatise, and an application of his principles to the determination of the advantage of the banker at pharaon, hazard, and other games, and to some questions relating to lotteries. The body of the work does not contain any new principle, but there are some remarks in the preface, which, considering the time at which they were written, deserve attention, and show how justly the author had apprehended the nature of his subject. "It is impossible," says he, "for a die with such determined force and direction not to fall on such a determined side, only I do not know the force and direction which make it fall on such a determined side, and therefore I call that chance, which is nothing but want of art."—"There are very few things which we know, which are

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* Histoire de l'Académie, 1716.

not capable of being reduced to a mathematical reasoning; and when they cannot, it is a sign our knowledge of them is very small and confused; and where a mathematical reasoning can be had, it is as great folly to make use of any other, as to grope for a thing in the dark when you have a candle standing by you."—" There is likewise a calculation of the quantity of probability founded on experience, to be made use of in wagers about any thing. The yearly bills of mortality are observed to bear such proportion to the live people as 1 to 30 or 26; therefore it is an even wager that one of thirteen dies within a year (which may be a good reason, though not the true, of that foolish piece of superstition,) because at this rate if 1 out of 26 dies, you are no loser."

71. Long before mathematics had been applied to this science, Kepler had formed the same accurate notion of the real meaning of chance as is here expressed by Motte. In his dissertation on the new star which appeared in 1604, after mentioning that some were of opinion it came by chance, and illustrated their meaning by supposing a set of dice to be thrown an infinite number of times, in which it would necessarily happen (according to them) that any required number would at last be thrown, he says that, even in the case adduced, those are very unthinking who look upon the events as entirely without a cause. " Why does six fall in one throw and ace in another? Because this last time the player took up the die by a different side, shut his hand upon it differently, shook it, threw it in a different manner, or because the wind was blowing differently upon it, or it fell on a different part of the board. There is nothing in all this, which is without its proper cause, if any one could investigate such niceties."*

72. The bills of mortality, mentioned in Motte's book, are registers which began to be kept in 1592, of the annual number of deaths in the city of London, which, with some intermission between 1594 and 1603, have been regularly returned to the present time. They were first intended to make known the progress of the plague; and it was not till 1662 that Captain Graunt, a most acute and intelligent man, conceived the idea of rendering them subservient to the ulterior objects of determining the population and growth of the metropolis; as before his time, to use his own words, " most of them who constantly took in the weekly bills of mortality, made little or no other use of them than so as they might take the same as a text to talk upon in the next company; and withal, in the plague time, how the sickness increased or decreased, that so the rich might guess of the necessity of their removal, and tradesmen might conjecture what doings they were like to have in their respective dealings." Graunt was careful to publish with his deductions the actual returns from which they were obtained, comparing himself, when so doing, to " a silly schoolboy, coming to say his lesson to the world (that peevish and tetchie master,) who brings a bundle of rods, wherewith to be whipped for every mistake he has committed." Many subsequent writers have betrayed more fear of the punishment they might be liable to on making similar disclosures, and have kept entirely out of sight the sources of their conclusions. The immunity they have thus purchased from contradiction could not be obtained but at the expense of confidence in their results.

73. These researches procured for Graunt the honour of being chosen a fellow of the Royal Society, and, to pass over Sir Wm. Petty's "Observations," as bearing less directly on our subject, were undoubtedly the cause which led Halley to consider the duration of human life, as he himself owns, in the

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* De Stellâ novâ. Pragæ, 1606.

paper published in the Philosophical Transactions in 1693. In this celebrated paper, from which we must date the commencement of real knowledge on the subject of life annuities and insurances in this country, Dr. Halley has made choice of a register of deaths which had been kept at Breslau, in Silesia, and which had been then recently communicated by Neumann (probably at Halley's request,) through Justell, to the Royal Society, in whose archives it is supposed that copies of the original registers are still preserved. Before continuing our notice of this very interesting branch of the subject, we shall mention some other important works which appeared about the same time.

74. James Bernoulli had shown that he was not inattentive to the progress of this science by a problem which, according to the fashion of that time, he had published in the Journal des Sçavans for 1685, in the form of a challenge to his contemporaries. This problem was to determine the chances of A and B, who are each to score a certain number of points thrown on the dice, A beginning with one throw, B following also with one; A then being allowed two, and B two; A three, and B three, and so on till the conclusion of the game. Leibnitz answered the question, and undertook to divine the analysis which had conducted Bernoulli to the solution given by him, without demonstration, in the Journal de Leipsic for May, 1690.

75. There is also a treatise by Leibnitz, on Complexions, or, as we now more commonly call them, Combinations, but which was not written with any reference to the science of chances, in which this theory is so pre-eminently useful.

The distinctive names which Leibnitz adopts, of combinations, conternations, conquaternations, &c., to express what we now call combinations two by two, three by three, &c., are no longer in use, except among German writers, where we still meet with the terms binions, ternions, quaternions, &c. Leibnitz mentions Clavius as having been the first who gave, in 1583, a clear view of this theory, "not being able to find any traces of it in the Arithmetic of Cardan, to whom Schwenter refers it." Schwenter probably alluded to Cardan's book, "De Proportionibus," in which the figurate numbers are mentioned, and their use shown in the extraction of roots, as employed by Stifel, a German algebraist, who wrote in the early part of the sixteenth century.

76. It is not necessary to do more than mention an essay, by Craig, on the probability of testimony, which appeared in 1699, under the title of "Theologiæ Christianæ Principia Mathematica." This attempt to introduce mathematical language and reasoning into moral subjects can scarcely be read with seriousness; it has the appearance of an insane parody of Newton's Principia, which then engrossed the attention of the mathematical world. The author begins by stating that he considers the mind as a movable, and arguments as so many moving forces, by which a certain velocity of suspicion is produced, &c. He proves gravely, that suspicions of any history, transmitted through the given time (*cæteris paribus*), vary in the duplicate ratio of the times taken from the beginning of the history, with much more of the same kind with respect to the estimation of equable pleasure, uniformly accelerated pleasure, pleasure varying as any power of the time, &c. &c.

77. An anonymous essay in the Philosophical Transactions of the same year, and of not much greater value, may perhaps be attributed to the same author. The theory there laid down is, that a fraction of the doubt which always remains as to the truth of a narrated fact, after any number of concurrent witnesses, is always removed by an additional testimony. This obviously false theory was taken up at a much later period, by Bicquilley, in

a work entitled "Du Calcul des Probabilités," and by Condorcet in the article "Probabilité," in the French Encyclopædia.

78. James Bernoulli was employed in preparing a copious work on the science of chances, till his death in 1705, by which its appearance was delayed during ten years, after which his nephew, Nicolas Bernoulli, found leisure to superintend its publication, though in an unfinished state. In the mean time Montmort published his celebrated work, "Essai d'Analyse sur les Jeux de Hazard," the most extensive of the sort which had till then appeared, in which the conditions of all the principal games then in vogue are discussed at considerable length, and the theory of combinations extended and enriched with several new theorems. Immediately after the publication of Montmort's book, Demoivre, a Frenchman naturalized in England after the revocation of the Edict of Nantes, inserted, in the Philosophical Transactions for 1711, a short essay entitled "De Mensura Sortis," which, in 1716, he published in a greatly enlarged form, under the title of "The Doctrine of Chances." This work is far superior, both in research and elegance, to all which had preceded it; the collection of problems which it contains is far too extensive to admit any complete notice of it being given in this place: it will be sufficient to instance the doctrine of recurring series, and the theorems on the duration of play, which are to be met with in it for the first time, to show how much farther Demoivre carried his inquiries than those who had written on the subject before him. Montmort, who had been a personal friend of Demoivre, thought he had some reason to complain of the manner in which Demoivre spoke of his methods, and a coolness existed in consequence for some time between them.

79. The treatise by James Bernoulli, mentioned above, which was published in 1715, by his nephew Nicolas, and entitled "Ars Conjectandi," may be considered as belonging to the earlier period, at which unquestionably it was written. It is divided into four parts; the first consisting of Huyghens's treatise, with a commentary on most of the propositions. The second contains the abstract theory of combinations, in which are many new and elegant results; amongst others, the expression for the sum of the a^{th} powers of the natural numbers, in which series occur for the first time those remarkable coefficients since become so famous, under the name of Bernoulli's Numbers. A less profitable labour, which is also to be met with in this part of the work, is the curious analysis of the permutations of the celebrated Latin verse, *Tot tibi sunt dotes, Virgo, quot sidera cœlo*, which are determined to be 3312 in number, without transgressing the laws of Latin metre. It does not appear that James Bernoulli intended to publish this analysis, which was found by his nephew among his loose papers. The third part gives the application of the preceding principles to a variety of questions. The following problem deserves notice, because Bernoulli has given a false, though plausible solution of it, together with the true one, in order, as he says, to show what care is necessary to avoid error in the discussion of these questions. A is to throw a die, and to repeat his throw as many times as the number thrown the first time. If the sum of the points given by the latter set of throws be less than 12, A loses; if more, he wins; if they equal 12, he takes half the stake. His expectation is required. The true value of his expectation is found to be $= \frac{15295}{31104}$: rather less than $\frac{1}{2}$, the false solution is as follows:—A has $\frac{1}{6}$ chance of throwing an ace at the first trial, in which case he will have but one throw to reckon upon, and as this may equally give him any number from 1 to 6, his chance from it may be reckoned at

$\frac{1+2+3+4+5+6}{6} = 3\frac{1}{2}$. In the same manner, if he throw a deuce the first time, he cannot, in the two throws which this secures to him, score less than two or more than 12, and it is easy to see that the chances of each number equidistant from the mean throw, 7, are equal; 7 is, therefore, his expectation on this supposition. If 3, 4, 5, or 6 be thrown the first time, his expectation on each will similarly be found to be the means between 3 and 18, 4 and 24, 5 and 30, 6 and 36, or $10\frac{1}{2}$, 14, $17\frac{1}{2}$, and 21 respectively, so that the expectation of his throws will be $\frac{1}{6}\{3\frac{1}{2} + 7, + 10\frac{1}{2}, + 14 + 17\frac{1}{2} + 21\} = 12\frac{1}{4}$ points, from which it would appear that the odds are in his favour. From the manner in which Bernoulli dwells upon the plausibility of this solution, it seems not improbable that he had been himself deceived in the first instance, by the erroneous view which he here exposes to his readers. The error consists in not multiplying each chance separately by the gain or loss it could occasion to the player. The fourth part of Bernoulli's book, which had been expected with the greatest impatience, but which unfortunately was left incomplete at the time of the author's death, was intended to contain an application of the theory of probabilities to the examination of questions connected with civil and domestic life. Imperfect as this has been left, it must, undoubtedly, be considered as the foundation of whatever has been since done in this branch of the science. Bernoulli seems to have been the first to introduce the term "moral certainty," on which we have already remarked in the body of the treatise. He also, in imitation of Aristotle,* distinguishes between what he calls free and casual contingencies, classing under the former all those contingent events which depend on the will of a rational being. He also inculcates strongly the fundamental principle, from the neglect of which so much error and confusion have arisen, that "contingency or chance has reference merely to the state of our knowledge." After explaining the principal rules by which we should be guided in our investigations, he proceeds, among other things, to examine the method of determining probabilities *à posteriori*, that is to say, by often-repeated experiment: and shows, in the noted theorem which still bears his name, that the probability of attaining to the knowledge of the probability of an unknown event constantly increases with the number of experiments made upon it, so that we can always, by multiplying our experiments, reach a degree of probability as near certainty as we choose to fix upon, that the error of our estimation lies within given limits. With this proposition the work abruptly terminates. At the end of Bernoulli's book is an anonymous letter on the game of tennis, the author of which is not certainly known, but the theory, language, and notation, strongly mark it to belong, if not to James Bernoulli himself, at any rate to some one trained in the same school, and fully imbued with his ideas and opinions.

80. The first great step beyond what Bernoulli had suggested, showing the use to be made of experiments in estimating unknown probabilities, is contained in a posthumous paper by Bayes, inserted in the Philosophical Transactions for 1763, through the means of Dr. Price, so well known by his own publication on the subject of annuities. The problem which Bayes proposes to solve in this paper is the following:—Given the number of times in which an unknown event has happened and failed. Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named. When disencumbered of the geometrical form under which it was then the fashion to represent inte-

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* Physica, lib. ii. cap. 6.

grals, Bayes's theorem is in substance, that if a and A be any two fractions between 0 and 1, the probability that the happening of an event which depends on unknown causes, but which has been already observed to happen p times exactly in $p + q$ experiments, has a degree of probability not greater than a, and not less than A, will be equal to $\frac{\int x^p . (1 - x)^q . d\,x}{\int x^p . (1 - x)^q . d\,x}$, the integral in the numerator being taken between the limits a and A, and in the denomination between 0 and 1. This theorem rests on the more elementary one, that the probability of the existence of a supposed cause of any observed event is proportional to the probability of the event, derived from the supposition of that cause being known to be true. The rest of the paper is taken up with different methods of approximating to the values of this integral within required limits, with which we need not here occupy ourselves. Bayes, or perhaps we should rather say Price, seems to have confounded the probability thus determined, with the probability that an event which has been already observed m times in $p + q$ experiments, will happen again. The difference between the two is obvious; and the reader has already seen the process for determining the latter.

81. The celebrated question, known as the Petersburg Problem, has been already mentioned: this name was given, on account of its having been proposed by Daniel Bernoulli in the Petersburg Transactions; much of the discussion it occasioned might have been spared if the real meaning of the results of the calculations of probability had been kept steadily in view. The difficulty of that question was supposed to consist in this, that no person could be supposed willing to pay the amount which the condition of the game pointed out as equal to his expectation, which after all amounts to no more than saying, that a game can be contrived of too ruinous a nature for the taste even of the most inveterate gamester. It has been well remarked by Buffon, that the science of probabilities never professed to make the condition of a gambler the same as if he did not play; it only indicates the events of which we have most reason to expect the recurrence. Condorcet took away everything appearing paradoxical from the result, by an observation he made in a memoir on this subject in 1784. "It may often happen," says he, "that a reasonable man A will refuse to give B a sum b for the chance n of gaining a, although a be greater than $\frac{b}{n}$; and the reason may be, because A has not the opportunity of repeating the venture often enough to repair the loss which may accrue to him in a single trial, and because the sum ventured may be so great that its loss would occasion him an inconvenience, not at all counterbalanced by the advantages he could derive from his contingent gain."* These are motives for inducing A to refrain from venturing, but cannot be made elements of the calculation as between him and a speculator B on the opposite event. No underwriter diminishes, or ought to diminish, his premium, on account of the small fortune of the party whose indemnity he guarantees.

82. There have been, however, some writers of great celebrity, who have taken an opposite view of this question; and although there can be no doubt of the fallacy of their reasonings, a notice of them must not be omitted in an historical account. D'Alembert instanced this Petersburg problem as tending to throw doubt on the universally admitted rule, that in every game the

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* Histoire de l'Académie Royale, 1784

deposit ought to be inversely proportional to the risk, which therefore he proposed to examine. The result of his examination was, that, in his opinion, a very small probability should be considered as none, and might be entirely disregarded. He illustrates this, by supposing "Peter to play with James, on this condition, that if a tossed halfpenny fall head, in the hundredth toss, and not before, he is to receive from James 2^{100} crowns, in which case the ordinary rule would determine Peter to give James one crown at the beginning of the game. I say, Peter ought not to give this crown, because he will lose it *certainly*,—because head will fall *certainly* before the 100th toss, although not *necessarily*."* Again, he says, "We must distinguish between what is *metaphysically*, and what is *physically* possible. In the first class is everything whose existence does not imply an absurdity; in the second, everything whose existence not only does not imply an absurdity, but even implies nothing too extraordinary, and beyond the daily course of events. It is *metaphysically* possible that two sixes may be thrown on two dice a hundred times in succession; but it is impossible *physically*, because it has never happened, and never will happen." In the same memoir he advances the opinion, that the oftener an event has already happened in succession, as, for instance, the oftener a halfpenny has already successively fallen head, the less is the probability that it will fall head in the next trial. It is rather singular that he did not from the first observe, that the extension of this principle to its utmost limit, namely, to the case in which the halfpenny should always have fallen head, would oblige us, according to his own rule, to class the arrival of tail among the things physically impossible, which never have happened, and which therefore we have no reason to believe ever will happen; and yet, according to his present argument, it will be precisely in this case that tail will be most likely to happen in the next trial. A sounder principle might have suggested to him, that so far as our judgment is determined solely by the supposed repetition, we should be disposed, on that very account, to expect rather the recurrence of head, the oftener it has already appeared, because that very inequality would seem to point out an inequality in the sides favouring that event. The real cause why this effect in ordinary cases is not produced, is that we tacitly refer to, and are influenced by, the great number of times in which head and tail have followed each other indiscriminately, as well as to all the other reasons we have for believing the two sides similarly circumstanced, and the probability arising from this of the perfect indifference of the sides is far from being out weighed by the results of a few sequences. Another error, not less extraordinary, was made by the same celebrated writer in the consideration of repeated experiments. If a player undertook with a halfpenny to throw head in two trials, D'Alembert observed that there were but three possible cases: head in the first trial; tail in the first, and head in the second; tail in both. He, therefore, asserted that the chance in favour of the player should be taken at $\frac{2}{3}$, and not $\frac{3}{4}$, according to the ordinary rule, in which the combination of head thrown twice is taken into the account, "because as soon as head is thrown the first time, it is as useless as ridiculous to throw the piece again; for the result of the second throw has no effect upon the game, and is as foreign to it as if, instead of throwing the piece again, the players had gone to Rome." D'Alembert's mistake lies in supposing that his opponents were not as much aware of this as himself. It is true that in this game there would be only three possible cases, but they would not be similarly circumstanced, which it is necessary that they should be, before an enumeration of them can furnish us with a measure of probability; and the superior probability of head falling the first time, as compared with either of the other two combinations mentioned by him, can be accurately allowed for only by taking the sum of all the ways in which one of two conflicting events may occur in two experiments. As to D'Alembert's farther observations on the possible difference in the law of facility of sequence of any set of events, the only answer that can be given is, that when any such difference is observed, it ought undoubtedly to form an element of the calculation of probabilities; but to suppose, as he seems to have done, that until such law is determined we are unable to estimate them, is to misapprehend entirely the meaning of the results we profess to deduce from it.

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* Opuscules Mathématiques, vol. ii.

83. Laplace inserted several memoirs on the subject of probabilities in the Memoirs of the French Academy, which he afterwards embodied in his splendid work, "Théorie Analytique des Probabilités," in which he also gave the calculus of generating functions. The principal application which he there makes of it, is to the solution of the equations of differences to which he reduces the questions of probability, by considering how the chances vary at each succeeding step. This is, in fact, the method, of which one of the simplest instances may be seen in Pascal's solution of the problem of points, although it is there put in rather a different form. Laplace's work contains the application of the theory to a variety of most intricate and interesting questions; and independently of the results he has obtained, this book is in the last degree valuable, from the specimens of refined and beautiful analysis it affords. Besides the authors here mentioned, a great number have composed works on the subject of probability, not very remarkable for the introduction of new principles or methods. The principal attention of English writers has been directed to compilations on the subject of annuities; and it is very much to be regretted that, with a few exceptions, a wanton and barbarous scheme of notation should conceal whatever may be valuable in their writings, nearly as much as if they were written in an unknown language.

84. The first complete tables of life annuities constructed in this country from Halley's and Demoivre's theory were by John Richards, of which the edition in the British Museum is dated 1730.* It contains the following short but curious historical sketch of the erroneous methods it was intended to supersede. "In valuing three lives absolute in copyhold estates, the general rule was, formerly, to reckon it as a lease of twenty-one years certain, which, by the tables for that purpose, at 5 per cent., is worth, in ready money, 12·82 years value and no more for three lives, the first of which they esteemed worth 6 years, the second 4, and the third 2·82; so that to renew two lives in reversion of one would cost 7 years, or one in reversion of two, three years' value. And this was the constant expectation, what age soever the life or lives *in esse* were of at the time of renewing. Whether this estimation of the value of leases arose from the Act of 32 Henry VIII., or was in use before, I know not; but it is there enacted, that a lease for more than twenty-one years, or three lives, is void: by which it seems as though three lives and twenty-one years were reckoned an equal duration,† the contrary of which was very evident even before any experiments were made concerning the duration of life, and therefore this way of computing was corrected by another, which is likewise in several respects erroneous. By this other method (which is still in practice) a lease for one life may be reckoned equivalent to one of 9, 10, 11, or 12

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* Gentleman's Steward Instructed. London, 1730.

† It seems more likely that the framers of the Act were guided by the pre-existing popular prejudice, than that they gave rise to it.

years, &c.; that for two lives at 17, 19, 21, or 23 years, &c.; that for three lives as a lease of 24, 27, 30, or 33 years, &c.; and though this latter method is a little more plausible than the former, on account of the steward's liberty of choosing which of these proportions he pleases, yet I cannot see any analogy that this bears to the reason of the thing. So that at best it is but only groping in the dark."

85. The old tables, to which Richards here alludes, are frequently referred to in the treatises of that day, under the name of Æcroid's Tables: the time at which their author lived was not even then accurately known, but is conjectured to have been about the time of Henry VIII.'s reign, as the interest of money when they were compiled was rather more than 10 per cent. The slightly improved method which Richards mentions as then still in practice, was suggested in an anonymous treatise entitled "Tables for renewing and purchasing Leases of Lives," first published at Cambridge about 1685. This is the book which is often cited as Newton's "Treatise on Life Annuities," but with which he was no otherwise concerned than as it bears his approbation as Lucasian Professor on the title-page, which is couched in the following terms:—"The method of this book is correct, and the numbers computed with sufficient accuracy, as I judge from re-calculating several of them."* There are no other traces that Newton meddled with this subject. In this treatise Æcroid's tables are said to have been calculated at 11*l.* 3*s.* $6\frac{6}{17}$*d.* per cent. The tables which Richards published are calculated at different rates, from 4 to 8 per cent., and are given for all ages, from five to five years on one life, and from ten to ten on two, and on three joint lives. Since that period the principal improvement of such tables has consisted in more careful and extensive registers of deaths to furnish the requisite data for their construction: the only addition to their theory has been in the suggestion, that such registers ought only to be considered as furnishing the results of a number of experiments: consequently, that the ratios given by them ought not to be immediately employed as probabilities *à priori*, but used as in the theory given in page 37.† An excellent account may be found of the authors who have treated of questions connected with annuities in the article MORTALITY, in the supplement to the "Encyclopædia Britannica."

86. The first tables of mortality were, in fact, formed by Dr. Halley from the registers of Breslau in Silesia, and are given in the Transactions of the Royal Society for 1693, art. 1. The next author who treated of this subject is William Kerseboom, who published at the Hagne, in 1730, a tract, entitled, "Eerste Verhandeling tot een Proeve om te weeten de probable menigte des volks in de provintie van Hollandt en Westvrieslandt." An account of this work is given by Mr. Eames in the Philosophical Transactions for 1738. In 1742 Kerseboom published two other tracts upon the same subject, an account of which is given by Mr. Van Rixtel in the Philosophical Transactions for 1743. Kerseboom's table of mortality was formed from registers of many thousand life annuitants in Holland and West Friesland, which had been kept there from 125 to 130 years previous to the date of his publication.

87. M. Desparcieux published, in 1746, his "Essai sur les Probabilités de la Durée de la Vie Humaine," in which he gave several tables of mortality, constructed from the lists of nominees in the French tontines and from the

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* Methodus hujus libri rectè se habet; numerique, ut ex quibusdam ad calculum revocatis, judico, satis exactè computantur. Is. Newton, Math. Prof. Luc.

† When insurances on lives began to be established in the beginning of the last century, their true principles were so little understood or acted upon, that every insurer, of whatever age he might be, paid the same premium, the only restriction being, that his life should lie between five and sixty years.

mortuary registers of different religious houses. The Northampton Tables, as they are called, were long the only tables in use in this country; they were given by Dr. Price in his "Observations on Reversionary Payments," which was published in 1771: the following extract will serve to explain the manner in which they were formed, vol. 2, p. 94. "In this town (Northampton), "containing four parishes, namely, All Saints', St. Sepulchre's, St. Giles's, and "St. Peter's, an account has been kept ever since the year 1741 of the number "of males and females that have been christened and buried, dissenters in-"cluded, in the whole town. And in the parish of All Saints, containing the "greatest part of the town, an account has been kept since 1735 of the ages "at which all have died there.

$$\text{Christened}\left\{\begin{matrix}\text{males} & 2152\\ \text{females} & 2066\end{matrix}\right\}4220 \qquad \text{Buried}\left\{\begin{matrix}\text{males} & 2377\\ \text{females} & 2312\end{matrix}\right\}4689$$

Of these died

Under two years of age	1529	50 and 60....	384
Between 2 and 5....	362	60.... 70....	378
5....10....	201	70.... 80....	358
10....20....	189	80.... 90....	199
20....30....	373	90....100....	22
30....40....	329		
40....50....	365	Total....	4689.

"The XVIIth Table in this volume is the genuine table of observations for "Northampton, from which may be calculated the true probabilities and "values of lives in that town." To the preceding paragraph is added this note. "In the fourth edition of this treatise the following corrections were made in this "table; first, the table printed in the first three editions having been formed "from the Northampton bills for 36 years, this table was rendered a little "more correct in consequence of being formed from the same bill for 46 "years. Secondly, the bills give the number dying annually between 20 and "30 greater than between 30 and 40; but this being a circumstance which "does not exist in any other register of mortality, and, undoubtedly, owing to "some accidental and local causes, the decrements were made equal between "22 and 40; preserving, however, the total of deaths between 20 and 40 "the same that the bills give them. Thirdly, the bills giving only the totals of "deaths under 2 years of age and between 2 and 5, the proportions of "deaths for every particular year between 2 and 5, and for every quarter of "a year after birth till one year of age, were made the same nearly that the "Chester register makes them."

Such are the alterations which Dr. Price made in the data which were presented to him, and he reduced the table of mortality to the radix 11650; why he chose this number in preference to any other we have not been able to discover. Dr. Price has also neglected to inform us what method he made use of to interpolate the living at those ages, between every ten years, which are not given by the observations.

88. Such is the history of the Northampton Tables. We shall now quote Dr. Price's observations on the Chester Tables.

"Chester is a healthy town, of moderate size, where the births had for "many years a little exceeded the burials; and the register to which I refer "had the particular advantage of being under the direction of Dr. Haygarth, "its founder as well as conductor. As it gives an accurate account of the "distempers of which all the inhabitants die in every season and at every age, "it contains much physical instruction; but my views lead me only to take "notice of that part of it which gives the law according to which human life

" wastes in all its different stages, both among males and females. Concerning these tables it is necessary I should make the following observations.

" The table for females must be considered as particularly correct, because " the number of females born and buried in Chester are very nearly equal. " On the contrary, the number of males born being about an eighth greater " than the number buried, it follows that, in the table of decrements for males " the number of the living, and, consequently, the probabilities of living at " every age for the last 10 or 15 of the first years of life, must be given too " low."

89. Dr. Price characterises Dr. Haygarth as an able and ingenious physician, and it appears that he made a survey of the ten parishes of Chester with great care in 1774, at which time the population consisted of 6697 males and 8016 females.

The table of mortality formed by Dr. Haygarth, from the observations at Chester, is given at length in Dr. Price's Treatise on Reversionary Payments, vol. ii. It distinguishes the sexes, which the Northampton Table does not; it contains 4006 observations, while Carlisle only furnished to Mr. A. Milne 1840, a number too small to admit of subdivision. The Chester observations were, probably, communicated to Dr. Price after those at Northampton, which may have been the reason why he made more use of the latter, probably also as they were rather the more favourable of the two, he wished to keep on the safe side. Dr. Haygarth has given different tables of observations in the Transactions of the Royal Society, in which the deaths are classed from 5 years to 5 years, and the diseases by which they were occasioned are also stated. As Dr. Haygarth was a physician practising in Chester at the time he collected these observations, he had great opportunities of obtaining exact information. Dr. Haygarth states distinctly, that all the numbers dying at every age were taken just as the register gave them, except in the case of 22 females above the age of 80, of whom the age was not closely specified.

It is much to be regretted that no registry of births, marriages, and deaths exists in this country, which would furnish very valuable statistical information. The act of the 52 Geo. III., for the better regulating and preserving parish and other registers of births, baptisms, marriages, and burials in England, has, indeed, a clause by which any person making false entries, or false copies of entries, is to be adjudged guilty of felony, and transported for fourteen years. Another clause which follows immediately after, directs that one half of the penalties levied in pursuance of this act shall go to the informer and the remainder to the poor of the parish. The returns of the weekly burials, which are made by the clerks within the bills of mortality, do not appear to be sent regularly to the parish clerk's office, so that it is difficult to ascertain the effect of the seasons or the weather in producing deaths. In these returns the number of marriages is not stated, nor are the burials of males and females discriminated.

90. We have before alluded to the error which arises in a table of mortality considered as furnishing the probabilities of life, from the supposition that the population has been stationary during the century previous to the observation, and we have explained the method which should be adopted in order to get rid of this when we know the actual increase. The accuracy of the Chester observations is such as to make it desirable to take into account this circumstance. The births in all England in the year 1700, according to the Parliamentary Reports, were 138,979, and in 1780, 201,310, making the mean annual rate of increase 1.0046; in the county of Chester, taken by itself, in 1700, they were 2650, and in 1780, 4592, making the mean

annual rate of increase 1.0061. We may, therefore, suppose the births to have increased in geometrical progression, during the century previous to Dr. Haygarth's observations, at the rate of 1.005, without fear of an error which can disturb the accuracy of our results. The deaths at the same time were about equal to the deaths forty years previously, a result which is confirmed by direct calculation. These data are sufficient to correct the table of mortality, and it is obvious that the error of our hypothesis is altogether of an order to be neglected, for a small inequality in the rate of increase will not affect the result, unless it be of long period.

91. Table (1), page 56, contains the observations of deaths by Dr. Haygarth at Chester.

Table (2) has been calculated upon this hypothesis, namely, that the births increased during the century previous to Dr. Haygarth's observation in a geometric progression, of which the common ratio was 1.005.

Table (3) shows the values of annuities on such lives.

Table (4) shows the values of single premiums for insuring £1. payable at death; and

Table (5) shows the values of annual premiums for insuring £1. payable at death.

Tables (3), (4), and (5), have been calculated from Table (2) by Mr. David Jones of the Royal Exchange Assurance Company.

92. Mr. Finlaison has recently published very extensive tables of mortality, formed from the government tontines and annuitants, which are rendered extremely valuable by the accuracy of the materials from which they have been deduced, and by the very great care and attention which have been bestowed on them by the author.

Mr. Finlaison (in his valuable report to the Lords of the Treasury) explains at length the manner in which he made use of the records of the tontines. Mr. Finlaison observes, "that the facts shown in these observations, bear conclusive testimony that the rate of mortality in England has, "during the last century, diminished in a very important degree, on each sex "equally, but not by equal gradations, nor equally at all periods of life; and "that while in regard to the males it seems in early and middling life to have "remained for a long time as it stood about fifty years ago, in respect of the "females it has during the same time visibly and progressively diminished "to this day by slight but still sensible gradations."

Whether life has generally improved or not, it is certain that epidemics are of much less frequent occurrence now than they were formerly, which circumstance must have a very favourable influence.

93. Mr. Griffith Davies has published tables of annuities taken from statements of Mr. Morgan, in his addresses to the general courts of the Equitable Society, and in notes added by him to the latter editions of Dr. Price's "Observations on Reversionary Payments." In Mr. Morgan's address to the general court held on the 24th April, 1800, he stated that the decrements of life among the members of the Equitable for the preceding thirty years, had been to those of the Northampton

from 10 to 20 as 1 to 2
20 .. 30 .. 1 .. 2
30 .. 40 .. 3 .. 5
from 40 to 50 as 3 to 5
50 .. 60 .. 5 .. 7
60 .. 80 .. 4 .. 5

which statement is confirmed in his subsequent addresses. In a recent publication, Mr. Morgan admits that he was not then aware of the great number of instances in which there are several policies upon one and the same life, but this circumstance cannot very materially affect Mr. Davies's calculations." Such statements as these, although not so detailed as might be wished, sufficiently prove that in the Equitable Society, the rate of mortality is considerably less than that given by the Northampton Table.

94. Mr. Babbage, in a work entitled "A Comparative View of the various Institutions for the Assurance of Lives," has examined the advantages which are presented by the different insurance offices in this metropolis. It is not our intention to follow him in this inquiry, which is rendered very intricate from the complicated manner in which some of the offices make returns to the assured of a portion of the immense profits which they accumulate, instead of charging, which is obviously a simpler method, the real value at first.

The offices which use the Northampton Table as the basis of their calculations are the

Albion,
Atlas,
Eagle,
Exchange, Royal,
Globe,
Imperial,
Law Life,
London Life Association,
Pelican,
Provident,
Rock,
Westminster.

95. The doctrine of fire and sea insurances seems to be at present nearly in the same state in which that of life insurances was at the beginning of the last century. Montucla mentions a treatise on the subject of ship insurances by Montandouin, a merchant of Nantes, of which he speaks in terms of commendation, and seems to intimate that the publication of this work drew the attention of the Académie des Sciences. That learned body proposed the theory of maritime insurances as a prize question in 1783, 1785, and in 1787, but without much success. None of the essays received were thought to deserve the prize, but, on the last occasion, half the prize of 6000 livres was divided between Lacroix and Bicquilly, two of the competitors. The remaining 3000 livres were intended to be offered, in 1791, for the best tables of premiums for maritime insurance, but the revolution intervened to prevent any adjudication of it. All our present knowledge on this subject seems to be confined to the personal experience of the underwriters.

96. Another extensive application of the theory of probability has been made by Condorcet, at the instance of the enlightened financier Turgot. In a work entitled "Essai sur la Probabilité des Décisions," Condorcet has investigated and compared the probabilities of error in the decisions pronounced by tribunals more or less numerous, and various schemes for determining the verdict. Connected with the same question is the inquiry into the best mode of collecting votes in elections, in which more than two conflicting propositions are presented to each elector. Condorcet has examined in detail the respective advantages and disadvantages of electing by a simple majority, by a majority exceeding a given number, or by a number proportional to the whole number of voters, with many others. He arrives at the conclusion that the best mode of electing is by a majority not below a given number of a single assembly.

TABLE I.—DR. HAYGARTH'S OBSERVATIONS at CHESTER, as given by Dr. Price in his work on Reversionary Payments, Vol. ii.

Age.	MALES.		FEMALES.		Age.	MALES.		FEMALES.	
		Deaths.		Deaths.			Deaths.		Deaths
0	1927	438	2139	368	50	558	16	752	15
1	1489	180	1771	181	51	542	16	737	14
2	1309	107	1580	127	52	526	16	723	14
3	1202	67	1463	77	53	519	16	709	14
4	1135	34	1386	53	54	494	15	695	14
5	1101	30	1333	30	55	479	14	681	13
6	1071	24	1303	10	56	465	14	668	13
7	1047	18	1285	11	57	451	14	655	13
8	1029	11	1274	9	58	437	14	642	15
9	1018	8	1265	7	59	423	16	627	15
10	1010	6	1258	6	60	407	19	612	20
11	1004	5	1252	6	61	388	22	592	25
12	999	5	1246	7	62	366	22	567	25
13	994	6	1239	7	63	344	22	542	25
14	988	6	1232	8	64	322	20	517	21
15	982	7	1224	9	65	302	16	496	17
16	975	9	1215	10	66	286	13	479	15
17	966	10	1205	11	67	273	11	464	15
18	956	11	1194	12	68	262	11	449	16
19	945	11	1182	11	69	251	13	433	20
20	934	11	1171	10	70	238	16	413	25
21	923	11	1161	10	71	222	22	388	30
22	912	12	1151	10	72	200	22	358	30
23	900	12	1141	11	73	178	21	328	30
24	888	12	1130	12	74	157	18	298	27
25	876	13	1118	16	75	139	15	271	23
26	863	13	1102	16	76	124	12	248	22
27	850	13	1086	16	77	112	11	226	21
28	837	12	1070	16	78	101	11	205	21
29	825	11	1054	16	79	90	10	184	21
30	814	10	1038	13	80	80	10	163	21
31	804	9	1025	13	81	70	10	142	21
32	795	10	1012	13	82	60	9	121	21
33	785	10	999	13	83	51	8	100	21
34	775	10	986	13	84	43	7	79	18
35	765	11	973	14	85	36	6	61	12
36	754	11	959	14	86	30	5	49	8
37	743	12	945	14	87	25	4	41	6
38	731	12	931	14	88	21	4	35	4
39	719	13	917	15	89	17	3	31	4
40	706	13	902	15	90	14	3	27	4
41	693	14	887	15	91	11	3	23	4
42	679	14	872	15	92	8	3	19	4
43	665	15	857	14	93	5	2	15	4
44	650	15	843	15	94	3	2	11	4
45	635	15	828	15	95	1	1	7	3
46	620	15	813	15	96			4	3
47	605	15	798	15	97			1	1
48	590	16	783	16					
49	574	16	767	15					

TABLE II.—TABLE OF MORTALITY formed from the Observations of Dr. Haygarth at CHESTER, corrected for the increase of Population during the Century previous to the Observations upon the supposition that the Births increased yearly in a geometrical progression of which the common ratio was 1.005.

Age.	MALES.		FEMALES.		Age.	MALES.		FEMALES.	
	Living.	Deaths.	Living.	Deaths.		Living.	Deaths.	Living.	Deaths.
0	10000	1778	10000	1351	50	3675	92	4302	77
1	8222	739	8649	670	51	3583	93	4225	72
2	7483	445	7979	474	52	3490	94	4153	73
3	7038	283	7505	291	53	3396	94	4080	73
4	6755	149	7214	203	54	3302	89	4007	73
5	6606	133	7011	118	55	3213	84	3934	69
6	6473	108	6893	73	56	3129	84	3865	69
7	6365	84	6820	47	57	3045	85	3796	70
8	6281	55	6773	40	58	2960	85	3726	80
9	6266	42	6733	32	59	2875	97	3646	80
10	6184	35	6701	28	60	2778	113	3566	105
11	6149	30	6673	29	61	2665	131	3461	130
12	6119	30	6644	33	62	2534	131	3331	131
13	6089	35	6611	33	63	2403	131	3200	132
14	6054	34	6578	37	64	2272	121	3068	112
15	6020	40	6541	41	65	2151	100	2956	93
16	5980	48	6500	45	66	2051	83	2863	83
17	5932	53	6455	49	67	1968	72	2780	83
18	5879	57	6406	54	68	1896	72	2697	89
19	5822	57	6352	50	69	1824	84	2608	110
20	5765	58	6302	46	70	1740	102	2498	136
21	5707	59	6256	46	71	1638	137	2362	163
22	5648	63	6210	46	72	1501	137	2199	164
23	5585	63	6164	51	73	1364	132	2035	165
24	5522	63	6113	55	74	1232	116	1870	150
25	5459	69	6058	72	75	1116	98	1720	129
26	5390	69	5986	72	76	1018	81	1591	124
27	5321	69	5914	73	77	937	76	1467	120
28	5252	64	5841	73	78	861	76	1347	120
29	5188	61	5768	73	79	785	70	1227	121
30	5127	56	5695	61	80	715	71	1106	121
31	5071	51	5634	61	81	644	71	985	122
32	5020	57	5573	62	82	573	65	863	123
33	4963	57	5511	62	83	508	59	740	123
34	4906	57	5449	62	84	449	54	617	107
35	4849	62	5387	67	85	395	47	510	74
36	4787	62	5320	67	86	348	42	436	52
37	4725	68	5253	67	87	306	35	384	41
38	4657	68	5186	68	88	271	36	343	30
39	4589	73	5118	73	89	235	30	313	30
40	4516	73	5045	73	90	205	29	283	31
41	4443	79	4972	73	91	176	30	252	31
42	4364	79	4899	73	92	146	30	221	31
43	4285	84	4826	69	93	116	24	190	32
44	4201	85	4757	74	94	92	24	158	32
45	4116	85	4683	75	95	68	17	126	26
46	4031	86	4608	75	96	51	7	100	26
47	3945	86	4533	75	97	44	7	74	14
48	3859	92	4458	80	98	37	7	60	8
49	3767	92	4378	76	99	30	7	52	8

TABLE III.—PRESENT VALUE of an ANNUITY of 1*l*. payable during the Life of a Male or Female. CHESTER RATE of MORTALITY.

Age.	3 per cent. $a_m, \quad r = \frac{1}{1.03}.$		4 per cent. $a_m, \quad r = \frac{1}{1.04}.$		5 per cent. $a_m, \quad r = \frac{1}{1.05}.$		6 per cent. $a_m, \quad r = \frac{1}{1.06}.$		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	m
0	16·0427	17·7180	13·3465	14·6363	11·3697	12·4024	9·8755	10·7291	0
1	19·0973	20·1002	15·8820	16·5994	13·5198	14·0566	11·7318	12·1496	1
2	20·6128	21·4417	17·1484	17·7130	14·5978	14·9988	12·6638	12·9597	2
3	21·5735	22·4798	17·9620	18·5850	15·2968	15·7434	13·2723	13·6049	3
4	22·1517	23·0882	18·4631	19·1081	15·7346	16·1974	13·6581	14·0030	4
5	22·3308	23·4694	18·6347	19·4478	15·8939	16·4997	13·8041	14·2729	5
6	22·4734	23·5873	18·7783	19·5720	16·0315	16·6213	13·9330	14·3883	6
7	22·5404	23·5549	18·8608	19·5727	16·1187	16·6392	14·0196	14·4149	7
8	22·5271	23·4299	18·8776	19·4969	16·1510	16·5924	14·0595	14·3858	8
9	22·4078	23·2762	18·8061	19·3972	16·1084	16·5255	14·0348	14·3395	9
10	22·2368	23·0890	18·6912	19·2694	16·0286	16·4346	13·9779	14·2725	10
11	22·0343	22·8815	18·5495	19·1243	15·9259	16·3288	13·9009	14·1923	11
12	21·8066	22·6708	18·3861	18·9761	15·8042	16·2200	13·8072	14·1095	12
13	21·5715	22·4675	18·2157	18·8336	15·6761	16·1160	13·7077	14·0307	13
14	21·3471	22·2576	18·0539	18·6852	15·5551	16·0067	13·6142	13·9472	14
15	21·1117	22·0549	17·8821	18·5426	15·4251	15·9021	13·5125	13·8676	15
16	20·8905	21·8599	17·7217	18·4059	15·3047	15·8026	13·4191	13·7924	16
17	20·6913	21·6727	17·5797	18·2756	15·1999	15·7084	13·3393	13·7218	17
18	20·5041	21·4936	17·4478	18·1520	15·1038	15·6200	13·2672	13·6564	18
19	20·3260	21·3266	17·3233	18·0385	15·0143	15·5404	13·2009	13·5989	19
20	20·1428	21·1407	17·1944	17·9089	14·9209	15·4468	13·1313	13·5292	20
21	19·9580	20·9350	17·0639	17·7622	14·8261	15·3385	13·0606	13·4464	21
22	19·7714	20·7228	16·9318	17·6096	14·7301	15·2247	12·9889	13·3588	22
23	19·5943	20·5038	16·8077	17·4506	14·6410	15·1052	12·9235	13·2660	23
24	19·4124	20·2951	16·6795	17·3001	14·5485	14·9928	12·8552	13·1792	24
25	19·2255	20·0937	16·5469	17·1554	14·4522	14·8854	12·7838	13·0968	25
26	19·0558	19·9455	16·4290	17·0562	14·3691	14·8176	12·7243	13·0496	26
27	18·8819	19·7939	16·3078	16·9544	14·2831	14·7479	12·6626	13·0010	27
28	18·7039	19·6426	16·1829	16·8530	14·1943	14·6789	12·5987	12·9533	28
29	18·5027	19·4879	16·0378	16·7489	14·0879	14·6079	12·5194	12·9042	29
30	18·2845	19·3298	15·8778	16·6422	13·9683	14·5349	12·4285	12·8538	30
31	18·0410	19·1253	15·6952	16·4953	13·8287	14·4268	12·3196	12·7726	31
32	17·7710	18·9147	15·4889	16·3428	13·6677	14·3140	12·1915	12·6871	32
33	17·5144	18·7013	15·2934	16·1878	13·5158	14·1988	12·0714	12·5996	33
34	17·2495	18·4815	15·0900	16·0268	13·3565	14·0784	11·9444	12·5076	34
35	16·9758	18·2550	14·8780	15·8597	13·1892	13·9524	11·8098	12·4106	35
36	16·7115	18·0395	14·6736	15·7019	13·0280	13·8345	11·6806	12·3210	36
37	16·4387	17·8177	14·4608	15·5382	12·8589	13·7115	11·5439	12·2268	37
38	16·1791	17·5893	14·2588	15·3685	12·6990	13·5831	11·4152	12·1278	38
39	15·9114	17·3577	14·0489	15·1956	12·5315	13·4518	11·2794	12·0263	39
40	15·6537	17·1371	13·8470	15·0321	12·3708	13·3287	11·1494	11·9324	40
41	15·3882	16·9104	13·6375	14·8629	12·2028	13·2007	11·0126	11·8340	41
42	15·1368	16·6772	13·4398	14·6878	12·0449	13·0672	10·8846	11·7310	42
43	14·8783	16·4374	13·2350	14·5064	11·8803	12·9281	10·7504	11·3691	43
44	14·6311	16·1761	13·0397	14·3054	11·7237	12·7714	10·6233	11·4990	44
45	14·3812	15·9246	12·8413	14·1128	11·5641	12·6219	10·4933	11·3815	45
46	14·1250	15·6693	12·6366	13·9162	11·3984	12·4687	10·3574	11·2608	46
47	13·8659	15·4064	12·4285	13·7123	11·2292	12·3088	10·2182	11·1339	47
48	13·6002	15·1356	12·2137	13·5007	11·0534	12·1416	10·0726	11·0005	48
49	13·3503	14·8746	12·0125	13·2973	10·8895	11·9817	9·9378	10·8736	49

TABLE III.—continued.

Age.	3 per cent. $a_m, \quad r = \frac{1}{1.03}$.		4 per cent. $a_m, \quad r = \frac{1}{1.04}$.		5 per cent. $a_m, \quad r = \frac{1}{1.05}$.		6 per cent. $a_m, \quad r = \frac{1}{1.06}$.		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	m
50	13·0950	14·5914	11·805^{7}	13·0735	10·7202	11·8030	9·7978	10·7297	50
51	12·8342	14·3031	11 593^{2}	12·8442	10·5453	11·6190	9·6523	10·5807	51
52	12·5715	13·9876	11·378^{2}	12·5895	10·3676	11·4115	9·5041	10·4100	52
53	12·3070	13·6650	11·160^{9}	12·3274	10·1873	11·1964	9·3531	10·2320	53
54	12·0371	13·3314	10·937^{8}	12·0541	10·0012	10·9704	9·1966	10·0435	54
55	11·7417	12·9861	10·690^{4}	11·7688	9·7921	10·7327	9·0184	9·8437	55
56	11·4186	12·6145	10·4166	11·4581	9·5577	10·4705	8·8161	9·6206	56
57	11·0856	12·2291	10·1320	11·1330	9·3125	10·1939	8·6029	9·3832	57
58	10·7460	11·8326	9·8399	10·7959	9·0589	9·9047	8·3809	9·1339	58
59	10·3956	11·4550	9·5360	10·4741	8·7930	9·6281	8·1464	8·8935	59
60	10·0814	11·0633	9·2637	10·1374	8·5551	9·3363	7·9368	8·6386	60
61	9·8241	10·7409	9·0428	9·8628	8·3637	9·1005	7·7697	8·4347	61
62	9.6420	10·4949	8·8907	9·6576	8·2359	8·9285	7·6616	8·2897	62
63	9.4726	10·2523	8·7504	9.4551	8·1191	8·7587	7·5641	8·1468	63
64	9·3194	10·0142	8·6251	9.2563	8·0166	8·5923	7·4802	8·0072	64
65	9·1389	9·7055	8·4747	8·9913	7·8910	8·3638	7·3751	7·8092	65
66	8·8720	9·3213	8·2434	8·6547	7·6895	8·0672	7·1987	7·5466	66
67	8·5236	8·8876	7·9347	8·2696	7·4145	7·7235	6·9525	7·2382	67
68	8·1127	8·4360	7 5655	7·8651	7·0809	7·3592	6·6495	6·9086	68
69	7·6859	7·9856	7·1787	7·4588	6·7284	6·9909	6·3266	6·5731	69
70	7·2987	7·5874	6·8263	7·0988	6·4059	6·6637	6·0300	6·2743	70
71	6·9858	7·2649	6·5414	6·8078	6·1450	6·3997	5·7898	6·0337	71
72	6·8521	7·0376	6·4240	6·6050	6·0412	6·2178	5·6974	5·8698	72
73	6·7665	6·8328	6·3520	6·4227	5·9803	6·0548	5·6458	5·7234	73
74	6·7163	6·6588	6·3139	6·2690	5·9521	5·9185	5·6257	5·6021	74
75	6·6368	6·4567	6·2489	6·0884	5·8994	5·7564	5·5831	5·4561	75
76	6·4940	6·1897	6·1245	5·8453	5·7907	5·5343	5·4878	5·2524	76
77	6·2670	5·9142	5·9201	5·5930	5·6058	5·3022	5·3200	5·0381	77
78	6·0248	5·6343	5·7004	5·3349	5·4056	5·0633	5·1369	4·8162	78
79	5·8063	5·3709	5·5024	5·0909	5·2254	4·8364	4·9723	4·6044	79
80	5·5660	5·1373	5·2827	4·8738	5·0239	4·6342	4·7867	4·4146	80
81	5·3651	4·9414	5·0997	4·6914	4·8568	4·4632	4·6333	4·2544	81
82	5·2108	4·8092	4·9609	4·5687	4·7313	4·3488	4·5198	4·1471	82
83	5·0538	4·7768	4·8195	4·5413	4·6035	4·3253	4·4040	4·1266	83
84	4·8894	4·9009	4·6709	4·6644	4·4689	4·4469	4·2817	4·2462	84
85	4·7246	5·1070	4·5218	4·8688	4·3338	4·6488	4·1591	4·4453	85
86	4·5236	5·1530	4·3379	4·9229	4·1651	4·7098	4·0040	4·5118	86
87	4·2988	5·0263	4·1306	4·8132	3·9736	4·6149	3·8268	4·4302	87
88	3·9996	4·7960	3·8506	4·6041	3·7111	4·4249	3·5803	4 2573	88
89	3·7507	4·4133	3·6181	4·2471	3·4936	4·0914	3·3765	3·9453	89
90	3·4285	4·0275	3·3135	3·8853	3·2051	3·7514	3·1029	3·6253	90
91	3·1133	3·6587	3·0138	3·5378	2·9199	3·4235	2·8310	3·3156	91
92	2·8656	3·2971	2·7785	3·1954	2·6959	3·0989	2·6175	3·0075	92
93	2·7149	2·9561	2·6369	2·8654	2·5627	2·7848	2·2921	2·7080	93
94	2·5258	2·6539	2·4578	2·5835	2·3928	2·5162	2·3307	2·4519	94
95	2·5198	2·4278	2·4582	2·3692	2·3992	2·3130	2·3425	2·2591	95
96	2·4605	2·1508	2·4088	2·1047	2·3589	2·0601	2·3107	2·0172	96
97	1·9375	1·9937	1·9037	1·9579	1·8708	1·9232	1·8390	1·8896	97
98	1·3731	1·5327	1·3544	1·5113	1·3360	1·4905	1·3182	1·4703	98
99	·7443	·8215	·7372	·8136	·7302	·8059	·7233	·7983	99

TABLE IV.—Single Premium required to secure the payment of 1*l.* at the end of the year in which the life shall fail.

Age.	3 per cent.		4 per cent.		5 per cent.		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	*m*
0	·50361	·45482	·44821	·39860	·41097	·36179	0
1	·41464	·38543	·35070	·32310	·30858	·28302	1
2	·37050	·34636	·30198	·28027	·25725	·23815	2
3	·34252	·31612	·27069	·24673	·22396	·20270	3
4	·32568	·29840	·25142	·22661	·20311	·18108	4
5	·32046	·28730	·24482	·21355	·19553	·16668	5
6	·31631	·28387	·23930	·20877	·18898	·16089	6
7	·31436	·28481	·23612	·20874	·18482	·16004	7
8	·31475	·28845	·23548	·21166	·18329	·16227	8
9	·31822	·29293	·23823	·21549	·18531	·16545	9
10	·32320	·29838	·24265	·22041	·18911	·16978	10
11	·32910	·30442	·24810	·22599	·19400	·17482	11
12	·33573	·31056	·25438	·23169	·19980	·18000	12
13	·34258	·31648	·26093	·23717	·20590	·18495	13
14	·34911	·32259	·26716	·24288	·21166	·19016	14
15	·35597	·32850	·27377	·24836	·21785	·19514	15
16	·36241	·33418	·27993	·25362	·22359	·19988	16
17	·36822	·33963	·28540	·25863	·22858	·20436	17
18	·37367	·34485	·29047	·26338	·23315	·20857	18
19	·37885	·34971	·29526	·26775	·23741	·21236	19
20	·38419	·35513	·30022	·27273	·24186	·21682	20
21	·38957	·36112	·30523	·27838	·24638	·22198	21
22	·39501	·36730	·31032	·28425	·25095	·22740	22
23	·40017	·37368	·31509	·29036	·25519	·23308	23
24	·40546	·37975	·32002	·29615	·25960	·23844	24
25	·41091	·38562	·32512	·30172	·26418	·24355	25
26	·41585	·38994	·32965	·30553	·26814	·24678	26
27	·42092	·39435	·33432	·30945	·27223	·25010	27
28	·42610	·39876	·33912	·31335	·27646	·25339	28
29	·43196	·40327	·34470	·31735	·28153	·25677	29
30	·43832	·40787	·35085	·32145	·28722	·26024	30
31	·44541	·41383	·35788	·32710	·29387	·26539	31
32	·45327	·41996	·36581	·33297	·30154	·27076	32
33	·46075	·42618	·37333	·33893	·30877	·27625	33
34	·46846	·43258	·38115	·34512	·31636	·28198	34
35	·47643	·43917	·38931	·35155	·32432	·28798	35
36	·48413	·44545	·39717	·35762	·33200	·29360	36
37	·49208	·45191	·40535	·36392	·34005	·29945	37
38	·49964	·45856	·41312	·37044	·34767	·30557	38
39	·50744	·46531	·42120	·37709	·35564	·31182	39
40	·51494	·47174	·42896	·38338	·36330	·31768	40
41	·52267	·47834	·43702	·38989	·37130	·32378	41
42	·53000	·48513	·44462	·39662	·37881	·33013	42
43	·53753	·49211	·45250	·40360	·38665	·33676	43
44	·54473	·49973	·46001	·41133	·39411	·34422	44
45	·55200	·50705	·46764	·41874	·40171	·35134	45
46	·55947	·51449	·47552	·42630	·40960	·35863	46
47	·56701	·52214	·48352	·43414	·41766	·36625	47
48	·57475	·53003	·49178	·44228	·42603	·37421	48
49	·58203	·53763	·49952	·45010	·43383	·38182	49

TABLE IV.—continued.

Age.	3 per cent.		4 per cent.		5 per cent.		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	m
50	·58947	·54588	·50747	·45871	·44190	·39033	50
51	·59706	·55428	·51565	·46753	·45022	·39910	51
52	·60471	·56347	·52392	·47733	·45869	·40898	52
53	·61242	·57286	·53227	·48741	·46727	·41922	53
54	·62028	·58258	·54085	·49792	·47613	·42998	54
55	·62888	·59264	·55037	·50889	·48610	·44130	55
56	·63829	·60346	·56090	·52084	·49725	·45379	56
57	·64799	·61469	·57185	·53335	·50893	·46696	57
58	·65788	·62623	·58308	·54631	·52100	·48073	58
59	·66809	·63723	·59477	·55869	·53367	·49390	59
60	·67724	·64864	·60524	·57164	·54500	·50780	60
61	·68473	·65803	·61374	·58220	·55411	·51902	61
62	·69004	·66520	·61959	·59009	·56020	·52721	62
63	·69497	·67226	·62498	·59788	·56576	·53530	63
64	·69943	·67920	·62980	·60553	·57064	·54322	64
65	·70469	·68819	·63559	·61572	·57662	·55410	65
66	·71247	·69938	·64448	·62867	·58621	·56823	66
67	·72261	·71201	·65636	·64348	·59931	·58460	67
68	·73458	·72517	·67056	·65903	·61520	·60194	68
69	·74701	·73828	·68543	·67466	·63198	·61948	69
70	·75829	·74988	·69899	·68851	·64734	·63506	70
71	·76740	·75927	·70995	·69970	·65976	·64763	71
72	·77130	·76590	71446	·70750	·66470	·65630	72
73	·77379	·77186	·71723	·71451	·66760	·66406	73
74	·77525	·77693	·71870	·72042	·66895	·67055	74
75	·77757	·78281	·72120	·72737	·67146	·67827	75
76	·78173	·79059	·72598	·73672	·67663	·68884	76
77	·78834	·79862	·73384	·74642	·68544	·69990	77
78	·79539	·80677	·74229	·75635	·69497	·71127	78
79	·80176	·81444	·74991	·76573	·70355	·72208	79
80	·80876	·82124	·75836	·77408	·71315	·73170	80
81	·81461	·82695	·76540	·78110	·72110	·73985	81
82	·81910	·83080	·77073	·78582	·72708	·74530	82
83	·82368	·83174	·77617	·78687	·73317	·74641	83
84	·82846	·82813	·78189	·78214	·73958	·74062	84
85	·83326	·82213	·78762	·77428	·74601	·73101	85
86	·83912	·82079	·79470	·77220	·75404	·72810	86
87	·84567	·82448	·80267	·77642	·76316	·73262	87
88	·85438	·83118	·81344	·78446	·77566	·74167	88
89	·86163	·84233	·82238	·79819	·78602	·75755	89
90	·87101	·85357	·83409	·81210	·79976	.77374	90
91	·88020	·86431	·84562	·82547	·81333	·78936	91
92	·88741	·87484	·85467	·83864	·82400	·80481	92
93	·89180	·88477	·86012	·85133	·83035	·81978	93
94	·89731	·89358	·86701	·86217	·83844	·83256	94
95	·89748	·90016	·86699	·87042	·83813	·84224	95
96	·89921	·90823	·86889	·88059	·84005	·85428	96
97	·91444	·91280	·88832	·88623	·86330	·86080	97
98	·93088	·92623	·90945	·90341	·88876	·88140	98
99	·94920	·94695	·93318	·93025	·91761	·91400	99

TABLE V.—ANNUAL PREMIUM required to secure 1*l.* at the end of the year in which the life shall fail.

Age.	3 per cent.		4 per cent.		5 per cent.		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	*m*
0	·02955	·02429	·03124	·02549	·03322	·02699	0
1	·02063	·01826	·02078	·01836	·02125	·01880	1
2	·01714	·01543	·01664	·01498	·01649	·01489	2
3	·01517	·01346	·01428	·01260	·01374	·01211	3
4	·01406	·01238	·01292	·01127	·01214	·01053	4
5	·01373	·01174	·01247	·01045	·01157	·00952	5
6	·01347	·01154	·01210	·01015	·01109	·00913	6
7	·01335	·01160	·01189	·01015	·01080	·00907	7
8	·01337	·01180	·01185	·01033	·01069	·00922	8
9	·01359	·01206	·01203	·01057	·01083	·00944	9
10	·01390	·01238	·01232	·01088	·01110	·00974	10
11	·01428	·01274	·01269	·01123	·01146	·01009	11
12	·01472	·01312	·01312	·01160	·01189	·01045	12
13	·01517	·01348	·01358	·01196	·01235	·01080	13
14	·01562	·01387	·01402	·01234	·01278	·01118	14
15	·01609	·01424	·01450	·01271	·01326	·01154	15
16	·01655	·01461	·01495	·01307	·01371	·01189	16
17	·01697	·01498	·01536	·01342	·01411	·01223	17
18	·01737	·01533	·01575	·01375	·01448	·01255	18
19	·01776	·01566	·01612	·01406	·01482	·01284	19
20	·01817	·01604	·01650	·01443	·01519	·01318	20
21	·01858	·01646	·01690	·01484	·01557	·01359	21
22	·01901	·01690	·01731	·01528	·01595	·01401	22
23	·01943	·01737	·01770	·01574	·01631	·01447	23
24	·01986	·01783	·01810	·01618	·01670	·01491	24
25	·02031	·01828	·01853	·01662	·01710	·01533	25
26	·02073	·01861	·01892	·01692	·01745	·01560	26
27	·02117	·01896	·01932	·01723	·01781	·01588	27
28	·02162	·01931	·01974	·01755	·01819	·01616	28
29	·02215	·01968	·02023	·01788	·01866	·01645	29
30	·02273	·02006	·02079	·01822	·01919	·01675	30
31	·02339	·02056	·02144	·01870	·01982	·01720	31
32	·02414	·02108	·02219	·01920	·02056	·01768	32
33	·02488	·02163	·02292	·01972	·02127	·01817	33
34	·02567	·02220	·02369	·02027	·02204	·01870	34
35	·02650	·02280	·02452	·02085	·02286	·01926	35
36	·02733	·02339	·02534	·02141	·02367	·01979	36
37	·02821	·02401	·02622	·02202	·02454	·02036	37
38	·02908	·02466	·02708	·02263	·02538	·02095	38
39	·03000	·02534	·02799	·02329	·02628	·02158	39
40	·03092	·02601	·02889	·02392	·02717	·02217	40
41	·03189	·02670	·02986	·02458	·02812	·02280	41
42	·03284	·02744	·03079	·02528	·02904	·02347	42
43	·03385	·02822	·03179	·02603	·03002	·02418	43
44	·03484	·02909	·03277	·02688	·03097	·02499	44
45	·03588	·02996	·03379	·02771	·03197	·02579	45
46	·03699	·03086	·03487	·02858	·03304	·02663	46
47	·03814	·03182	·03601	·02951	·03415	·02752	47
48	·03936	·03284	·03722	·03050	·03534	·02847	48
49	·04056	·03386	·03839	·03148	·03649	·02941	49

TABLE V.—continued.

Age.	3 per cent.		4 per cent.		5 per cent.		Age.
m	Males.	Females.	Males.	Females.	Males.	Females.	*m*
50	·04182	·03501	·03963	·03260	·03770	·03049	50
51	·04316	·03622	·04095	·03377	·03900	·03463	51
52	·04455	·03759	·04233	·03513	·04035	·03295	52
53	·04602	·03906	·04377	·03657	·04177	·03437	53
54	·04757	·04065	·04531	·03814	·04328	·03592	54
55	·04935	·04237	·04708	·03986	·04504	·03761	55
56	·05139	·04432	·04913	·04181	·04710	·03956	56
57	·05361	·04646	·05137	·04396	·04935	·04171	57
58	·05600	·04880	·05379	·04632	·05180	·04408	58
59	·05862	·05116	·05645	·04869	·05449	·04647	59
60	·06111	·05377	·05897	·05133	·05704	·04913	60
61	·06326	·05604	·06111	·05360	·05917	·05139	61
62	·06484	·05786	·06265	·05537	·06066	·05310	62
63	·06636	·05974	·06410	·05719	·06204	·05485	63
64	·06778	·06166	·06543	·05904	·06329	·05663	64
65	·06950	·06428	·06708	·06163	·06485	·05918	65
66	·07217	·06776	·06973	·06512	·06746	·06267	66
67	·07587	·07201	·07346	·06942	·07122	·06701	67
68	·08060	·07685	·07829	·07434	·07613	·07201	68
69	·08600	·08216	·08381	·07976	·08177	·07752	69
70	·09137	·08732	·08931	·08502	·08741	·08287	70
71	·09609	·09187	·09414	·08962	·09234	·09752	71
72	·09822	·09529	·09624	·09303	·09440	·09093	72
73	·09963	·09854	·09756	·09626	·09564	·09413	73
74	·10047	·10144	·09827	·09911	·09622	·09692	74
75	·10182	·10498	·09949	·10262	·09732	·10039	75
76	·10431	·10996	·10190	·10763	·09964	·10542	76
77	·10848	·11550	·10605	·11322	·10376	·11105	77
78	·11322	·12160	·11078	·11940	·10849	·11731	78
79	·11779	·12783	·11533	·12572	·11301	·12372	79
80	·12317	·13381	·12071	·13179	·11839	·12988	80
81	·12798	·13918	·12548	·13724	·12313	·13542	81
82	·13188	·14301	·12930	·14111	·12686	·13934	82
83	·13606	·14398	·13338	·14200	·13084	·14016	83
84	·14067	·14034	·13788	·13808	·13523	·13597	84
85	·14555	·13462	·14264	·13193	·13986	·12941	85
86	·15191	·13339	·14888	·13037	·14599	·12752	86
87	·15960	·13681	·15645	·13356	·15344	·13048	87
88	·17089	·14340	·16770	·13998	·16464	·13672	88
89	·18137	·15560	·17808	·15212	·17492	·14879	89
90	·19668	·16977	·19337	·16624	·19019	·16284	90
91	·21399	·18552	·21068	·18191	·20749	·17845	91
92	·22957	·20359	·22620	·19990	·22295	·19635	92
93	·24006	·22403	·23650	·22025	·23306	·21660	93
94	·25450	·24455	·25074	·24059	·24712	·23678	94
95	·25498	·26260	·25071	·25834	·24657	·25422	95
96	·25985	·28825	·25490	·28363	·25010	·27916	96
97	·31130	·30490	·30593	·29962	·30071	·29447	97
98	·39227	·36571	·38628	·35973	·38046	·35390	98
99	·54417	·51987	·53719	·77515	·53036	·50613	99

The following tables will serve to show how nearly different tables of mortality and different tables of annuities agree.

Tables of Mortality.

Age.	(1.) Desparcieux.	(2.) Northampton, Dr. Price.	(3.) Carlisle, Mr. Milne.	(4.) Equitable Experience, Mr. Davies.	(5.) Mr. Finlaison's Tables.		(6.) Chester Table.		Age.
					Male.	Female.	Male.	Female.	
	Living.	Living.	Living.	Living.	Living.	Living.	Living.	Living.	
0		1000	1000		1000	1000	10000	10000	0
10	830	487	646	2844	896	903	6184	6701	10
20	814	440	609	2705	837	848	5765	6302	20
30	734	415	564	2501	732	777	5127	5695	30
40	657	312	507	2236	644	700	4516	5045	40
50	581	245	439	1937	561	623	3675	4302	50
60	463	174	364	1524	440	539	2778	3566	60
70	310	105	240	1028	288	412	1740	2498	70
80	118	40	95	480	125	210	715	617	80
90	11	3	14	65	11	52	205	283	90

Annuities at 3 per cent.

Age.	(1.) Desparcieux.	(2.) Northampton, Dr. Price.	(3.) Carlisle, Mr. Milne.	(4.) Equitable Experience, Mr. Davies.	(5.) Mr. Finlaison's Tables.		(6.) Chester Table.		Age.
					Male.	Female.	Male.	Female.	
0			17·320				16·042	17·718	0
10	22·766	20·663	23·512	23·768	20·524	22·312	22·237	23·089	10
20	21·168	18·638	21·694	21·795	19·223	20 720	20·143	21·140	20
30	19·492	16·922	19·556	19·671	17·082	19·174	18·284	19·330	30
40	17·183	14·848	17·143	17·351	14·011	16·111	15·654	17·137	40
50	13·899	12·436	14·303	14·477	10·777	12·560	13·095	14·591	50
60	10·522	9·777	10·491	11·539	7·425	8·638	10·081	11·063	60
70		6·734	7·123	8·285			7·299	7·587	70

These numbers are extracted from the following works:—

(1.) The Doctrine of Life Annuities, by Mr. Francis Bailey, vol. ii. p. 532.
(2.) Observations on Reversionary Payments, by Dr. Price, vol. ii. p. 314.
(3.) A Treatise on the Valuation of Annuities, by Mr. Milne, vol. ii. p. 594.
(4.) Tables of Life Contingencies, by Mr. Griffith Davies.
(5.) These tables are taken from the Report of the House of Commons on Friendly Societies, 1827.
(6.) These values are extracted from Table III. p. 58.

The Table of Mr. Milne agrees very closely with the mean of the columns for males and females in Table (1), and, in fact, it would have been sufficient to have supposed the rate of increase 1·007 in the formation of Table (2) during the last century to render the difference almost insensible.